Oil Spill Accidents off the Coast of São Paulo State: History, Modelling and Simulations

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Abstract
In this work we present a classical evolution equation for the movement of oil slicks in marine water in its second phase (Fay, 1969), and, setting it in its variational formulation, discretize it with a view towards the use of the Finite Element Method (using first degree approximations for oil concentration). For simulating marine currents, special upwinding techniques are adopted so as to eliminate main oscillations caused by numerical options. These currents are given by the solution of Stokes' equation using second order finite elements for velocities and discontinuous piecewise constant elements. Boundary conditions are obtained from data presented in (Furtado, 1978). Resulting currents were then used for a qualitative simulation so as to verify the model comparing it to registered information (CETESB, 1996).

Keywords: oil spills, numerical simulation, finite elements, Stokes' equation, diffusion equation.

1 Introduction

Off the northern coast of São Paulo State, in south-eastern Brazil, there is a channel separating the isle of São Sebastião and the continent (see figure 1). This channel, also called São Sebastião, is where the Brazilian national petroleum company (Petrobrás S.A.) operates the most important oil terminal in the country, through which passes over 55% of all transported oil in Brazil. The São Paulo state agency for environmental protection (CETESB) registered about 350 oil spill accidents in the last 24 years.

Given the severe damages that these accidents have, in some cases, provoked, there began, in 1994, a joint effort between protection agency, petroleum company and universities (as well as local authorities) in order to create and standardise the necessary procedures for protection and clean-up in the case of future accidents. A relevant part of this project consisted in the formulation of appropriate mathematical models for the definition of numerical codes that would undertake the qualitative prediction of oil spill movements in the region. These were intended to provide the possibility of establishing strategies for protection and cleanup activities. In principle, therefore, a manual considering the most probable scenarios was to be put together, in which the mathematical models and computer simulations were to play the part of oil slick movement prediction. In other words, this part of the general project was responsible for anticipating the movement of the oil spill, as well as its effects (by evaluating oil concentration levels) on coastal ecossystems – and to help in the decision-making as to which equipment and work teams to deploy in which areas.

The mathematical models that were adopted were of deterministic nature, using a diffusive-advective partial differential equation, capable of describing the movement of oil spills in coastal waters in their so-called second phase (Benqué, 1980; Cuesta, 1990; Fay, 1969; Meyer, 1993; Meyer, 1998). Convenient border conditions for modelling open sea and coastal situations were adopted (Carbonel, 1995). In previous work, the circulation models were created by interpolating observed measures registered by oceanographical research (Furtado, 1978). The present strategy consists of coupling a Stokes' model in order to numerically obtain the components of a vector field which describes fixed scenarios for local circulation patterns. Once a circulation model has been obtained, using second-order finite elements, these are used to furnish parameters for the second part of the program, in which first-order finite elements with streamline-upwind/Petrov-Galerkin techniques (Brooks, 1982) are used to obtain the evolutionary description of the oil spill movements in the proposed situation. In temporal approximations, the implicit classical Crank-Nicolson method is adopted. Computer simulations are undertaken using initial data obtained from CETESB registers, and results which are remarkably similar to historical descriptions are obtained (CETESB, 1996). These are presented with the use of special software for visualisation.

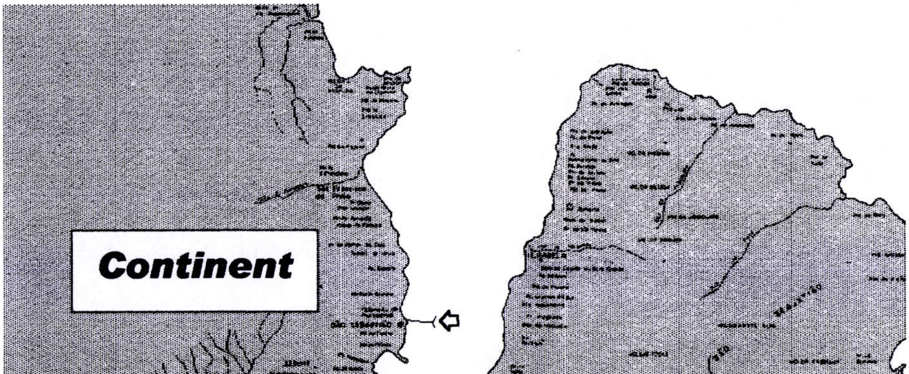


Fig. 1: São Sebastião Channel, along with indications to the oil terminal (horizontal arrow) and the pumping systems (vertical arrow).

2 Mathematical Model

2.1 Oil Spill Model

The classical characterization of oil spills, due to Fay (1969), leads us quite naturally to the adoption of the so-called diffusive-advective, or Second phase, which describes the oil spill from a few hours after its occurrence to about two weeks later. During this phase, the oil spill can have an area of several square kilometers, while maintaining a width between 2 and 5 centimeters. These dimensions indicate the natural adoption of only 2 space variables, a decision sustained by most of the authors who work with the modelling during this period (Benqué, 1980; Cuesta, 1990; Psarafits, 1985). Therefore, the mathematical formulation of the described problem, for a concentration u of oil at time t and at a point (x,y) is given by $u = u(x,y;t)$, for $(x,y) \in \Omega$ and $t \in (0,T]$ such that

$$\frac{\partial u}{\partial t} + \operatorname{div}(-\alpha \nabla u) + \operatorname{div}(\mathbf{V}u) + \sigma u = f(x,y;t) \tag{1}$$

such that

$$\left.\frac{\partial u}{\partial \eta}\right|_{\Gamma_0} = 0 \text{ and } -\alpha \left.\frac{\partial u}{\partial \eta}\right|_{\Gamma_1} = k\mathbf{V}_\eta u, \text{ for } \Gamma_0 \cup \Gamma_1 = \partial\Omega, \ \forall t \in (0,T] \tag{2}$$

where k is a constant representing the fraction of pollutant leaving Γ_1 in the direction of the normal velocity $\mathbf{V}_\eta$, and

$$u(x,y;0) = u_0(x,y) \text{ for all } (x,y) \in \Omega. \tag{3}$$

The involved parameters also describe the considered phenomena:

- The model considers effective diffusion and, for coastal situations, a diffusivity, α, which is constant in space and time (Stolzenbach *et alii*, 1977, Meyer, 1993).
- In the advective term, $\mathbf{V} = \mathbf{V}(x,y)$ describes the resultant velocity due to tides, wind and currents, and accounts for transport of the spill, varying in space, and div(**V**) is assumed to be null.
- The several decay processes are considered in the model as a linear function of the concentration of oil, σu (even though this can be modified, in order to have σ varying in space and time).
- Boundary conditions indicate that no oil crosses part Γ_0 of the boundary, which describes the land boundaries, or the shores along the channel margins, and that along Γ_1 the model accounts for oil leaving the domain in proportion to the normal component of resulting ocean currents.
- Finally, this formulation also includes possible oil sources (this situation actually occurred in the studied region when one of the main pipelines was accidentally perforated).

Taking into account the discontinuity of initial conditions, as well as that of many cases of sources, the above formulation in terms of a classical Partial Differential Equation is transformed into a variational, or "weak" formulation, which continues to hold in a generalized sense. It is also very appropriate for the use of Galerkin's Method and Finite Elements. As usual, we will denote

$$(f \mid g) = \iint_{\Omega} f(x,y)\, g(x,y)\, ds\,, \tag{4}$$

$$(\mathbf{f} \parallel \mathbf{g}) = (f_1 \mid g_1) + (f_2 \mid g_2)\text{, and} \tag{5}$$

$$\langle f \mid g \rangle_{\Gamma} = \int_{\Gamma} f(x,y)\, g(x,y)\, d\gamma\,. \tag{6}$$

We then have, with convenient use of Green's Theorem as well as of the boundary conditions:

$$(\frac{\partial u}{\partial t} \mid v) + \alpha(\nabla u \parallel \nabla v) + (\mathbf{V}.\nabla u \mid v) + \sigma(u \mid v) + \langle p\mathbf{V}_{\eta} u \mid v \rangle_{\Gamma_1} = (f \mid v), \quad \forall v \in H. \tag{7}$$

where

$$H = H^1(\Omega) \tag{8}$$

and

$$W = \left\{ v \in L^2((0,T], H^1(\Omega) : \forall t \in (0,T], \frac{\partial v}{\partial t} \in L^2(\Omega) \right\}. \tag{9}$$

This theoretical lay-out guarantees existence and uniqueness of the desired solution, the analytic form of which is quite impossible to express. Nevertheless, these results set the basis for appropriate numerical approaches.

2.2 Stokes' Model for Marine Currents

The above described oil spill model assumes the existence of a vector field $\mathbf{V}$ that characterizes the region currents. This information will be mathematically modeled by a Stokes' system, chosen due to the characteristics of the Channel (Furtado, 1978). Therefore, the mathematical formulation, for a velocity field $\mathbf{V}$ constant in time at a point (x, y) is such that

$$\begin{aligned} -\mu\Delta\mathbf{V} + \nabla p = \mathbf{f}, &\quad (x,y) \in \Omega \\ \operatorname{div}(\mathbf{V}) = 0, &\quad (x,y) \in \Omega \\ \mathbf{V} = \mathbf{V}_{\partial\Omega}, &\quad (x,y) \in \partial\Omega \end{aligned} \tag{10}$$

where $\mathbf{V} = (V_1, V_2)$ is the velocity field, μ stands for water kinematic viscosity, p for pressure, $\mathbf{V}_{\partial\Omega}$ for the boundary velocity distribution and $\mathbf{f} = (f_1, f_2)$ for a forcing term.

Defining the spaces

$$H_0 = H_0^1(\Omega) = \{ v \in H^1(\Omega) : v|_{\partial\Omega} = 0 \}, \tag{11}$$

$$L_0^2(\Omega) = \{ p \in L^2(\Omega) : \int_{\Omega} p(x)\, ds = 0 \} \tag{12}$$

and the following bilinear forms

$$a(\mathbf{V},\mathbf{U}) = \mu(\nabla\mathbf{V} \| \nabla\mathbf{U}) = \mu\sum_{i=1,2}(\nabla V_i \| \nabla U_i), \tag{13}$$

$$b(p,\mathbf{U}) = (\nabla p \| \mathbf{U}) = -(p \mid \operatorname{div}(\mathbf{U})) \tag{14}$$

we obtain, in a straightforward manner the variational formulation for (10)

$$\begin{aligned} & a(\mathbf{V},\mathbf{U}) + b(p,\mathbf{U}) = (\mathbf{f} \mid \mathbf{U}), && \forall \mathbf{U} \in H_0 \times H_0. \\ & b(q,\mathbf{V}) = 0, && \forall q \in L_0^2(\Omega), \text{ and} \\ & \mathbf{V} = \mathbf{V}_{\partial\Omega}, && \text{in } \partial\Omega \end{aligned} \tag{15}$$

with $\mathbf{f} \in H^{-1}(\Omega) \times H^{-1}(\Omega)$ and $\mathbf{V}_{\partial\Omega} \in H^{1/2}(\partial\Omega) \times H^{1/2}(\partial\Omega)$ in order to guarantee the well-posedness of the problem (Girault, 1986).

3 Approximate Solutions

3.1 Oil Spill Model

The boundary conditions have been made part of formulation (7), and, as previously mentioned, the velocity field should be conservative (Brooks, 1982), that is, $\operatorname{div}(\mathbf{V}) = 0$. This expression permits the choice of an approximation via Galerkin's Method by choosing a subspace V_h of H, which is finite-dimensional and generated by chosen finite elements φ_j. This choice separates space and time variables, by the use of

$$u_h = \sum_{j=1}^{N} c_j(t)\varphi_j(x,y), \quad \forall \varphi_j \tag{16}$$

from a convenient basis $\beta = \{\varphi_1, \ldots, \varphi_N\}$ of V_h. This will transform equation (7) into a system of Ordinary Differential Equations.

Besides the choice for (14), we will also make an option for a second-order approximation method in time: Crank-Nicolson, and we will, therefore, define

$$u_h(x_j, y_j; t_n) \cong \sum_{j=1}^{N} c_j(t_n)\,\varphi_j(x,y) \cong \sum_{j=1}^{N} c_j^{(n)}\varphi_j(x,y), \qquad \varphi_j \in \beta \subset V_h. \tag{17}$$

The use of this expression in the weak formulation given by (7) besides the choice for $c_j^{(0)} = u_0(x_j, y_j)$, leads us to a linear system of equations in the unknowns $c_i^{(n+1)}$, $i = 1, \ldots, N$:

$$\mathbf{A}\mathbf{c}^{n+1} = \mathbf{B}\mathbf{c}^{n} + \mathbf{d} \tag{18}$$

where

$$\begin{aligned} a_{ij} = {} & \left[1 + \sigma\frac{\Delta t}{2}\right](\varphi_j \mid \psi_i) + \alpha\frac{\Delta t}{2}(\nabla\varphi_j \| \nabla\psi_i) + p\mathbf{V}_\eta\langle\varphi_j \mid \varphi_i\rangle_{\Gamma_1} + \\ & + \frac{\Delta t}{2}\left[\sum_l V_x^l\left(\frac{\partial\varphi_j}{\partial x}\varphi_l \mid \psi_i\right) + \sum_m V_y^m\left(\frac{\partial\varphi_j}{\partial y}\varphi_m \mid \psi_i\right)\right], \end{aligned} \tag{19}$$

$$b_{ij} = \left[1-\sigma\frac{\Delta t}{2}\right]\left(\varphi_j \mid \psi_i\right) - \alpha\frac{\Delta t}{2}\left(\nabla\varphi_j \parallel \nabla\psi_i\right) - p\mathbf{V}_\eta\left\langle\varphi_j \mid \varphi_i\right\rangle_{\Gamma_1} -$$

$$-\frac{\Delta t}{2}\left[\sum_l V_x^l\left(\frac{\partial\varphi_j}{\partial x}\varphi_l \mid \psi_i\right) + \sum_m V_y^m\left(\frac{\partial\varphi_j}{\partial y}\varphi_m \mid \psi_i\right)\right] \text{ and}$$

$$d_i = \Delta t\left((f_j^{n+1} + f_j^n)/2 \mid \varphi_i\right)_S$$

with $i = 1,\dots,N$ and for given $\mathbf{c}^{(0)} = (c_1^{(0)}, c_2^{(0)}, \dots, c_N^{(0)})$.

Some special aspects of the above formulation should be mentioned.

1. Trial functions, namely ψ_i, are defined by $\psi_i = \varphi_i + \tau\mathbf{V}\nabla\varphi_i$, where τ stands for the upwinding factor. Its calculation is explicitly presented in Codina (1992). If $\tau = 0$, (18) expresses the traditional Galerkin method.
2. When this kind of upwinding is used, a divergence-free condition must be observed (Brooks, 1982).
3. Depending on their magnitude, source terms may induce a damped oscillation, as well.
4. Since the available velocity field data is defined pointwise, not a very convenient situation for evaluating the inner products in (7), an interpolation of the vector field $\mathbf{V}$ using the same base functions of V_h is therefore performed.

3.2 Stokes' Model

In order to obtain the numerical solution of Stokes' system of equations we use a mixed finite element formulation on the São Sebastião discretized domain.

Stokes' system poses an additional restriction in the choice of discrete approximation spaces: the inf-sup condition (Brezzi, 1991). A convenient pair of spaces respecting the inf-sup condition is $V_h = P_2 \times P_2$, the second order polynomial finite elements for velocity and $\Pi_h = P_0$, non-conforming constant polynomial finite elements for pressure.

Denoting:

- The finite element basis of P_2 by $\vartheta = \{\phi_1,\dots,\phi_N\}$;
- $V_{h,k} = \sum_{j=1}^{N} c_{k,j}\phi_j$, $k = 1,2$ as the aproximation for V_k, $k = 1,2$;
- The approximated pressure by $\mathbf{p} = \mathbf{p}_h = (p_1,\dots,p_{Nel})$ with *Nel* being the number of elements (remembering that pressure is constant within each element).

Evaluating the inner products with $V_{h,k}$ and p_h we obtain the linear system:

$$\begin{pmatrix} \mathbf{A} & & \mathbf{B}_1^t \\ & \mathbf{A} & \mathbf{B}_2^t \\ \mathbf{B}_1 & \mathbf{B}_2 & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{c}_1 \\ \mathbf{c}_2 \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \mathbf{0} \end{pmatrix} \tag{20}$$

where

$$\begin{array}{ll} a_{1,ij} = a_{2,ij} = a(\phi_j, \phi_i), & 1 \le i, j \le N \\ b_{1,ij} = b(p_i, \frac{\partial \phi_j}{\partial x}), & 1 \le i \le Nel, \quad 1 \le j \le N \\ b_{2,ij} = b(p_i, \frac{\partial \phi_j}{\partial y}), & 1 \le i \le Nel, \quad 1 \le j \le N \\ d_{1,i} = (f_{1,j} \mid \phi_i), d_{2,i} = (f_{2,j} \mid \phi_i), & 1 \le i, j \le N. \end{array} \tag{21}$$

Note that the pair $(\mathbf{c}_1, \mathbf{c}_2)$ corresponds to the approximation of V_1, V_2 and $\mathbf{p}$ to the pressure.

The resultant linear system (20) was solved with a special implementation of the conjugate gradient method, taking in consideration the matrix structure.

4 Numerical Experiment

The numerical experiment we present reproduces an oil spill that happened in 1994, registered as "TEBAR V" (CETESB, 1996). This spill ocurred due to a severe failure in the pumping systems, resulting in about $2.700\,m^3$ of oil being spilled into the channel waters. All parameters and variables are dimensionless in the usual way.

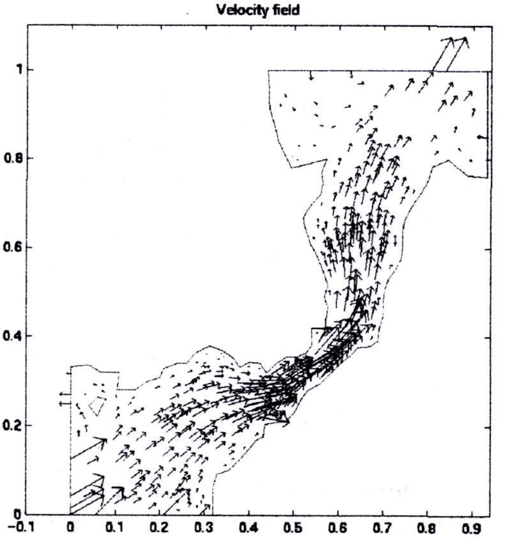
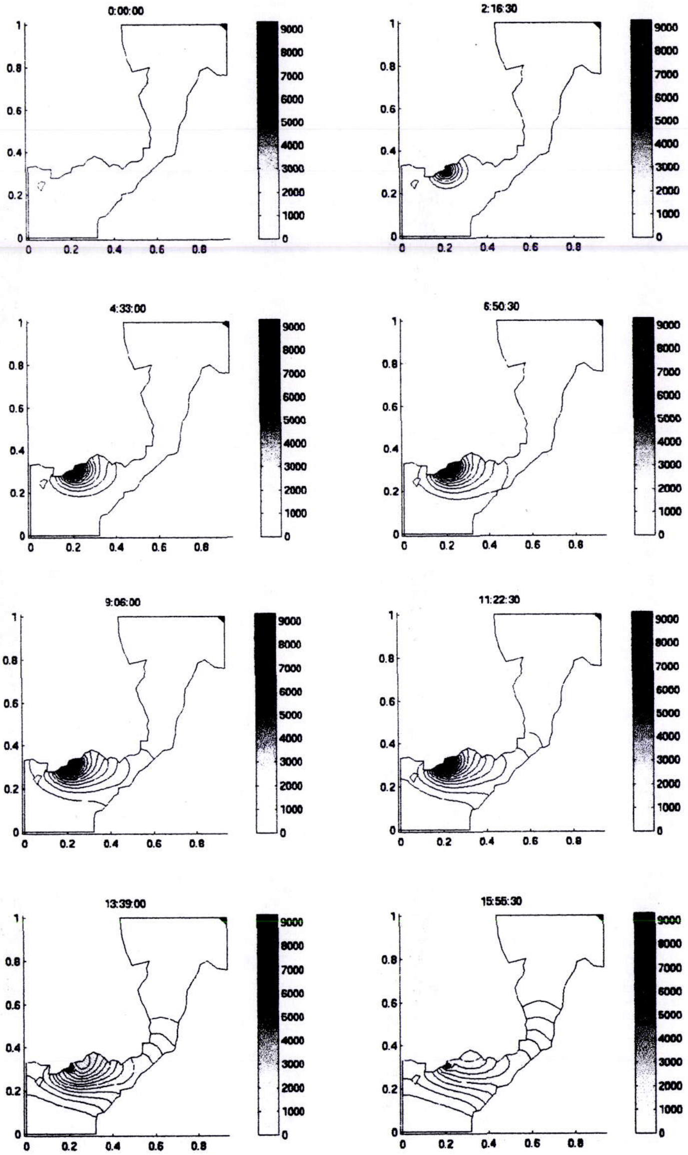
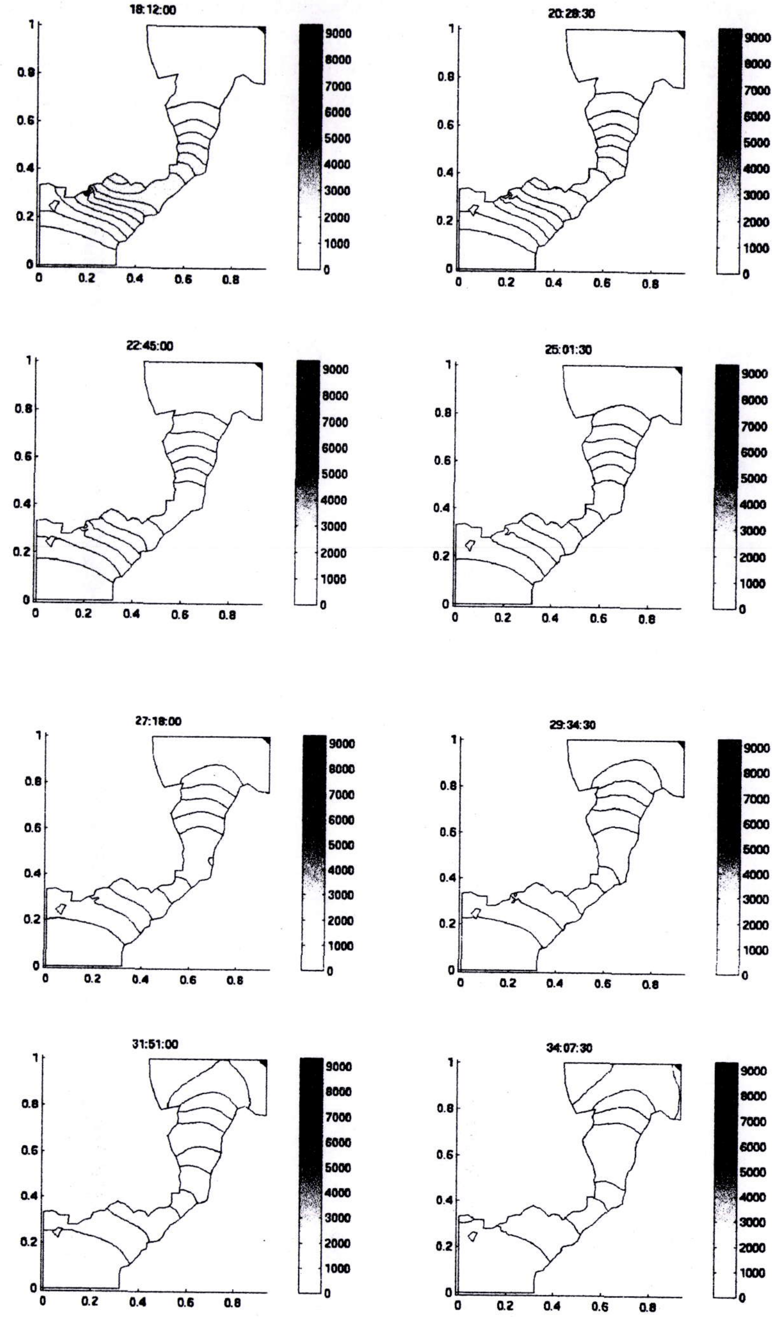


Fig. 2: Velocity field obtained from Stokes' equation.

Figure 2, illustrates part of the vector field obtained by the Stokes' equation. Not all vectors are presented to keep the figure visually clearer.

The next set of figures illustrates de modelled spill in several time steps (in hours). The predicted oil behaviour shows quite a good approximation to historical registers (CETESB, 1996).




18:12:00
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Conclusions

After a series of numerical simulations of oil spill movements in the case we here present, São Sebastião Channel, as well as in other critical coastal situations in Brazil (Guanabara Bay, for example, where the January 18 accident occurred, and in the Angra Bay region, the closest oil terminal to the newly discovered super reservoir "Superpoço da bacia de Santos"), and considering the efficient approximation results obtained with the use of these techniques for the first two mentioned cases (models, approximation strategies, computational characteristics), we have sufficient material as to justify the use of the models and simulating described methods for new marine scenarios on other locations on the southamerican coast where environmental impact risks due to spills are present.

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