Time Suboptimal Pole Assignment Control via Relative Degree Reduction

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Abstract
This paper builds on the previous results achieved in designing pole assignment controllers for the 1st and the 2nd order systems with constrained control signal. By examining properties of the control loop with the reduced relative degree of the original plant, it prepares methodology for applying the achieved results for controlling higher order systems with constrained input. The success of the relative degree reduction by introduction of a fictive output that involves derivatives of the original output is shown to be dependent on the anticipation of the working range of the controlled system.
Keywords: Control signal saturation, minimum time pole assignment control, exact linearization, relative degree reduction.

1 Introduction

Control signal saturation represents on of the most important nonlinearities in the control design. For many decades it is attracting attention of the researchers, but despite many different approaches, still its role in the control design is not sufficiently clarified. In a serious of papers, authors and their co-workers are developing new approach based on dynamical classes of control. The newly developed controllers have been denoted as the minimum time pole assignment (MTPA) controllers. It was shown (Huba et al., 1999a) that for the 1st order systems the MTPA controllers correspond to simple P-controller. Here, the control signal saturation does not cause any serious problems, like e.g. overshoot, or oscillations. It just prolongs the transient responses and restricts the allowable set point, disturbance and initial state values.
For the 2nd order systems, the output of the linear pole assignment controller already cannot be constrained to an arbitrary value. For linear 2nd order systems (with the relative degree 2), it is possible to derive minimum time pole assignment controllers (MTPA), whereby the complexity of the controllers does not reasonably exceed simple linear PD-controllers. However, the complexity of the analytical design of the MTPA controllers reasonably increases already in controlling the 3rd order systems. So, there is a high motivation for looking for suboptimal simplified solutions. In order to explain the problems faced in the suboptimal control of higher order systems, we will firstly analyse MTPA and suboptimal control of the linear 2nd order systems. Then, we will extend our attention to the nonlinear and higher order systems.

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2 Minimum Time Pole Assignment (MTPA) Control Algorithms

2.1 Continuos MTPA Control Algorithms

For the 1st order system represented by the simple integrator

$$\frac{dy(t)}{dt} = K_S u(t) \tag{1}$$

the continuous P-controller can be developed as

$$u = -K_R y; \qquad K_R = -\gamma / K_S \tag{2}$$

Ideally, it enables to assign the close loop pole to any value $\gamma \in \langle -\infty, 0 \rangle$. This controller fulfills the condition of decreasing the output towards zero, whereby the closed loop pole γ represents the coefficient of the decrease defined by the differential equation

$$\frac{dy(t)}{dt} = \gamma\, y \tag{3}$$

Due to the control saturation given usually in the form

$$u \in \langle U_1, U_2 \rangle \tag{4}$$

the last equation can not be fulfilled from an arbitrary initial point. There exists the so-called **proportional band of control (PB)**, over which the condition given by eq. 3 is fulfilled. The points outside this zone are influenced by the fact that the P-controller eq. 2 is followed by the limiter guaranteeing eq. 4. So, outside of the proportional zone, the MTPA control algorithm holds the control signal at one of the limit values up to the moment of reaching PB, when the control signal starts to decrease exponentially, fulfilling eq. 3.

For the continuous control of the double integrator, the minimum time pole assignment controller have been proposed (Huba 1999b) guaranteeing a regular speed of the distance decrease of the reference braking curve. The distance is measured in a defined direction between the representative point and the chosen reference braking curve. The quotient of such a decrease is specified by one of the closed loop poles. The 2nd pole characterizes the speed of the distance decrease along the reference braking curve to the origin. For a chosen double real pole γ, sampling period T, given control signal constraints eq. 4, y being the output with the required state shifted to the origin and $\dot{y} = dy/dt$ its derivative, the anticipatory system is accomplished when designing the control algorithm by the following formulas

if $y < 0$ *then* $U_j = U_1$ *else* $U_j = U_2$

if $\left(y < 0 \; AND \; \dot{y} > \frac{U_j}{\gamma} \right) OR \left(y > 0 \; AND \; \dot{y} < \frac{U_j}{\gamma} \right)$ *then*

$$u=\left[1-\gamma\frac{y-\frac{1}{2}\left(\frac{\dot{y}^2}{U_j}+\frac{U_j}{\gamma^2}\right)}{\dot{y}}\right]U_j \qquad (5)$$

$else\ u=r_0 y+r_1\dot{y} \qquad r_0=-\gamma^2\ ;\ r_1=2\gamma$

if $u<U_1$ $then\ u=U_1$

if $u>U_2$ $then\ u=U_2$

Due to the last limitation, the control signal take the non-saturated values just in the proportional zone surrounding the reference braking curve. In Figs.1-2, this is outlined by the line segments (corresponding to the linear algorithm applied for lower velocities) and by the parabolic curves corresponding to the limit values $u=U_1$ and $u=U_2$ for higher velocities.

Repeating the procedure outline in (Huba, 1999b) for the I_1T_1 system described by the differential equation

$$\ddot{y}=\frac{1}{T_0}(u-\dot{y}) \qquad (6a)$$

with the corresponding transfer function

$$F(s)=\frac{1}{s(T\ s+1)} \qquad (6b)$$

one gets the reference braking line

$$y=\frac{1}{\gamma_1}\dot{y} \qquad (7)$$

for velocities lying between the values

$$\dot{y}_1=\frac{U_1}{\gamma_1 T_0+1}\ ;\ \dot{y}_2=\frac{U_2}{\gamma_1 T_0+1} \qquad (8)$$

Out of this range, the reference braking curve is described as

$$y=\dot{y}\gamma_1 T_0+\left[\gamma_1 T_0\ln\frac{(U_j-\dot{y})(\gamma_1 T_0+1)}{U_j\gamma_1 T_0}-1\right]U_j \qquad (9)$$

The control algorithms guaranteeing desired speed of approaching the nonlinear segment of the reference braking curve is given for the double real pole γ as

$$u=\gamma\left\{y+\dot{y}T_0-\frac{y+T_0(U_j-\dot{y})\ln\left[\frac{(U_j-\dot{y})(\gamma T_0+1)}{U_j\gamma T_0}\right]-\frac{U_j}{\gamma}}{\dot{y}}U_j-T_0U_j\right\} \qquad (10)$$

else, in measuring the distance from the linear segment of the RBC

$$u = r_0 y + r_1 \dot{y} \qquad r_0 = -\gamma^2 T_0 \;;\; r_1 = 2\gamma T_0 - 1 \tag{11}$$

Output of the controller has again to be limited to the allowed limits given by eq. 4. The corresponding proportional band and transient responses are shown in Fig.3-4.

Comparison of the both systems shows that in the case of the I_1T_1 systems and relatively slow poles the shape of the nonlinear part of the proportional band is much closer to its linear segment. So, it is to expect that the linear design procedure will give satisfactory results in such a case.

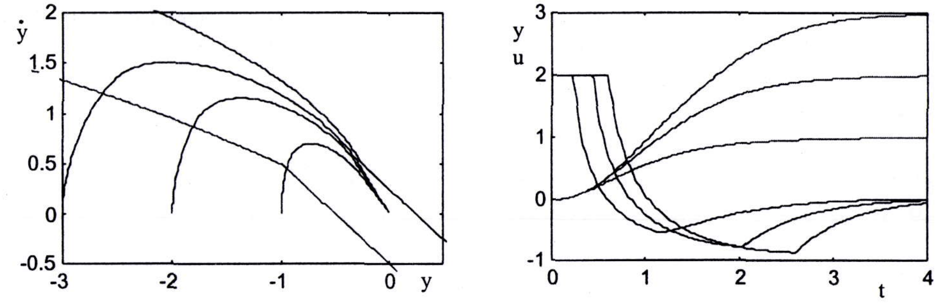


Fig. 1: MTPA controller for I_2-system. Proportional band and the system trajectories in the phase plane for 3 different reference signal steps (left) and the corresponding output and control signal time responses. $\gamma = -2; U_{min} = -1; U_{max} = 2;$

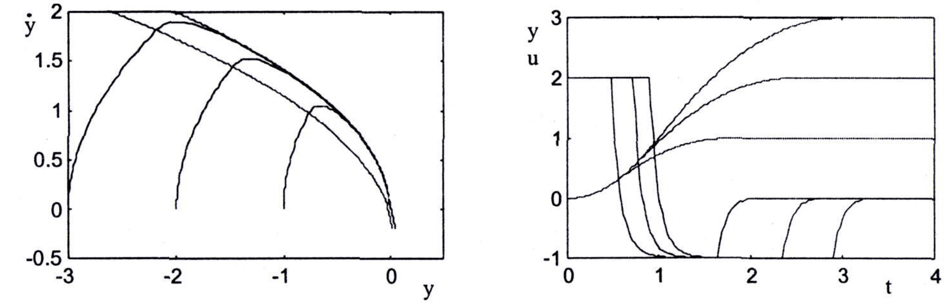


Fig. 2: MTPA controller for I_2-system. Proportional band and the system trajectories in the phase plane for 3 different reference signal steps (left) and the corresponding output and control signal time responses. $\gamma = -10; U_{min} = -1; U_{max} = 2;$

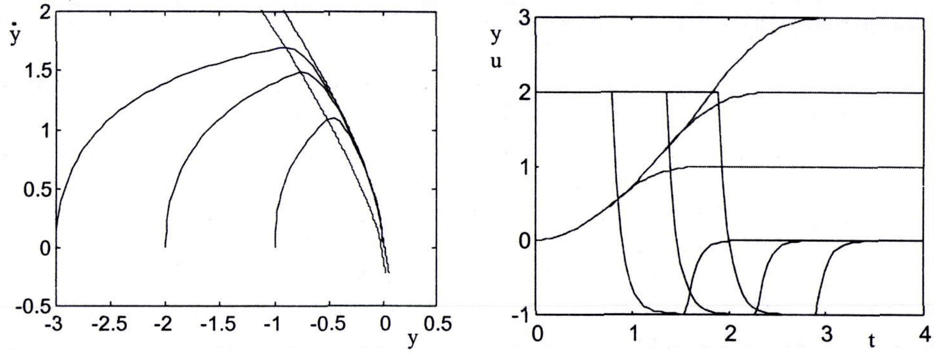


Fig. 3: MTPA controller for I_1T_1-system. Proportional band and the system trajectories in the phase plane for 3 different reference signal steps (left) and the corresponding output and control signal time responses.

$\gamma = -2; U_{min} = -1; U_{max} = 2; T_0 = 1$

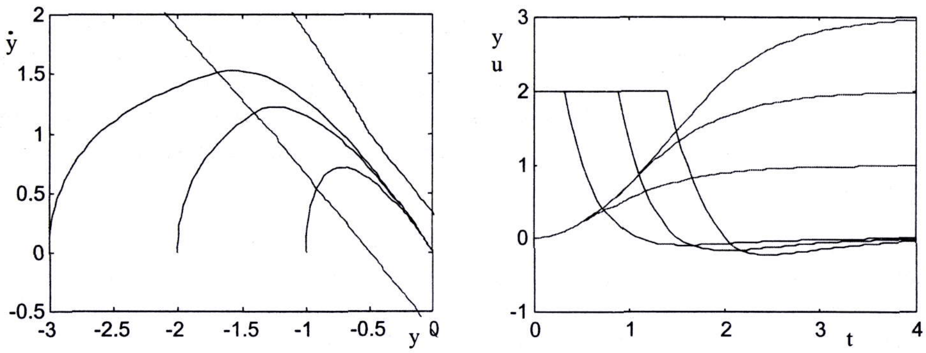


Fig. 4: MTPA controller for I_1T_1-system. Proportional band and the system trajectories in the phase plane for 3 different reference signal steps (left) and the corresponding output and control signal time responses.

$\gamma = -10; U_{min} = -1; U_{max} = 2; T_0 = 1$

2.2 Discrete MTPA Control Algorithms

While the continuous algorithms are given by simpler formulas, from the practical point of view it is easier to work with their discrete time counterparts. The reasons are given by the facts that the nonlinear control laws are much easier to be implemented by digital techniques than by the older analogous one. By an appropriate choice of the sampling period it is furthermore possible to achieve additional effects, e.g. to reduce the influence of the neglected dead time.

Before showing the more sophisticated discrete algorithm for the 2nd order integrator it is useful to show the basic idea on the example of the 1st order system represented by eq. 1 where the control signal limits eq. 2 are to be considered.

For such a simple integrator the discrete P-controller with the sampling period T can be designed

$$u(n) = -K_R y(n); \qquad K_R = (1-\lambda)/K_S T \tag{12}$$

where choosing parameter γ allows to assign the close loop pole from the value $\lambda \in \langle 0,1 \rangle$. This controller fulfills the condition of decreasing the output λ-time in each step.

$$y(n+1) = \lambda y(n) \tag{13}$$

Because of the control saturation the last condition can only be held for a limited range of initial points called proportional zone of control. For the points outside this zone the control law is given by the P-controller eq. 12 and consequently constrained by the limiter eq. 2. This means that the suboptimal control algorithm is created by the limit values of the control signal until the condition of the maximal decrease eq. 13 could be fulfilled when it produces values of P-controller.

For the discrete time control of the double integrator, the minimum time pole assignment controller have been proposed (Huba et al. 1998) guaranteeing a regular decrease of the representative point from a chosen reference braking curve. The quotient of such a decrease is specified by one of the closed loop poles. The 2nd pole is characterizing the distance decrease along the last segment of the reference braking curve to the required state. For a chosen double real pole λ, sampling period T, given control signal constraints eq. 2, y being the output and $d = dy/dt$ its derivative and the required state shifted to the origin, the anticipatory system is accomplished when designing the control algorithm by the following formulas

$$p = 2y + T\frac{1+\lambda}{1-\lambda}d$$

If (y < 0) $U_j = U_1$

else $U_j = U_2$

If $\quad \dfrac{p}{U_j T^2} > \dfrac{2\lambda}{(1-\lambda)^2}$

$$N = IP\left\{\frac{1-3\lambda}{2(1-\lambda)} + \sqrt{\frac{1-6\lambda+\lambda^2}{4(1-\lambda)^2} + \frac{p}{U_j T^2}}\right\} \tag{14}$$

else N = 1

Resultant controller corresponds to a piecewise linear PD-controller

$$u_k = sat\{r_0 y + r_1 d + r_c\} \tag{15}$$

with parameters depending on the integer parameter N

$$r_0 = -\frac{(1-\lambda)^2}{T^2[1+(N-1)(1-\lambda)]}$$
$$r_1 = -\frac{(1-\lambda)[3+\lambda+2(N-1)(1-\lambda)]}{2T[1+(N-1)(1-\lambda)]} \tag{16}$$
$$r_c = -\frac{(1-\lambda)[(N-1)(1+\lambda)+(N-1)^2(1-\lambda)]U_j}{2[1+(N-1)(1-\lambda)]}$$

3 Reduction of the Relative Degree by Defining a New Output

Let us start our explanation by taking the I_1T_1 system

$$\ddot{y} = \frac{1}{T_0}(u - \dot{y}) \tag{17}$$

After defining a new (fictive) output that includes the derivative of the original output

$$y_f = y + T_f\dot{y} \tag{18}$$

and expressing from the last equation

$$\ddot{y} = \frac{1}{T_f}(\dot{y}_f - \dot{y}) \tag{19}$$

for the new output the system equation will be

$$\dot{y}_f = \frac{T_f}{T_0}u + \dot{y}\left(1 - \frac{T_f}{T_0}\right) \tag{20}$$

This anticipatory system can be interpreted as the 1st order system with a "disturbance" caused by the velocity of the original system. It is interesting to note that for $T_f = T_0$ from the point of view of the new output the systems behaves like the 1st-order - single integrator system. So, it can be controlled by the P-controller eq.2 as

$$u = -K_R(y_f - w); \quad K_R = -\gamma T_0 / T_f \tag{21}$$

Transient processes consist of two phases:

- transient towards the required value w of the fictive output y_f and

- non-controlled dynamics described by the homogenous differential equation $0 = y + T_f \dot{y}$.

In order to achieve sufficiently fast dynamics of the final phase of control, T_f should be as small as possible. However, by decreasing T_f the effective gain $K_s = T_f / T_0$ of the fictive system decreases and the up to now "neglected" disturbance caused by the signal $\dot{y}$ is becoming on intensity, what can finally result in instability. So, in applying this approach to the control design of 2nd order systems by using the control algorithms derived for the 1st order systems, one has to look for optimal balancing of these two counteracting demands. After expressing the control algorithm in terms of the real output as

$$u = -K_R\left(y - w - T_f \dot{y}\right) \tag{22}$$

it is to see that the new algorithm is actually the PD-controller with the time constant of the derivative action T_f. The proportional band of this controller will be outlined by two lines

$$U_1 = -K_R\left(y - w - T_f \dot{y}\right);\; U_2 = -K_R\left(y - w - T_f \dot{y}\right) \tag{23}$$

Obviously, by choosing constant values K_R, T_f, it is not possible to approximate the proportional band of the above MTPA controller in a broader extent. So, the success of this approach is depending on the anticipation of the range of possible initial states that have to be considered by the designer.

These conclusions are yet much important in the case of controlling pure double integrator (I_2-system)

$$\ddot{y} = u \tag{24}$$

After defining the new (fictive) output eq. 18 and expressing the 2nd derivative according to eq. 19, for the new output the system equation will be

$$\dot{y}_f = T_f u + \dot{y} \tag{25}$$

Here, the output derivative acts as an outer disturbance of the fictive system for each value of T_f. The P-controller processing the fictive output

$$u = -K_R\left(y_f - w\right);\; K_R = -\gamma / T_f \tag{26}$$

variable leads finally again to the PD-controller given by eq.22 with the striplike proportional band eq. 23.

A decrease of T_f necessary for faster dynamics of the final phase of control decreases the effective gain of the fictive system and so makes it harder to eliminate the influence of the "outer disturbance". Again, for each initial condition, it is possible to find appropriate control loop parameters, but the dependence on the initial state is much stronger as in the previous case. The anticipation of the possible working area becomes now one important stage of the controller design.

4 Exact Linearization of Nonlinear Systems and Reduction of Relative Degree

The nonlinear system is given in the form

$$x^{(n)} = f(\mathbf{x}) + b(\mathbf{x})u \tag{27}$$

$$y = h(\mathrm{x})$$

After appropriate coordinate transformation it could be expressed in the controllability canonical form

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix} \mathrm{x}_1 \\ \vdots \\ \mathrm{x}_{n-1} \\ \mathrm{x}_n \end{bmatrix} = \begin{bmatrix} \mathrm{x}_2 \\ \vdots \\ \mathrm{x}_n \\ \mathrm{f}(\mathbf{x}) + \mathrm{b}(\mathbf{x})\mathrm{u} \end{bmatrix} \tag{28}$$

$$\mathrm{y} = \mathrm{x}_1$$

that has the relative degree equal to the order of the system. In the linear systems the relative degree expresses the difference between the order of the denominator and numerator of a transfer function. In fact, it corresponds to the number of derivation of the output until the input appears.

In the case the relative degree differs from the order of the system there exists the so called "zero dynamics" that is not influenced by the input and therefore it is desirable the zero dynamics to be stable. Choosing the dummy output in the form

$$y_a = x_1 + \sum_{i=1}^{a} T_i x_1^{(i)} \tag{29}$$

where $\mathrm{T_i} \in R^+$ (because of stable zero dynamics) it is possible to decrease the relative degree. Then it would be necessary to derive the new output y_a *(n-a)* times in order the input appears in the derivation. Thus the relative degree was reduced by the number a. The control value that cancels nonlinear behavior and introduces the desired closed loop poles can be expressed

$$u = \frac{1}{b(\mathrm{x})}\left(-f(\mathrm{x}) + \frac{\nu}{T_a}\right) \tag{30}$$

where ν represents the polynomial of x

$$\nu = -x_1^{(n-a)} - \sum_{i=n-a+1}^{n-1} T_{i-n+a} x_1^i - k_0(y_a - w) - k_1\dot{y}_a - \ldots - k_{n-a-1} y_a^{(n-a-1)} \tag{31}$$

with appropriate chosen coefficients k_i so that the polynomial

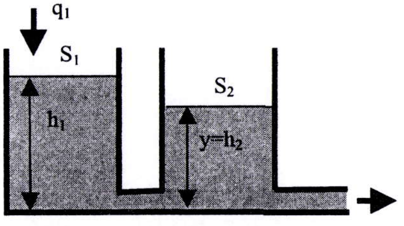
$$p(s) = s^n + k_{n-1}s^{(n-1)} + \ldots + k_0 \tag{32}$$

will have all roots in the left half plane with exponential stable dynamics.

This approach splits the control of the given dynamical systems into two phases. First, the fictive output y_a is controlled to the reference value and later the original output is damped out according to chosen time constants T_i.

5 Application of Designed Algorithm to Control of Two-Tank System

The quality of the designed control method was proved on the real two-tank system (Fig.5) that can be modeled by the following nonlinear differential equations:



$$\frac{dh_1}{dt} = \frac{1}{S_1} q_1 - \beta_1 \sqrt{h_1 - h_2}$$
$$\frac{dh_2}{dt} = \beta_2 \sqrt{h_1 - h_2} - \beta_3 \sqrt{h_2}$$
$$y = h_2$$

Fig. 5: Two tank system (33)

where h_1 and h_2 are state variables corresponding to the heights of liquid level in the first and second tank respectively and q_1 is the input representing the amount of liquid flowing into the first tank per time unit. Parameters β_i represent hydraulic resistance constants ($\beta_1 = \beta_2 = 0.02214$, $\beta_3 = 0.01163$) and parameters $S_{1,2}$ denote the areas of tanks (S_1=S_2=2.025 10^{-3} m^2).
The given nonlinear system can be transformed into the controllability canonical form

$$y = h_2 = z_1 \tag{34}$$

$$\dot{z}_1 = \dot{y} = \dot{h}_2 = \beta_2 \sqrt{h_1 - h_2} - \beta_3 \sqrt{h_2} = z_2 \tag{35}$$

$$\dot{z}_2 = \ddot{y} = \frac{dh_1}{dh_1}\frac{dh_1}{dt} + \frac{dh_2}{dh_2}\frac{dh_2}{dt} = a(h_1, h_2)q_1 + b(h_1, h_2) \tag{36}$$

whereby

$$a(h_1, h_2) = \frac{\beta_2}{2S_1\sqrt{h_1 - h_2}} \quad \text{and} \quad b(h_1, h_2) = -\beta_1^2 + \frac{\beta_2\beta_3}{2}\sqrt{\frac{h_2}{h_1 - h_2}} - \frac{\beta_2\beta_3}{2}\sqrt{\frac{h_1}{h_2} - 1} + \frac{\beta_3^2}{2}$$

Then the exact linearization method can be applied that linearizes the given nonlinear model and consequently the suboptimal control algorithm is used that gives the results shown in Fig.6:

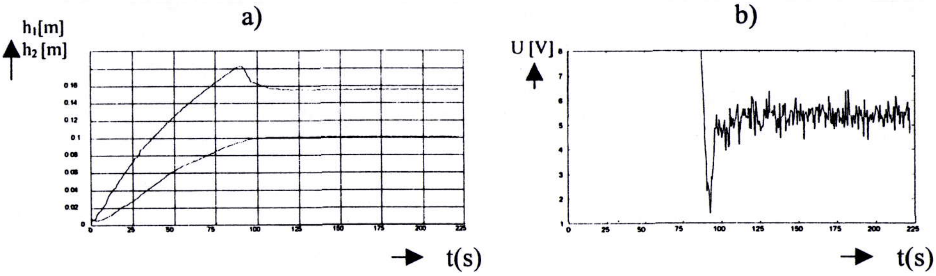


Fig. 6: Responses from the real experiment for $\lambda_{1,2} = 0.8$
a) output $y=h_1$ and the state h_2 b) control variable

These can be compared with those achieved using the relative degree reduction method. However, in this case it was not necessary to apply the more sophisticated suboptimal control algorithm derived the 2nd order systems, but this was replaced by the simple suboptimal PD-controller. A decrease of the controller gain leads to a softer control, but simultaneously increases the permanent control error and the settling time.

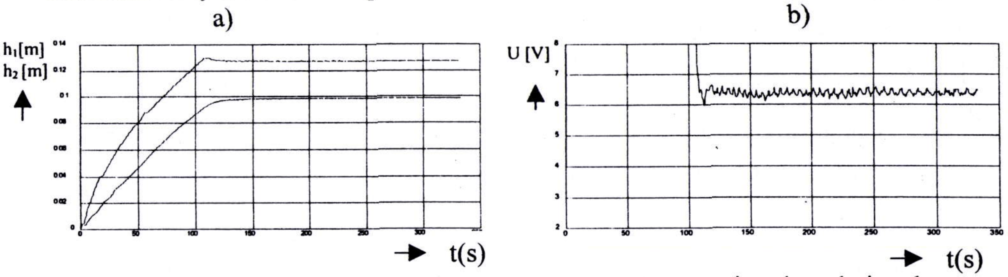


Fig. 7: Responses from the real experiment using the relative degree reduction method (parameters: $\lambda_{1,2} = 0.8$, $T_I=10$)
a) output $y=h_1$ and the state h_2 b) control variable

6 Pendulum Control

Problem of pendulum control can often be met e.g. in cargo crane control, but it is simultaneously frequently used for testing different control algorithms. In this paper, the attention will be focussed on the controller design respecting given control signal constraints.

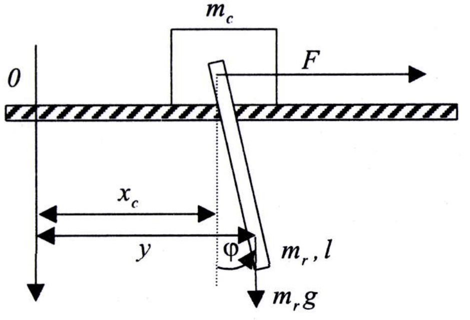


Fig.8: Pendulum

State variables:

x_c - cart position; φ - rod angle; F - constrained force ($\pm$ 50N)

Parameters:

$m_c = 21.8\ kg$ - cart mass; $m_r = 0.215\ kg$ - rod mass; $l = 0.165\ m$ - rod length

$g = 9.81\ ms^{-2}$

Under neglecting Coulomb and viscose friction the plant behaviour can be described as

$$\ddot{x}_c = \frac{F + m_r \sin\varphi (g\cos\varphi + l\dot{\varphi}^2)}{m_c + m_r \sin^2\varphi} \tag{37}$$

$$\ddot{\varphi} = \frac{-\ddot{x}_c \cos\varphi}{l} - \frac{g \sin\varphi}{l} \tag{38}$$

The task is to control the load position

$$y = x_c + l\sin\varphi \tag{39}$$

For a relatively small deviation φ the output equation can be linearized

$$y = x_c + l\varphi \tag{40}$$

After introducing state vector x and measurement vector y_m

$$x = \begin{bmatrix} x_c \\ \dot{x}_c \\ \varphi \\ \dot{\varphi} \end{bmatrix}; \qquad y_m = \begin{bmatrix} x_c \\ \varphi \end{bmatrix} \tag{41}$$

in the neighbourhood of a chosen operating point $x_p=0$ the system can be described by linear state equations

$$\dot{x} = Ax + bu$$

$$y_m = Cx, \qquad y = [1 \quad l]y_m = c^T x; \quad c^T = [1 \quad 0 \quad l \quad 0] \tag{42}$$

whereby

$$A=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & \dfrac{m_r}{m_c}g & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -\dfrac{m_c+m_r}{m_c}\dfrac{g}{l} & 0\end{bmatrix};\qquad b=\begin{bmatrix}0\\ \dfrac{1}{m_c}\\ 0\\ -\dfrac{1}{m_c l}\end{bmatrix};\qquad C=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$$

The transfer function corresponding to the defined output y is

$$G_{y/u}(s)=\frac{K}{s^2(s^2+\omega^2)};\qquad K=\frac{g}{lm_c};\qquad \omega^2=\frac{m_c+m_r}{m_c}\frac{g}{l} \tag{43}$$

It may be transformed to the form

$$G_{y/u}(s)=\frac{K_S}{s^2(T_0^2 s^2+1)};\qquad K_S=\frac{K}{\omega^2}=\frac{1}{m_c+m_r}:\qquad T_0^2=\frac{1}{\omega^2} \tag{44}$$

Since for a 4th order system a requirement of the strictly minimum time control leads to very sophisticate algorithms, it will be shown how the design can be based on the minimum time pole assignment controller (see e.g. Huba, 1998) and an appropriate relative degree reduction.

Introducing a fictive output defined as

$$y_f=y+(T_1+T_2)\dot{y}+T_1T_2\ddot{y} \tag{45}$$

the corresponding transfer function is

$$G_{y_f/u}(s)=\frac{K_S(1+(T_1+T_2)s+T_1T_2s^2)}{s^2(T_0^2s^2+1)} \tag{46}$$

For $t\to 0$, when $s\to\infty$, the step response of y_f can be approximated by a double integrator with the gain $\dfrac{K_S T_1 T_2}{T_0^2}$, since the approximation error e(t) approaches zero:

$$e(s)=\lim_{s\to\infty}\left[\frac{K_S T_1 T_2}{T_0^2 s^2}-\frac{K_S s^2\left(\dfrac{1}{s^2}+\dfrac{T_1+T_2}{s}+T_1T_2\right)}{s^4\left(T_0^2+\dfrac{1}{s^2}\right)}\right]=0 \tag{47}$$

However, with increasing time this approximation becomes less precise, therefore, under sampled data control, it can be used just for relatively short sampling periods. In order to avoid multiple differentiation procedures, following signals will be computed from a measured signal y_m:

$$\begin{aligned}y&=c^T x\\ \dot{y}&=c^T\dot{x}=c^T Ax\\ \ddot{y}&=c^T\ddot{x}=c^T A^2x\\ \dddot{y}&=c^T\dddot{x}=c^T A^3x\end{aligned} \tag{48}$$

These derivatives are then substituted into the fictive state variables y_f and $\dot{y}_f$, which are finally led to the MTPA-controller designed for the double integrator.
Extensive simulation and experimental work has been done in order to investigate dependence of the control quality from the (double) closed loop pole λ, sampling period T and the time constants T_1 a T_2. Simulations carried out in MATLAB/Simulink give results very close to real experiments. They show that it is necessary to work with relatively short sampling periods (in the range of 10ms) and with the time constants $T_1, T_2 = 2T_0$. The value of the closed loop pole λ has to be taken greater than 0.8, in order to compensate parasitic time lags and also to compensate the parasitic influence of the signal quantisation. Oscillation noticeable at the settling of the transient responses are mostly due to the quantisation of the rod angle (0.0015 rad), where it could be appropriate to use IRC with a higher resolution. Examples of the achieved transients are shown in following figures:

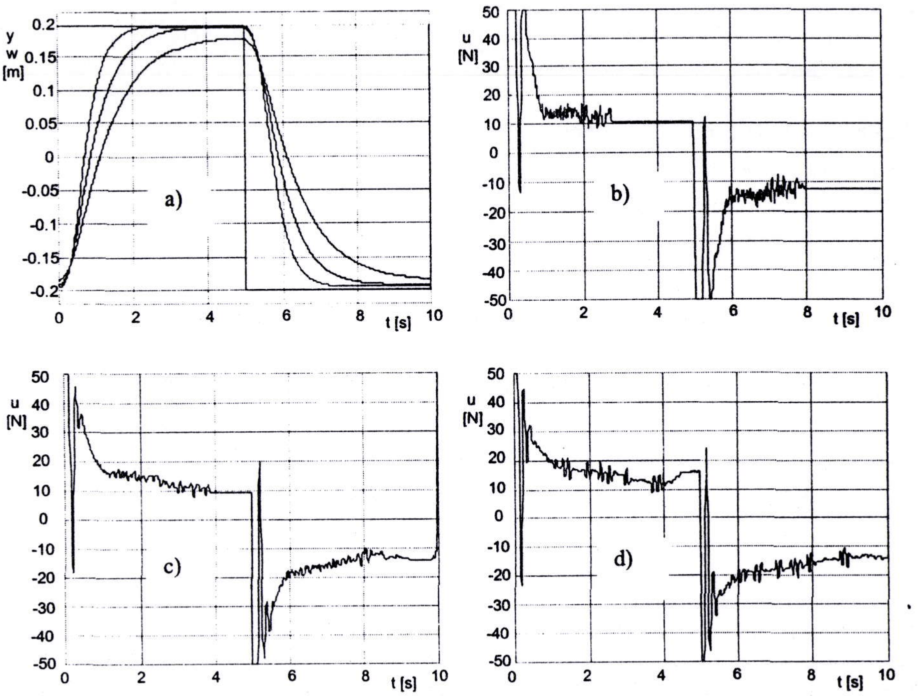


Fig. 9: Pendulum control. The output transients are increasingly damped for increasing the time constants $T_{1,2}=2T_0$, $3T_0$, $4T_0$ (a) and the corresponding control sequences (b-d)

7 Conclusion

From the comparison of the control responses in the case of the two tank system one can see that when limited to a broader set of initial conditions one can also use simpler suboptimal controller based on lower order systems. The disadvantage of this method can be the absence of a procedure how to tune the parameters T_i. Also the model of the system must be identified more accurately in order to avoid a permanent error. But adding I-action into the control law could solve this problem.

Similar conclusions can also be done in controlling pendulum. In both cases the carried out experiments and simulations shown good correspondence between the used model and the real plant and also that between the achieved and expected results what was due to the respecting the saturation limits. So, the control signal saturation represents one of the most important components of the complex anticipatory systems. The dominant factor in the controller design showed to be the quantisation and measurement noise, whose effect should be taken also in the final decision in looking for an appropriate controller.

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