Modelling the Approximation Hierarchy to Optimisation Problems Through Category Theory

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Abstract.

Aiming at developing a theoretical framework for the formal study of NP-hard optimisation problems, we have focused on structural properties of optimisation problems related to approximative issue. From the observation that, intuitively, there are many connections among categorical concepts and structural complexity notions, in this work we present a categorical approach to cope with some questions originally studied within Computational Complexity Theory. After defining the polynomial time soluble optimisation problems category OPTS and the optimisation problems category OPT, we introduce a comparison mechanism between them following the basic idea of *categorical shape theory*, in such way the hierarchical structure of approximation to each optimisation problem can be modelled.

Keywords: Structural Complexity, Category Theory, NP-Hard Optimisation Problems, Approximation Hierarchy.

1 Introduction

According to Holmberg (2000 [10]), *modelling* emerges as a core concept in anticipation and anticipative processes, and it becomes evident already from the Rosen's work (1985 [19]). However, modelling is an enormously rich concept.

Motivated by the current interest in Computer Science on Approximative Algorithms Theory as a feasible alternative to those optimisation problems considered computationally intractable, this paper has explored possible new avenues of future research and development of theoretical framework for *modelling* structural properties of optimisation problems related to approximative issue. Heuristic techniques for solving such problems in an approximate way have always been used throughout

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the history of computing, but without any guarantee on approximation quality. On the other hand, approximative algorithms satisfying some specific quality criterion provide an approximate solution which is probably close to an optimum one. Here we are assuming the concept of an approximative algorithm formalized in the early seventies by Johnson (1974 [11]), which is necessarily polynomial and is evaluated by the worst case possible relative error over all possible instances of the problem. Approximative algorithms have developed in response to the impossibility of solving a great variety of important optimisation problems classified as NP-hard. The analysis of approximative algorithms always involves deriving estimates on the value of the optimum. As such, approximative algorithms and their analysis could be useful in anticipating good feasible solutions through a properly approximation process. According to Dubois (2000 [5]), anticipation is a very complex concept, and the anticipatory effects, in many cases, can be obtained mathematically. In optimisation problems context, Hochbaum (1997 [9]) explored the limits of polynomial approximative algorithms for NP-hard problems and investigated some criteria for evaluating the quality of approximative algorithms. It is noted that various problems differ quite substantially in terms of the quality of the ratio or worst case error bound. It is tempting to assess the difficulty of a hard problem as proportional to the possible approximation.

According to Bovet and Crescenzi (1994 [3]), after the original success in obtaining approximative algorithms to various problems, a great research effort has been devoted in trying to find a uniform structure to deal with the notion of approximability to optimisation problems, under the Complexity Theory point of view. As theoreticians continue to seek more powerful methods for proving problems intractable, parallel efforts focusing on learning more about the ways in which problems are interrelated with respect to their difficulty and comparing the complexity of different combinatorial optimisation problems have been an extremely active research area during the last twenty years. The different behaviour of NP-hard optimisation problems with respect to their approximability properties is captured by means of the definition of approximation classes and, under the "P $\neq$ NP" conjecture, these classes form a strict hierarchy whose levels correspond to different degrees of approximation.

Structural Complexity Theory is often concerned with the inter-relationships between complexity classes. However, it seems that an attempt of organizing all these results in a unified framework as general as possible is lacking. The aim of this paper is to make a first step in this direction. Starting from the observation that, intuitively, there are many connections among categorical concepts and structural complexity notions, in Leal et. al. (2000 [12]) we have defined two categories: the OPT category of optimisation problems and the APX category of approximation problems. In this direction, a preliminary version of this paper appeared in Leal et. al. (2001 [13], 2001 [14]), where a comparison mechanism between OPT and APX categories has been introduced. The basic idea is based on categorical shape theory due to Cordier and Porter (1990 [4]) and is motivated by previous works by C. Rattray (1994 [16], 1995 [17]).

Category theory is likely to be useful in providing framework within which to explain basic universal notions from Structural Complexity such as "completeness", "hardness" and "best approximation". According to D. Ellerman (1988 [7]), "the category theory's foundational relevance is that it provides universality concepts to characterize the important structures throughout mathematics." In this context, the notion of *universal* for a property represents the *essential* characteristics of such a property without any imperfections, and category theory is a precise mathematical theory of *concrete universals.*

In this paper, optimisation problems categories OPT and APX are redefined, a brief overview of a general theory of universals is presented, and we discuss how these properties can be applied to explain universality within structural complexity, by means Category Theory.

This paper is organized as follows. In section 2 category theory is presented as a suitable mathematical foundation to deal with the structural aspects of optimisation problems, and in the context of a general theory of universals, category theory is identified as a theory of concrete universals. In section 3 are introduced the OPTS category - the polynomial time soluble optimisation problems category, and the OPT category - the optimisation problems category. After that, the connections with categorical shape theory are presented and a system approximation to each optimisation problem is outlined in section 4. Finally, some conclusions are sketched and directions for further works are suggested.

2 Mathematical Foundations

In order to make the paper self-contained, this section gives some basic categorical concepts following the literature (Barr and Wells,1990 [2], Goldblat, 1986 [8]), and introduces briefly the Theory of Universals, based on the paper by D. Ellerman (1988 [7]).

But, why *category theory*?

According to Barr and Wells (1990 [2]), there are various view on what category theory is about, and what it is good for. Category theory is a relatively young branch of mathematics stemming from algebraic topology, and designed to describe various *structural* concepts from different mathematical fields in a *uniform* way. Indeed, category theory provides a bag of concepts (and theorems about those concepts) that form an abstraction of many concrete concepts in diverse branches of mathematics, including computing science. Hence, it will come as no surprise that the concepts of category theory form an abstraction of many concepts that play a role in structural complexity.

2.1 Category

Quoting Goldblat (1986 [8]): "A category may be thought of in the first instance as a universe for a particular kind of mathematical discourse. Such a universe is determined by specifying a certain kind of *objects*, and a certain kind of *arrow* that links different objects."

Definition 1. *A* category **C** *is specified by a collection ob***C***, disjoint sets* **C***(A,B) for A, B* $\in$ *ob***C***, and an associative operation* $\circ$*, such that (i)*$(f \circ g)$ *is defined for* $g \in \mathbf{C}(A,B)$*,* $f \in \mathbf{C}(C,D)$ *if and only if B=C; (ii) for each* $A \in ob\mathbf{C}$*, there exists* $1_A \in \mathbf{C}(A,A)$ *such that* $(1_A \circ f) = f$ *and* $(g \circ 1_A) = g$*, whenever the composition is defined.*

2.2 Functor

A *functor* is a mapping from one category to another that preserves the categorical structure, that is, it preserves the property of being an object, the property of being a morphism, the typing, the composition, and the identities. Functors are the mathematically type of transformation between categories, and form a categorical tool to deal with structured objects.

Definition 2. *A* functor $F{:}\mathbf{C} \longrightarrow \mathbf{D}$ *for the categories* **C** *and* **D** *maps* ob**C** *into* ob**D** *and sets* **C***(A,B) into* **D***(FA,FB) such that it preserves (i) units, that is,* $1_{FA} = F(1_A)$*, for each object of* **C***; (ii) composition, that is,* $F(f \circ g) = (Ff \circ Fg)$*, whenever* $(f \circ g)$ *is defined.*

2.3 Comma Category

Following (Goldblat, 1986 [8]), comma category can be thought of a particular kind of *arrow categories* (categories denoted by $\mathbf{C}^{\rightarrow}$ whose objects are all the **C**-morphisms, for a given category **C**), in which the arrows have a fixed domain or codomain.

Definition 3. *Let* **C** *be a category, and B any object of* **C***. The* comma category $\mathbf{C} \downarrow B$ *is the category of objects over B such that it has* **C***-morphisms with codomain B as objects, and as morphisms from* $f : A \longrightarrow B$ *to* $g : A' \longrightarrow B$ *the* **C***-morphisms* $k : A \longrightarrow A'$*, where* $g \circ k = f$*.*

2.4 Theory of Universals

The *Theory of Universals* due to D. Ellerman (1988 [7]) is originally concerned to explain many of the ancient philosophical ideas about universals, such as: (1)the Platonic notion that all the instances of a property have the property by virtue of participating in the universal, and (2)the notion of the universal as showing the essence of a property without any imperfections.

The notion of *universality* is fundamental to the category theory. The foundational role of category theory is to characterize what is important in mathematics by exhibiting its concrete universality properties. The concrete universal for a property represents the *essential* characteristics of the property without any imperfections, and category theory provides the concepts to stress the universal instance from among all the instances of a property. All the objects in category theory with universal mapping properties such as *limits* and *colimits* (see, for example, (Barr and Wells, 1990 [2])) are concrete universals for universal properties. Thus the universal objects of category theory can typically be presented as the limit (or colimit) of a process of filtering out to arrive at the *essence* of the property.

Definition 4. *A mathematical theory is said to be a* theory of universals *if it contains a binary relation* μ *and an equivalence relation* $\approx$ *so that with certain properties* F *there are associated entities* u_F *satisfying the following conditions: (i)* universality*: for any* x, $(x \ \mu \ u_F)$ *iff* $F(x)$, *and (ii)* uniqueness*: if* u_F *and* u'_F *are universals for the same* F, *then* $u_F \approx u'_F$.

A universal u_F is said to be *abstract* if it does not participate in itself, i.e., $\sim (u_F \ \mu \ u_F)$. Alternatively, a universal u_F is *concrete* if it is self-participating, i.e., $u_F \ \mu \ u_F$.

2.5 Set Theory: Theory of Abstract Universals

Set theory readily is qualified as a theory of abstract universals. The universal representing a property F is the set of all elements with the property: $u_F = \{x \mid F(x)\}$.

The participation relation is the set membership relation usually represented by $\in$. The universality condition in set theory is the called the (naive) *comprehension axiom*: there is a set y such that for any x, $x \in y$ iff $F(x)$.

Set theory also has an *extensionality axiom* which states that two sets with the same members are identical: for all x, $(x \in y$ iff $x \in y')$ implies $y = y'$.

The naive comprehension axiom lead to inconsistency for some properties, yielding contradictions such as Russel's Paradox. Thus, set theory cannot qualify as a general theory of universals.

2.6 Category Theory: Theory of Concrete Universals

For the concrete universals of category theory, the participation relation is the *uniquely-factors-through* relation. It can always be formulated in a suitable category as: "$x \ \mu \ u$" means "there exists a unique arrow $x \longrightarrow u$".

In the universality condition it has that, for any x, $(x \ \mu \ u)$ iff $F(x)$.

The existence of the *identity arrow* to u is the self-participation of the concrete universal which corresponds with $F(u)$, the application of the property to u.

In category theory, the equivalence relation used in the uniqueness condition is the *isomorphism*.

Making a comparison, category theory as the theory of concrete universals has quite a different flavor from set theory, the theory of abstract universals. Given the collection of all the elements with a property, set theory can *postulate* a more abstract entity, the set of those elements to be universal, while category theory must *find* its universals among the entities with the property.

3 Optimisation Problems Categories

In this section a categorical approach to optimisation problems is presented in such way that the notion of *reduction* from a problem to another one appears, naturally, in the conceptual sense of *morphism* between two objects. Reductibility provides the key-concept to this approach. The recognition that the only structure that an object has is by virtue of its interaction with other object leads to focus on structural aspects of optimisation problems. A preliminary and short version of this idea appeared in (Leal et. al., 2000 [12]).

The introduction of an appropriate notion of reductibility between optimisation problems allows to formally state that an optimisation problem is as hard to approximate as another one. In particular, the notion of approximation-preserving reductictibility orders optimisation problems with respect to their difficult of being approximated. *Hard problems* are the maximal elements in a class, with respect to this order, and capture the *essential* properties of that class. In this sense, NP-hard problems are *universal* to NPO class.

We assume that the basic concepts of computational complexity theory are familiar. We are following the notation of Garey and Johnson (1979 [6]), which is universally accepted, as such as the known books by (Ausiello et. al., 1999 [1], Bovet and Crescenzi, 1994 [3], Papadimitriou, 1994 [15]). In the following, we briefly review the basic terminology and notation.

3.1 NP-Optimisation Problem

On the analogy of the theory of NP-completeness, it there has been more interest in studying a class of optimisation problems whose feasible solutions are short and easy-to-recognize. In an optimisation problem, one is given an instance to which a finite set of feasible solutions is associated. A properly defined measure function attributes an integer cost to any such solution, and the goal is to find a feasible solution of optimum cost. Hundreds of natural problems from Computer Science, Operations Research and Engineering (but also Discrete Mathematics, Theoretical Physics, Molecular Biology,...) are formulated in this way.

Definition 5. *An NP-optimisation problem* $\mathbf{p}$ *is a triple* $\mathbf{p} = (I, S, Opt)$, *such that* $Opt : (I, S) \longrightarrow Z^+$, *where: (i)The set of instances* I *is recognizable in polynomial time; (ii)Given an instance* x *of* I, *all the feasible solutions of* x *belonging to the set* S_x *are short, that is, a polynomial* p *exists such that, for any* $y \in S_x, y \leq p(x)$.

Moreover, it is decidable in polynomial time whether, for any x and for any y such that $y \leq p(x)$, $y \in S_x$. (iii) The objective function Opt is computable in polynomial time.

Definition 6. *The NPO* class *is the set of all NP-optimisation problems, and the PO stands to the class of NPO problems that admits a polynomial algorithm to find their optimum solution.*

Similarly to "P=NP?" fundamental question in Computational Complexity, also it is not known whether "PO=NPO".

By means of the notion of reductibility between optimisation problems it is possible to define *hardness* to NPO class.

3.2 Reductions

In general, within complexity theory, a *reduction* from a problem A to a problem B specifies some procedure to solve A by means of an algorithm solving B. In the context of approximation, the reduction also should guarantee that an approximate solution of B can be used to obtain an approximate solution for A.

Definition 7. *A* reduction *between the optimization problems* $\mathbf{p} = (I, S, Opt)$ *and* $\mathbf{q} = (I', S', Opt')$ *is a pair of polynomial time computable functions* (f, g), *where* $f : I \longrightarrow I'$ *and* $g : (I', S') \longrightarrow (I, S)$ *are such that the diagram in the figure 1 commutes.*

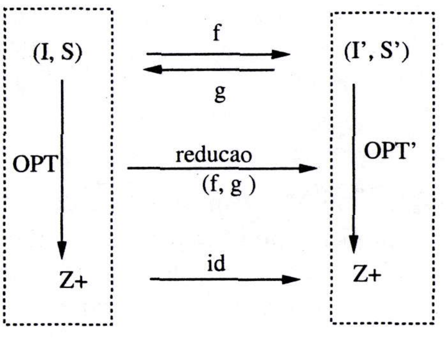


Fig. 1. Reduction between Optimisation Problems

The meaning of that diagram commutes is that, in order to obtain the optimum solution for the problem $\mathbf{p}$, it is possible firstly to reduce the problem $\mathbf{p}$ to the problem $\mathbf{q}$, and secondly to solve the problem $\mathbf{q}$. The solution obtained will be the same solution given by some procedure which solve the problem $\mathbf{p}$ directly.

Reductions are defined in such way that they are composable and they satisfy *transitivity* and *reflexivity* properties. Two problems are said *polynomially equivalent* whenever they reduce to each other. It follows that a reduction defines an equivalence relation, and thus it imposes a partial order on the resulting equivalence classes of problems.

Definition 8. *Given a reduction, an NPO problem* $\mathbf{p}$ *is said to be* NP-hard *respect to that reduction, if for all NPO problems* $\mathbf{p}'$ *we have that* $\mathbf{p}'$ *reduces to* $\mathbf{p}$.

It is important to observe that *hardness* means different things to different people. As a matter of convenience, the theory of NP-completeness was designed to be applied on decision problems. To other kind of problems, such as optimisation problems or those problems not belonging to the NP class, should be used the hardness term. On the other hand, hard and complete problems are defined for all kind of problems. Let $\wp$ be any class of problems and $\propto$ a given reduction. A problem p is said to be $\wp$-hard if for all problems q in $\wp$ it has $q \propto p$. A $\wp$-hard problem p is said to be $\wp$-complete if in addition p is in $\wp$.

An approximation-preserving reduction is defined as a reduction between optimisation problems adding some conditions that guarantee some property related with approximation.

Definition 9. *A* approximation-preserving reduction *between the NP optimization problems* $\mathbf{p} = (I, S, Opt)$ *and* $\mathbf{q} = (I', S', Opt')$ *is a triple of polynomial-time computable functions* (f, g, c), *where* $f : I \longrightarrow I'$, $g : (I', S') \longrightarrow (I, S)$ *and* $c : Z^+ \longrightarrow Z^+$ *are such that the correspondent diagram commutes.*

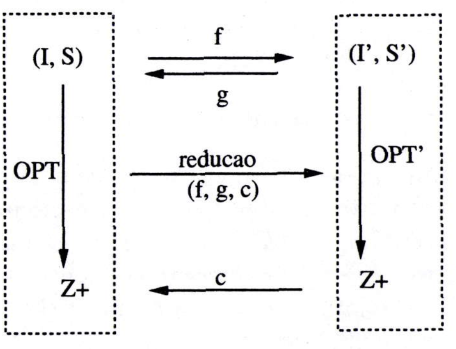


Fig. 2. Approximation-preserving Reduction

Here is introduced a function c, which role is that of preserving the quality of approximation. Depending on the imposing relation between quality approximation of problems, several different approximation-preserving reduction have been defined in the last fifteen years, according Trevisan (1997 [20]).

3.3 OPTS Category

The *polynomial time soluble optimisation problem category* OPTS has PO optimisation problems as objects and reductions between optimisation problems as morphisms.

Theorem 1. *OPTS is a category.*

Proof. Since reductions are defined as computable functions satisfying the reflexive and transitive properties, it has that identity morphisms are guaranteed by means of reflexivity, and composition with associativity is obtained by means of transitivity.

After we have given a first step in the categorical approach with the definition of the polynomial time soluble optimisation problems category, it is natural to pursue in this direction, aiming at extending to NPO optimisation problems considered intractable. Next, considering the notion of approximation-preserving reduction as morphisms between optimisation problems, it is possible to define an wider category.

3.4 OPT Category

The *optimisation problems category* OPT has optimisation problems as objects and approximation-preserving reductions as morphisms.

Analogously to OPTS category, is easily verified that OPT is really a category.

Theorem 2. *OPT is a category.*

Proof. Approximation-preserving reductions are defined as computable functions satisfying the reflexive and transitive properties. Thus it has that identity-morphisms and composition with associativity are guaranteed by means of reflexivity and transitivity, respectively.

Theorem 3. *NP-hard problems are concrete universals to OPT category.*

Proof. Given a reduction $\propto$, let U a NP-hard problem respect to $\propto$. We have to show that U is a concrete universal object for some participation relation μ and a equivalent relation $\approx$, according to Definition 4. Let the participating relation be: $(p \; \mu \; U)$ iff $(p \propto U)$, where $F(p) \equiv$ "p reduces to U", and the equivalence relation $\approx$ the polynomial equivalence relation defined in terms of the reduction $\propto$.

It has that, U satisfies the universality condition, by NP-hard problem definition, that is, for any NPO-problem p, $(p \; \mu \; U)$ iff $(p \propto U)$. Also, since the reduction induces an equivalence relation, it has that if U' is also a NP-hard problem, then $U \propto U'$ and $U' \propto U$, that is, U is polynomially equivalent to U'. Therefore U is a concrete universal object to OPT category, which concreteness condition corresponds to the reflexivity property of reduction.

4 OPTS Category × OPT Category

Having defined both the polynomial time soluble optimisation problems category and the optimisation problems category, the next step is to identify the relationships between them. We start from these basic questions:

1. How do OPTS and OPT categories interact with each other?
2. What does it mean to say that a problem A "approximates" an optimisation problem B?
3. What is it understood by the "best approximation" for such an optimisation problem?

The goal is now to provide mechanisms for the comparison between such categories. This will lead us to the categorical shape theory. A first version of this idea is showed in (Leal et. al., 2001 [14]).

4.1 Categorical Shape Theory

Very often we wish to find a mathematical model of a structure in order to explain its properties and predict its behavior in different circumstances. Related to the approximability issue to optimisation problems, it is likely that the categorical shape theory would be such a model. It does provide a comparison mechanism to establish the meaning of an approximation system, identifying the universal properties in the category theory sense, in order to describe how an object "best approximates" another object. This section has been motivated from previous works by Rattray (1994 [16], 1995 [17]). The basic idea of categorical shape theory due to Cordier and Porter (1990 [4]) is that, in any approximating situation, the approximation are what encode the only information that it can analyze.

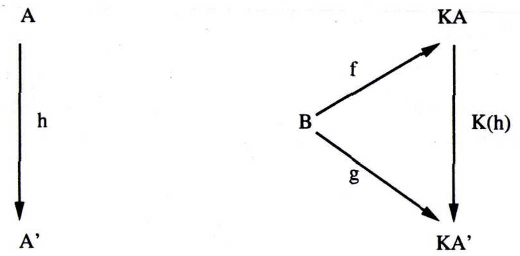
In the context of categorical shape theory it has:

1. a category **B** of objects of interest;
2. a category **A** of archetypes or model-objects;
3. a "comparison" of objects with model-objects, ie. a functor $K : \mathbf{A} \longrightarrow \mathbf{B}$.

Roughly speaking the idea behind categorical shape theory is that recognizing and understanding an object of interest B via a comparison $K : \mathbf{A} \longrightarrow \mathbf{B}$ requires the identification of the corresponding archetype A which best represents B.

Definition 10. *Given category* **A** *of archetypes, category* **B** *of objects of interest, and a comparison* $K : \mathbf{A} \longrightarrow \mathbf{B}$, *an* approximation *to an object* B *in* **B** *is the pair* (f, A), *where* A *in* **A** *is an archetype and* $f : B \longrightarrow KA$.

A morphism between approximations $h : (f, A) \longrightarrow (g, A')$ is a morphism $h : A \longrightarrow A'$ of the underlying archetypes, such that $K(h) \circ f = g$, ie. the triangle



commutes.

Approximations with their morphisms form a category $B \downarrow K$, the comma category of K-objects under B. The cone-like form of the morphisms in **B** giving the approximations for some object B, suggests that taking the limit object of the diagram would result in an archetype A* "as near as possible" to B. See figure 3 below.

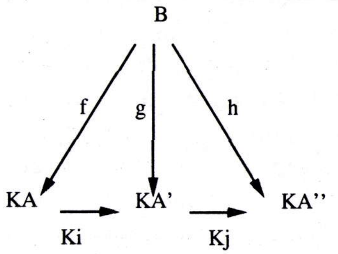


Fig. 3. Approximations to B

The notion of "most closely approximates" is given by a universal object.

Definition 11. *Let K : **A** $\longrightarrow$ **B** be a comparison functor. An archetype A of **A** is said to be* K-universal *for an object of interest B of **B** if there exists an approximation (f, A) to B such that, for each approximation (g, A') to B, with A' in **A**, there exists a unique morphism $h : A \longrightarrow A'$ in **A** with $g = K(h)^\circ f$.*

Definition 12. *Category **A** is said to be* K-universal *in **B** if every object of interest of **B** has a* K-universal *archetype in **A**.*

4.2 Connections with OPTS and OPT Categories

In the scenario of categorical shape theory, we may consider the OPT category as the category of objects of interest **B**, the OPTS category as the category of archetypes **A**, and K: OPTS$\longrightarrow$ OPT would be a comparison mechanism related to an approximation method (for instance by using the relaxation). Through this theory it is possible to identify the best approximation to an optimisation problem B, if it exists. In fact, the existence of optimisation problems not allowing any kind of approximation makes the proposition below consistent.

Proposition 1. *OPTS category is K-universal in OPT if and only if PO=NPO.*

4.3 Approximations Category

In order to characterize approximation degrees by means of categorical shape theory, the basic idea is the construction of a system approximation to each optimisation problem using limits (or colimits). A limit construction provides a means of forming complex objects from patterns (diagrams) of simpler objects. By using colimits, a hierarchical structure can be imposed upon the system of approximation.

Definition 13. *Given a comparison functor K: OPTS$\longrightarrow$ OPT and an optimization problem B in OPT category, an approximation problem to B in OPT is the pair (f, A), where A in $OPTS$ is a polynomial time soluble optimization problem and $f : B \longrightarrow KA$.*

In this case, the comma category of K-objects under B give the approximations for the problem B.

In this direction, other properties are being investigated at the present moment, regarding some aspects of categorical shape theory which have not been dealt here.

5 Conclusion

The main objective of this work is to develop a formal theory to approximative algorithms, considering them as a feasible alternative to intractable problems in such way that integrates the Structural Complexity's conceptions to the fundaments of suitable semantic model. The recognition that the notion of reductibility between problems substantiate in a specialization process of Category Theory, led to an investigation on structural aspects of approximation classes through of categorical approach, focusing over the approximation preserving reductions between optimisation problems.

Category theory provides basic notation and a universal language for explaining, investigating and discussing concepts and constructions from different fields, in a uniform way. It allows this from a viewpoint different to that of set theory in which an object is described in terms of its "internal" structure. In categorical terms the only structure that an object has is by virtue of its interaction with other objects, described in terms of "external" features. Through categorical approach we have improved our understanding and development of many concepts within structural complexity related to approximation for optimisation problems. Theory of universals has provided a mathematical foundation to explain basic universal elements from complexity theory in an elegant way.

A comparison between OPTS and OPT categories has been motivated from previous work by C. Rattray (1994 [16], 1995 [17]), based on categorical shape theory. The study that we have started in this paper is an attempt in this direction. Along this line, a number of important questions remain to be studied, and we think that in order to establish connections among optimisation problems and their approximability properties, it may be fruitful to find relationships with other results drawn from other approaches, at the same level of abstraction, such as the one developed in (Rattray, 1998 [18]). The work is still on-going and involves many aspects of categorical shape theory. The study of approximation hierarchy to optimisation problems identifying problems with intermediate degrees is in order for further works.

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