Theoretical Computer Simulation Studies of Group Bias in Random Populations

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Abstract
A mathematical model for a random population has been developed and implemented in computer simulations of group bias within populations. For uncorrelated categorical attribution and bias, although the average bias characterizing different populations may differ dramatically, groups within a given population are found to have remarkably similar biases. For correlated categorical attribution and bias, the tendency of the groups within a given population to have similar biases is reduced. That is, groups within a population now exhibit a greater range of biases. This latter model, which is based upon assumptions that are more realistic, thus leads to results that are more realistic.
Keywords: bias, prejudice, categorization, computer simulation, random populations.

1 Introduction

The Nature of Prejudice (Allport, 1954) is a classical work describing prejudice both phenomenologically and analytically. This work is sufficiently insightful and comprehensive to provide a basis for developing a mathematical model of prejudice. One of Allport's fundamental concepts is that of category.

In this manuscript, a mathematical model will be developed which incorporates these features of categorization and prejudice. Although this model is described in detail in the next section, a brief informal description is given here. It is assumed that there are a certain number of categories. Each category is either attributed to a given individual, or else another category, called the negation (or complement) of the original category, is attributed to the individual. The original category and the negation are of equal status; either could be designated as "the category" and the other as "the negation."

It may be difficult at first to understand the difference between the concepts of "categorical attribution" and "group." A mathematical analogy may clarify this distinction: The number "2" has the "categorical attribution" of being even. We may define the set (or "group") of all even numbers, and "2" is obviously a member of this set. However, even if the concept of set had never entered into the human imagination, "2" would still have the "categorical attribution" of being even. In his discussion of the *axiom schema of separation*, Suppes (1972, p. 6) explains that this axiom " . . . permits us to separate off the elements of a given set which satisfy some property and form the [sub]set consisting of just these elements. . ." In our case, we want to separate off the elements (individuals) of a given set (population) which satisfy some property (a particular categorical attribution) and form the subset (group) consisting of just these elements.

The real-world context of this theoretical model is exemplified at one extreme by members of one group trying to help members of another group, while at the other extreme, members of a group may commit genocide upon members of another group.

An expanded version of this manuscript is available at http://www.science.gmu.edu/~bzeeberg/CASYS2000/frames.html

2 The Model

2.1 Abstract Definition

A *population* consists of a number of *individuals*. Individuals are assumed to be living sentient humans. A *category* is the abstract product of the collective minds of the individuals within the population. To paraphrase Georg Cantor (Kamke, 1952, p. 1), it is a definite, well-distinguished object of our perception or of our thought. A category has three essential properties: (1) the *negation* of a given category is attributed to each individual to whom the given category is not attributed, (2) any individual can recognize the attribution of categories for any individual, and (3) a given category is associated with a *categorical bias* toward either another or the same category.

For example, consider a population consisting of four individuals (Ind 1, Ind 2, Ind 3, and Ind 4) with four categories (Cat 1, Cat 2, Cat ~1, Cat ~2), the last two being the negation of the first two (Table 1). If Cat 1 is "female," then Cat ~1 is "male." The number of categories will be symbolized by "N_CAT," which includes both the categories and their negations, so that in this example N_CAT = 4.

Table 1: Categories Attributed to Individuals

	Cat 1	Cat 2	Cat ~2	Cat ~1
Ind 1	1	1	0	0
Ind 2	1	0	1	0
Ind 3	0	1	0	1
Ind 4	0	0	1	1

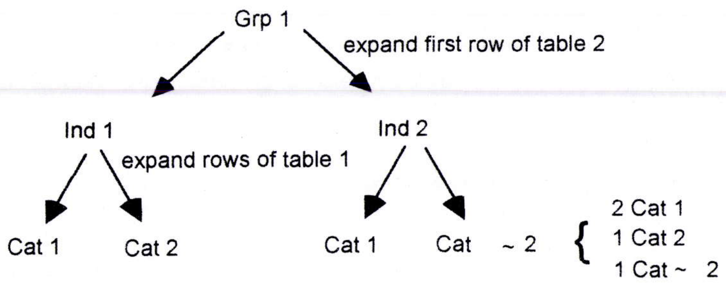
A *group* consists of those individuals to whom is attributed a given category. There are four groups (one for each category) resulting from this arrangement (Table 2). Each group name corresponds to the name of the category defining that group. For example, Grp 1 consists of those individuals to whom are attributed Cat 1. Thus, the category is an attribute, and the group is the set of individuals having that attribute.

There are three possible relationships between groups: (1) an *identity* relationship is exemplified by "Cat 1" in relationship to "Cat 1", (2) a *mutually exclusive* relationship is exemplified by "Cat 1" in relationship to "Cat ~1," and (3) a *general* relationship is any relationship that is neither identity nor mutually exclusive, exemplified by "Cat 1" in relationship to "Cat 2." Each individual in Table 2 is associated with two categories. Thus,

Table 2: Groups Corresponding to Table 1

	Ind 1	Ind 2	Ind 3	Ind 4
Grp 1	1	1	0	0
Grp 2	1	0	1	0
Grp ~2	0	1	0	1
Grp ~1	0	0	1	1

the first row in Table 2 corresponding to Grp 1 could be expanded as



and Table 2 can be reformulated in terms of the categorical composition of each group (Table 3). Note that the matrix associated with Table 3 is, of necessity, a symmetrical matrix. Each entry for a given group in Table 3 can be normalized (by dividing each row entry by the row sum) so as to reflect its fractional contribution to the group (Table 4). This process of normalization, in general, will remove the symmetry (although in this particular example the symmetry happens to remain).

Table 3: Categorical Composition of Groups

	Cat 1	Cat 2	Cat ~2	Cat ~1
Grp 1	2	1	1	0
Grp 2	1	2	0	1
Grp ~2	1	0	2	1
Grp ~1	0	1	1	2

Table 4: Fractional Categorical Composition of Groups

	Cat 1	Cat 2	Cat ~2	Cat~1
Grp 1	.50	.25	.25	0
Grp 2	.25	.50	0	.25
Grp ~2	.25	0	.50	.25
Grp ~1	0	.25	.25	.50

Finally, the *group bias* of the perceiver group toward the perceived group would be computed as the weighted average of the categorical biases. It should be emphasized that the group bias is not necessarily the same as the bias that a particular member of the group would exhibit; rather, it is the ensemble average bias for the group as a whole. Any two members of a group may very well exhibit biases that are different from the ensemble average and from each other.

2.2 Numerical Example

Suppose now that the categorical biases (including the net effect of several factors, such as visibility, strangeness, and oversimplification (Allport, 1954, pp. 129-136) happen to be as shown in Table 5.

To complete the hypothetical illustrative example set up in Tables 1 - 5, we define fract(i,k) as the entry in the i^{th} row and k^{th} column of Table 4 and cat bias(k,l) as the entry in the k^{th} row and l^{th} column of Table 5.

Table 5: Categorical Biases

	PERCEIVED			
PERCEIVER	Cat 1	Cat 2	Cat ~2	Cat ~1
Cat 1	+.12	-.28	-.14	+.02
Cat 2	-.13	-.34	+.21	-.07
Cat ~2	-.45	+.39	+.03	+.21
Cat ~1	-.11	-.27	+.09	-.35

Then the group bias for perceiver group 1 towards perceived group 2 can be calculated from Tables 4 and 5 as

group bias(1,2) = fract(1,1) x
[fract(2,1) x cat bias(1,1) + fract(2,2) x cat bias(1,2)
+ fract(2,3) x cat bias(1,3) + fract(2,4) x cat bias(1,4)]
+ fract(1,2) x
[fract(2,1) x cat bias(2,1)+ fract(2,2) x cat bias(2,2)
+ fract(2,3) x cat bias(2,3)+ fract(2,4) x cat bias(2,4)]
+ fract(1,3) x
[fract(2,1) x cat bias(3,1) + fract(2,2) x cat bias(3,2)
+ fract(2,3) x cat bias(3,3) + fract(2,4) x cat bias(3,4)]
+ fract(1,4) x
[fract(2,1) x cat bias(4,1) + fract(2,2) x cat bias(4,2)
+ fract(2,3) x cat bias(4,3) + fract(2,4) x cat bias(4,4)]
= .50 x [.25 x (+.12) + .50 x (-.28) + 0 x (-.14) + .25 x (+.02)]
+ .25 x [.25 x (-.13) + .50 x (-.34) + 0 x (+.21) + .25 x (-.07)]
+ .25 x [.25 x (-.45) + .50 x (+.39) + 0 x (+.03) + .25 x (+.21)]
+ 0 x [.25 x (-.11) + .50 x (-.27) + 0 x (+.09) + .25 x (-.35)]
= -.07375.

This represents a relatively small negative bias, since if all the entries in Table 5 had been the maximum value of +0.5, then group bias(1,2) would have been +0.5.

2.3 Formal Mathematical Development

In a mathematically formal sense, the group bias for perceiver group i toward perceived group j is computed as the weighted average of the categorical biases

$$\text{group bias}(i,j) = \sum \text{fract}(i,k) \sum \text{fract}(j,l)\ \text{cat bias}(k,l) \tag{1}$$

Eq. 1 can be conveniently expressed in a matrix representation as

$$G = FCF^T \tag{2}$$

where G, F, and C are matrices representing group bias, fractional categorical composition of groups, and categorical bias, respectively. Since the number of groups is equal to the number of categories, all the matrices involved in eq. 2 are square matrices of the same size. Completing the numerical example by applying eq. 2 to the matrices given in Tables 4 and 5 results in numerical values for the matrix G (Table 6). Note that $g_{1,2}$ is equal to the numerical value of -.07375 that had been computed above for group bias(1,2).

Table 6: Group Biases

	PERCEIVED			
	Grp 1	Grp 2	Grp ~2	Grp ~1
PERCEIVER				
Grp 1	-0.07688	-0.07375	-0.01500	-0.01188
Grp 2	-0.08500	-0.19875	+0.00125	-0.11250
Grp ~2	-0.09625	-0.02125	-0.04875	+0.02625
Grp ~1	-0.10438	-0.14625	-0.03250	-0.07438

The matrix representation G (when arranged as in Table 6) is particularly convenient, since the diagonal terms running from upper left to lower right ($g_{i,i}$) constitute the identity relationship, the diagonal terms running from upper right to lower left ($g_{i,-i}$) constitute the mutually exclusive relationship, and all other terms constitute the general relationship.

2.4 Inversion of $G = FCF^T$

The matrix F (eq. 2) can be expressed as

$$F = DS \tag{3}$$

where S is a symmetrical matrix that corresponds to Table 3, and D is a diagonal matrix with terms

$$d_{i,i} = 1/\sum s_{i,j} \tag{4}$$

Thus, eq. 2 can be written as

$$G = DSC(DS)^T = DSCS^TD^T = DSCSD \tag{5}$$

Since S is a real symmetrical matrix, by the Fundamental Theorem on Symmetric Matrices (Fiedler, 1986, p. 50), S can be expressed in the form

$$S = OD'O^T \tag{6}$$

where D' is a real diagonal matrix whose diagonal entries are eigenvalues of S, and O is an orthogonal matrix, whose k^{th} column is the eigenvector of S corresponding to the k^{th} diagonal entry of D'. Therefore, eq. 5 can be written as

$$G = DSCSD = D(OD'O^{T})C(OD'O^{T})D \tag{7}$$

By the definition of an orthogonal matrix

$$O^{T} = O^{-1} \tag{8}$$

and eq. 7 can be written as

$$G = D(OD'O^{-1})C(OD'O^{-1})D \tag{9}$$

Finally, eq. 9 can be inverted as

$$C = (OD'^{-1}O^{-1}D^{-1})G(OD'^{-1}O^{-1}D^{-1})^{T} \tag{10}$$

The significance of eq. 10 is that the categorical bias C, which might otherwise have been regarded as a mathematical abstraction, can in principle be calculated from the empirically determinable matrices G and F.

2.5 Random Populations

A *random population* consists of the two random matrices F and C corresponding, respectively, to Table 4 "Fractional Categorical Composition of Groups" and Table 5 "Categorical Biases." The underlying concept is based upon computer simulation studies of random fitness contributions of genetic loci reported in *The Origins of Order* (Kauffman, 1993, pp. 42-44). The general procedure is to generate a random population and determine the numerical values corresponding to certain statistical properties of that particular population. The whole procedure is repeated for a large number of random populations. Finally, the accumulated set of numerical values corresponding to the entire sequence of random populations is evaluated using appropriate statistical analyses. In this way, an "average" behavior for a random population can be estimated.

2.6 Overview of the Simulation Studies Performed

In the initial simulation studies, it was assumed that a given individual's categorical attributions were not statistically correlated. That is, for a given individual, the probability of attribution of category i was independent of attribution of category j for $j \neq \pm i$. Although this is not a particularly realistic assumption, the purpose of this assumption was to set a baseline behavior before introducing the effect of correlation of categorical attributions. In fact, one would expect that some categorical attributions might be correlated. For example, there may be data to support the speculation that differences in crime rates are statistically significant for different racial, sexual, and age categories. That is, attribution of a category "high crime rate" may not be statistically independent of attribution of the categories "race x," "sex y," and "age z."

Similarly, in the initial simulation studies, it was assumed that categorical biases were statistically independent. Again, although this is not a particularly realistic assumption, the purpose of this assumption was to set a baseline behavior before introducing the effect of correlation of categorical biases. In fact, one would expect that some categorical biases might be correlated. For example, someone in the category "nonsmoker" may have a stronger negative bias towards someone in the categories "smoker" and "under 20 years old" than towards someone in the categories "smoker" and "over 20 years old."

After these initial studies, the more realistic situation of correlated categorical attributions and correlated categorical biases were studied. In addition to evaluating the validity of Allport's assertion that every known difference between human groups fits into one of the four types, the results of these simulation studies were used to determine whether there are any statistical patterns of group prejudice occurring within random populations, and whether these patterns depend upon the assumptions about the degree of correlation of categorical attributions and the degree of correlation of categorical biases.

3 Methods

3.1 Dichotomous Attributes of Individuals

Fifty or one hundred different dichotomous attributes of individuals were assumed, which results in one hundred or two hundred categories, respectively, when the negations of the categories are taken into account.

3.2 Computer Implementation and Random Number Generator

Computer simulations using random variables were performed by implementing the model described above with code written in the C language (Kernighan and Ritchie, 1988). The random number generator ran2() provides perfect random numbers within the limits of its floating-point accuracy (Press, Teukolsky, Vetterling, and Flannery, 1992).

3.3 Uncorrelated Categorical Attributions

Uncorrelated categorical attributions were assigned for an individual by repeatedly allowing ran2() to generate a uniformly distributed random number between 0 and 1, once for each pair category/negation of category. If the random number was less than 0.5, then the category was attributed; otherwise, if the random number was greater than or equal to 0.5, then the negation of the category was attributed. This procedure was equivalent to the assumption of a fixed 50% chance of attribution of a category, and was repeated for each individual in the population. The result would be to generate a table which was a larger version of Table 1. As described in the example given above, the equivalent of Table 4 (that is, the matrix F) was calculated from this version of Table 1.

3.4 Uncorrelated Categorical Biases

Uncorrelated categorical biases were assigned for a perceiver category by repeatedly allowing ran2() to generate a uniformly distributed random number between 0 and 1, once for each perceived category. The numerical value of 0.5 was subtracted from each random number, so that the categorical biases ranged from -0.5 to +0.5. This resulted in a table which was a larger version of Table 5 (that is, the matrix C). Eq. 1 was used to calculate the group biases for the mutually exclusive relationship (that is, the elements of the matrix G corresponding to $g_{i,-i}$).

This entire procedure described in the preceding two paragraphs was then repeated to calculate the group biases for the mutually exclusive relationship for new random populations until it was determined that the statistical conclusions would not change significantly with additional populations.

3.5 Correlated Categorical Attributions

In further simulation studies, it was assumed that categorical attributions could be correlated. First, in order to dispose with the assumption of a fixed 50% chance of attribution of Cat 1, a single random number referred to as "P(1)" was generated from uniformly distributed random numbers between 0 and 1. P(1) remained constant for all of the individuals of the current population. That is, P(1) was a fixed characteristic of the entire population. For the population as a whole, P(1) can be interpreted as the probability of attribution of Cat 1. It follows that P(~1) = the probability of attribution of Cat ~1 = 1 - P(1).

For each individual in the population, attribution of Cat 1 was determined by generating a random number from a uniform distribution of random numbers between 0 and 1. If this random number for a given individual was less than P(1), then Cat 1 was attributed to this individual. For example, assume that P(1) = 0.87 for the population under consideration. Further assume that the random numbers for Ind 1 and Ind 2, respectively, were 0.52 and 0.92. Then Cat 1 would be attributed to Ind 1 (since 0.52 < P(1)), and Cat ~1 would be attributed to Ind 2 (since 0.92 > P(1)).

Next, Cat 1 was arbitrarily assumed to be the "independent" variable, and conditional probabilities (that is, conditional upon attribution of Cat 1 or Cat ~1) for all the other pairs of categories/negations of categories were randomly assigned. For example, for category i, the conditional probabilities P(attribution of Cat i given attribution of Cat 1) and P(attribution of Cat i given attribution of Cat ~1) were generated from uniformly distributed random numbers between 0 and 1. This assignment of conditional probabilities was kept constant for all the individuals of the current population. That is, this set of conditional probabilities was a fixed characteristic of the entire population.

Finally, for each individual in the population, and for each category (other than Cat 1 or Cat ~1), categorical attribution was determined by generating a random number from a uniform distribution of random numbers between 0 and 1. If Cat 1 had been attributed to the individual, then this random number was compared with the conditional probability P(attribution of Cat i given attribution of Cat 1). If the random number was less than or equal to P(attribution of Cat i given attribution of Cat 1), then Cat i was attributed to the individual; otherwise, Cat ~i was attributed to the individual. Similarly, if Cat ~1 had been attributed to the individual, then the random number was compared with the conditional probability P(attribution of Cat i given attribution of Cat ~1). If the random number was less than or equal to P(attribution of Cat i given attribution of Cat ~1), then Cat i was attributed to the individual; otherwise, Cat ~i was attributed to the individual.

For the population as a whole, the probability of attribution of Cat i can be designated as "P(i)." A simple expression for P(i) is given by

$$P(i) = P(1)\,P(\text{attribution of Cat i given attribution of Cat 1}) + P(\sim 1)\,P(\text{attribution of Cat i given attribution of Cat} \sim 1) \quad (11)$$

3.6 Correlated Categorical Biases

In addition to assuming that categorical attributions were correlated, in some simulation studies it was also assumed that the categorical biases were correlated. Categorical biases were generated as in **Uncorrelated Categorical Biases** and modified by adding α x P(attribution of Cat i given attribution of Cat 1), where P(attribution of Cat i given attribution of Cat 1) had already been generated as described in **Correlated Categorical Attributions**, and α is a user-defined deterministic value that controlled the magnitude of the effect of this modification. The set of conditional probabilities was kept constant for all the individuals of the current population. That is, this set of conditional probabilities was a fixed characteristic of the entire population.

4 Results and Discussion

4.1 Uncorrelated Categorical Attributions and Uncorrelated Categorical Biases

In the initial simulation studies, it was assumed that a given individual's categorical attributions were not statistically correlated with each other. That is, for a given individual, the probability of attribution of category i was independent of attribution of category j to that individual, for all $i \neq \pm j$. Similarly, it was assumed that categorical biases were statistically independent. Although these are not particularly realistic assumptions, their purpose was to set a baseline behavior before introducing the effect of correlation of categorical attributions and biases.

Table 7: Statistical Results for Four Population Structures in the Absence or Presence of Correlations

CORRELATIONS	no	no	no	no	yes
N_IND x 1000	5	5	10	10	10
N_CAT	100	200	100	200	200
N_POP	113	104	200	390	123
MEAN OF POP MEANS x 10^{-4}*	-3.30	-0.65	2.79	1.17	0.95
STD OF POP MEANS x 10^{-4}	29.0	15.0	31.2	14.6	17.2
MEAN OF POP STDS x 10^{-4}†	5.81	2.07	5.74	2.06	4.50
STD OF POP STDS x 10^{-4}	0.57	0.15	0.57	0.15	2.00
STD OF POP MEANS /MEAN OF POP STDS	5.00	7.25	5.43	7.10	3.85

*POP MEAN of a population is computed as the average mutually-exclusive group bias over the N_CAT groups within the population. MEAN OF POP MEANS is computed as the average of the N_POP POP MEANS. STD OF POP MEANS is computed as the standard deviation of the N_POP POP MEANS.

†POP STD of a population is computed as the standard deviation of the mutually-exclusive group bias over the N_CAT groups within the population. MEAN OF POP STDS is computed as the average of the N_POP POP STDS. STD OF POP STDS is computed as the standard deviation of the N_POP POP STDS.

As shown in Table 7, there were four sets of simulation studies done in the absence of correlations. These four sets represent different combinations of the number of individuals (N_IND) and the number of categories (N_CAT). Although the number of populations (N_POP) studied differed for each of the four sets, the number of populations studied was more than sufficient to ensure that the statistical conclusions did not change significantly with additional populations.

4.2 A Typical Result

A typical result is given in Fig. 1. The dashed line represents the pooled mutually exclusive group biases for 390 random populations, corresponding to column 4 of Table 7. The solid lines represent the mutually exclusive group biases for 2 random populations selected from these 390 random populations. Fig. 1 is intended to convey intuitively the finding that, although there is substantial opportunity for a given population's average feeling to deviate either positively or negatively from neutrality, groups within a given population tend to feel fairly similarly. Although the population means vary considerably, the population standard deviations are small and fairly constant (Fig. 2). These results are tabulated quantitatively in the fourth column of Table 7. For example, the mean of the population means ($1.17 \times 10^{-4} \pm 14.6 \times 10^{-4}$) was statistically indistinguishable from 0, as is apparent from the peak and the spread of the dashed line in Fig. 2, whereas the mean of the population standard deviations ($2.06 \times 10^{-4} \pm 0.15 \times 10^{-4}$) was small but statistically distinguishable from 0. The ratio (standard deviation of the population means)/(mean of the population standard deviations), equal to 7.10 (Table 7), provides a single numerical value that characterizes the narrow spread of the group biases within a population relative to the wide spread of the population means. Again, these results show that, although there is substantial opportunity for a given population's average feeling to deviate either positively or negatively from neutrality, within a given population, groups tend to feel fairly similarly.

4.3 Comparison of Four Different Underlying Population Structures

The statistical results of simulations utilizing four different underlying population structures (first four columns of Table 7) indicate that this behavior occurs generally for various population structures. The mean of the population means fluctuate in a statistically insignificant manner, whereas the mean of the population standard deviations were small but statistically distinguishable from 0.

Furthermore, certain trends could be discerned. For example, comparison of columns 1 and 3 (or comparison of columns 2 and 4) indicates that the standard deviation of the population means, the mean of the population standard deviations, the standard deviation of the population standard deviations, and the ratio (standard deviation of the population means)/(mean of the population standard deviations) were independent of the number of individuals in the population. On the other hand, comparison of columns 1 and 2 (or comparison of columns 3 and 4) indicates that the standard deviation of the population means and the mean of the population standard deviations vary inversely in a nearly linear manner with the number of categories. The standard deviation of the population standard deviations varies inversely in a nonlinear manner, and the ratio (standard deviation of the

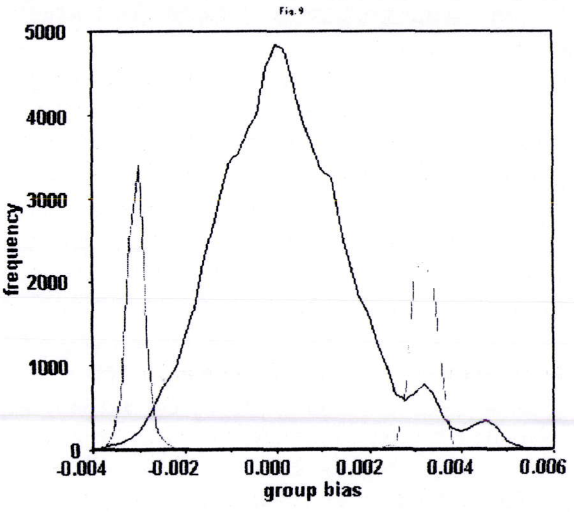


Fig. 1: Histograms of group bias for two representative random populations (solid lines), and of pooled group biases from a total of 390 random populations (dashed line), corresponding to the data reported in the fourth column of Table 7. The frequencies for the two representative random populations are multiplied by a factor of 40 for display purposes.

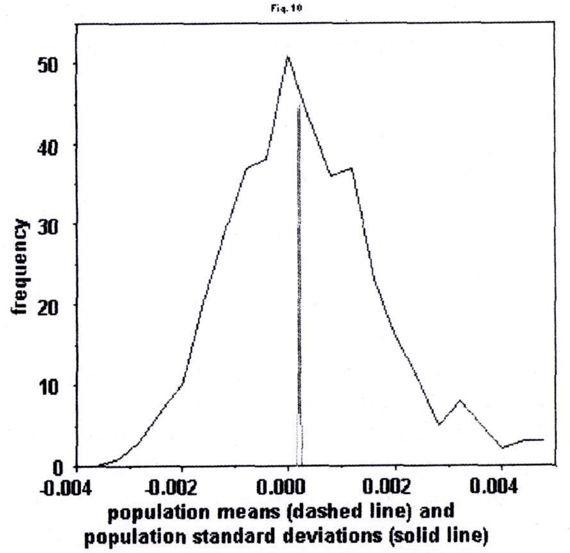


Fig. 2: Histograms of the population means (dashed line) and population standard deviations (solid line), corresponding to the data reported in the fourth column of Table 7.

population means)/(mean of the population standard deviations) varies in a nonlinear manner with the number of categories.

We would expect the mean of the population means to be close to neutrality, since there is no reason to prefer positive over negative or negative over positive deviations in a set of random populations. However, the large standard deviation of the population means gives rise to the opportunity noted above for a particular population to exhibit either a substantial positive or negative deviation from neutrality. The population standard deviations are very similar from one population to the next, and are small relative to the standard deviation of the population means. These properties account for the observation noted above that, within a given population, groups tend to feel fairly similarly.

4.4 Randomness Has Given Rise to a High Degree of Order

These results are somewhat surprising, since the random numbers constituting the random populations are uncorrelated, and yet the mutually exclusive group biases within a given population are highly correlated (that is, the standard deviations for the groups within a given population are small relative to the standard deviation of the population means for a set of populations). Stated slightly differently, randomness has given rise to a high degree of order.

The significance of these findings is that a population consists of groups that all mutually love each other, that all mutually hate each other, or that all mutually feel neutral towards each other. Thus, intergroup feelings would be fairly consistent within a given population. It is purely a matter of random chance as to whether a population mean corresponding to love or to hate happens to govern the particular population.

4.5 Comparison between Correlated and Uncorrelated Categorical Attributions

The assumptions that a given individual's categorical attributions were not statistically correlated and that categorical biases were statistically independent are not realistic assumptions. Therefore, in the remaining simulation studies the more realistic assumption was made that categorical attributions might be correlated (although for now the categorical biases were still assumed to be uncorrelated). To illustrate using a hypothetical example, suppose that Cat 1 is attributed to Ind 1, but not to Ind 2. Then the probability that Cat 2 is attributed to Ind 1 may be, for example, 0.80, but the probability that Cat 2 is attributed to Ind 2 may be only 0.30. Thus, throughout the entire population, there will be a positive correlation between attributions of Cat 1 and Cat 2.

The results of these simulation studies (Fig. 3) are qualitatively similar to those for uncorrelated categorical attributions (Fig. 1). The dashed lines represent the pooled mutually exclusive group biases for 123 random populations, corresponding to the last column of Table 7. The solid lines represent the mutually-exclusive group biases for 2 random populations selected from these 123 random populations. Fig. 3 is intended to convey intuitively the finding that although there is substantial opportunity for a given population's average feeling to deviate either positively or negatively from neutrality,

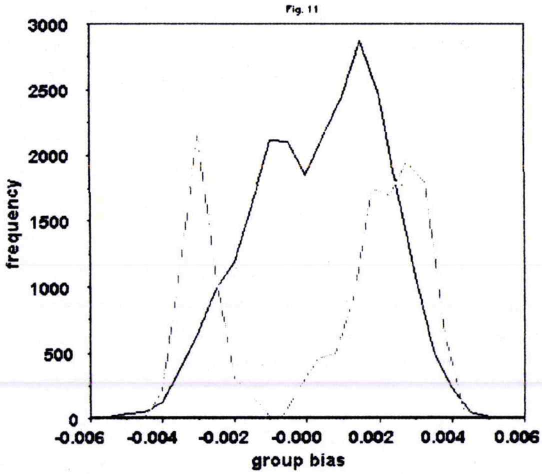


Fig. 3: Histograms of group bias for two representative random populations (solid lines), and of pooled group biases from a total of 123 random populations (dashed line), corresponding to the data reported in the last column of Table 7. The frequencies for the two representative random populations are multiplied by factors of 25 and 50 for display purposes.

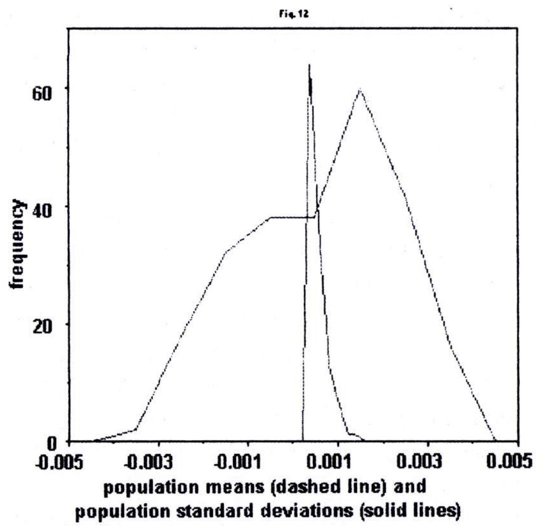


Fig. 4: Histograms of the population means (dashed line) and population standard deviations (solid line), corresponding to the data reported in the last column of Table 7. The frequencies for the population means are multiplied by a factor of 2 for display purposes.

within a given population, groups tend to feel fairly similarly, but not quite as similarly as for uncorrelated categorical attribution (Fig. 1). Although the population means vary considerably, the population standard deviations are small and fairly constant (Fig. 4), although somewhat larger than for uncorrelated categorical attribution (Fig. 2).

Quantitative comparison with the results of a similar population structure with uncorrelated categorical attributions (column 4 of Table 7) and the population with correlated categorical attributions (last column of Table 7) indicates the following significant differences: The mean and the standard deviation of the population standard deviations are greater for the correlated case, and the ratio (standard deviation of the population means)/(mean of the population standard deviations) is lower for the correlated case. Thus there is an apparent paradox that the effect of increasing the degree of correlation of the categorical attribution is to diminish the correlation of the group biases within a population.

4.6 Correlated Categorical Attributions and Correlated Categorical Biases

In addition to assuming that categorical attributions could be correlated, in some simulation studies it was also assumed that the categorical biases depended upon the conditional probability P(attribution of Cat i given attribution of Cat 1). That is, the higher this conditional probability for a given category, the higher the range of random values for bias towards that category. Thus, the biases towards a category tended to a higher positive value if that category was highly correlated with Cat 1. Recall that it is arbitrary as to whether a high positive value represents love or hatred. The degree of shifting of the range of random values was governed by the user-selected deterministic value for α. This shifting would permit an intensification of the bias towards a group containing many individuals to whom are attributed Cat 1 and the categories which are highly correlated with Cat 1. The mean of the population means was linear with α, whereas the standard deviation of the population means, mean of the population standard deviations, and standard deviation of the population standard deviations are linear only for high values of α, but nonlinear for low values of α. The ratio (standard deviation of the population means)/(mean of the population standard deviations) appears to approach 1.5 for very high values of α.

For $\alpha = 0.00$, half of the populations statistically have population means that are positive, and half of the populations statistically have population means that are negative (Fig. 1). It is only for very small values of α that a mixture of some positive and some negative values for population means will occur. For example, for $\alpha = 0.002$, 25% of the values for population means were negative, but for $\alpha = 0.01$, 0% of the values for population means were negative.

For $\alpha = 0.00$, the mean of the population means is much smaller than the standard deviation of the population means (Table 7). For $\alpha > 0.002$, the mean of the population means is no longer much smaller than the standard deviation of the population means. Thus, correlation of categorical attribution and bias reduces the tendency of the various groups within a given population to feel similarly. In some extreme cases, the standard deviation of a particular population can be equivalent to the standard deviation of

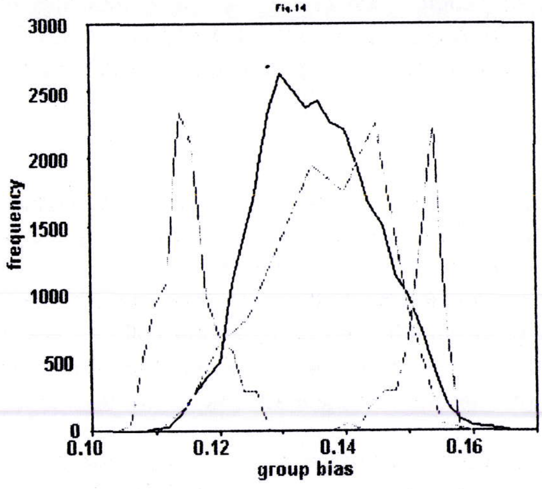


Fig. 5: Histograms of group bias for three representative random populations (solid lines), and of pooled group biases (dashed line) from a total of 156 random populations, corresponding to the data for $\alpha = 0.25$. The frequencies for the three representative random populations are multiplied by factors of 30 or 50 for display purposes.

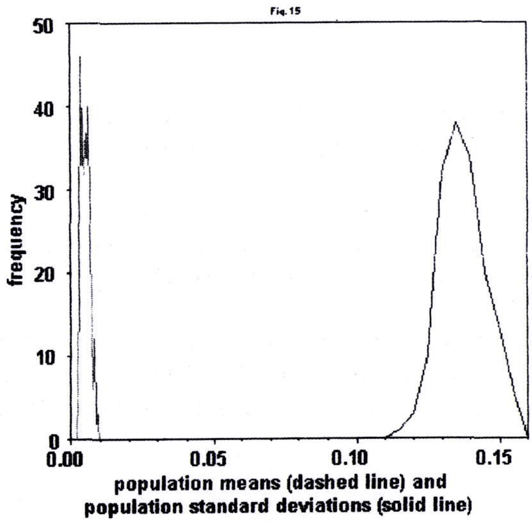


Fig. 6: Histograms of the population means (dashed line) and population standard deviations (solid line), corresponding to the data for $\alpha = 0.25$. The frequencies for the population standard deviations are multiplied by a factor of 2 for display purposes.

the pooled bias (Fig. 5). Comparison of Figs. 4 and 6 indicates the dramatic effect of correlated biases upon the overall dispersion of the group biases within the ensemble of populations relative to the dispersion of the group biases within individual populations.

5 Conclusions

For uncorrelated categorical attribution and bias, although the average feeling characterizing various populations may differ dramatically, groups within a given population are found to have remarkably similar feelings. These results are somewhat surprising, since the random numbers constituting the random populations are uncorrelated, and yet mutually exclusive group biases within a given population are highly correlated. Thus, randomness has given rise to a high degree of order. The significance of these findings is that mutually exclusive groups within a population all love each other, all hate each other, or all feel neutral towards each other.

For correlated categorical attribution and bias, the mean of the population standard deviations is no longer much smaller than the standard deviation of the population means. Thus, correlation of categorical attribution and bias reduces the tendency of the various groups within a given population to feel similarly. In some extreme cases, the standard deviation of a particular population can be equivalent to the standard deviation of the pooled bias. This latter model, which is based upon assumptions that are more realistic, thus leads to results which are more realistic.

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