Systemic Approach to Distribution Law Analysing in Problems of Metrological Accuracy Assurance

Zhagora N.A., Golovin A.N. and Kirpich S.V.
Belarusian State Institute of Metrology
Starovilenski trakt, 93, 220053, Minsk, Belarus
Fax: (375-17) 288-0938
E-mails: belgim@belgim.belpak.minsk.by
kirpich@rocketmail.com

Abstract
This article contains an original approach to the maintenance of objects metrological quality on an example of the measurements accuracy maintenance and assurance. The approach throws down a challenge to the usual practice of "classical" distribution laws application, using of which results in a methodical error, that frequently prevents to provide the metrological characteristics required. It is based on the probabilistic estimation of the parameters of random variables measurements with discrete and continuous "nature" providing information on the object under research. Proposed GANCO law generalizes some well-known laws of random variables distribution and uses analytical open models and simulation models which allow an adaptation to various objects with the purpose of the given metrological accuracy maintenance.
Keywords: Metrology, Accuracy, Models, Probability, Distribution

1 Introduction

The given paper is devoted to one of the main aspects of the quality metrological maintenance on the example of parameters measurement accuracy maintenance that characterize various consumer properties of the surveyed processes and systems in many people life activity spheres.

The basic attention is given to the problem of the measurements system development, which should provide reproduced conditions for measurements, their "transparency", and should also meet the harmonized requirements of Metric Convention, as well as the International Organization for Standardization according to the International systems of the units of measure. Nevertheless, the use of continuous distribution laws, which are "classical", leads to a methodical error, which may become a hindrance for the metrological accuracy maintenance. Such an error is caused when the idealized parameters description according to the well-known laws is inadequate to the actuality.

This paper contains the description of an original approach to the measurements unity maintenance, being one of the major components of metrological maintenance. This approach is based on the probabilistic estimation of the parameters of random variables measurements with discrete and continuous "nature".

International Journal of Computing Anticipatory Systems, Volume 15, 2004
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-930396-01-6

2 Research Problem Statement

Maintenance of metrological accuracy and measurements stability is one of the basic conditions of an effective development for many spheres of national economy of the country; furthermore evaluation and measurements accuracy prediction tasks are very significant in metrology. To resolve such tasks it is necessary to have a way providing system-defined measurements, which would be scientifically proved and characterized by the practical value.

The given work submits the formal (mathematical) approach, developed by one of the authors (professor Golovin A.N.). This approach is based on the probabilistic evaluation of the casual events measurements parameters, having custom distribution laws, and named as "discrete-continuous parameters distribution law" (called as GANCO law). The given approach represents a generalization of well-known parameters distribution laws, describing measurement process under research. This method contains a number of the generalized mathematical models used for analytical calculations, as well as for simulation of the process of metrological measurement of the various content values.

The mathematical apparatus, used in the given approach, allows to calculate or find, by the modelling, a full set of variants that covers all possible cases of combinations of random variables and their distribution laws, that are subject to research in metrological context.

3 Modelling Formal Task

The formal description of measurements metrological aspect consists in the following: there are four variants of a combination of two mutually opposite conditions or events (A and B):

- process under research begins in the condition (or characterized by the event) A and also comes to an end in the same condition;
- process begins in the condition A, but would come to an end in the condition B;
- process begins in the condition B, but comes to an end in the condition A;
- process begins in the condition B and comes to an end in the same condition.

As an illustration of the conditions, mentioned above, the following examples may be used:

a) in the technical systems work, for example, reference complexes, the condition A is considered as time period t, during which some given requirements (on reliability, accuracy etc.) are kept; the condition B is considered as the time T, during which the requirements (one, two etc.), made to the system, are not met;

b) during devices checking in the Centre of standardization and metrology the condition A is considered as the time t, during which a checking of devices is in progress, and the condition B is considered as the time T during which the Centre will not carry out devices checking for whatever reasons.

The analysis of similar examples shows that distribution laws of the specified events A and B may change from the elementary distribution law (Poisson law) with the most random values to the strictly determined distributions of values under investigation.

Duration's ratio of the events A and B may represent the random values varying from zero up to the "infinity". Thus, for an authentic analysis and "behaviour" prediction of the processes under research, it is necessary to know certain mathematical laws that would adequately describe the corresponding processes in respect of metrological specifications of the subject.

It is known that in classical probability theory there is no such distribution law that would take into account both discrete and continuous "nature" of the processes under research. Therefore, nowadays, the Poisson distribution law is frequently applied for the mathematical description of the specified processes: it is to note that the given law is received at the strong assumption, when duration of the event A is zero, and the interval between events A and B (i.e. duration of the event B) is distributed according "exponential" law that corresponds to the elementary stationary process.

For a high-grade and authentic research of the probabilistic processes when resolving metrological maintenance tasks in terms of the events A and B it very important to know the distribution laws of these events duration, as well as the distribution law of the quantity n_{guar} occurrence for the events A and B on the time interval under consideration, taking into account requirements relating to the allowable quantity n and the duration of each event T_{Aguar} and T_{Bguar}, and also, relating to their total duration. The specified durations depend on the probability of the given events occurrence on the time interval under consideration. For the analysis of the events A and B appearance laws, it is also necessary to have some generalized mathematical expression which would allow to take into account simultaneously three various conditions:

1) probabilities $\overline{R}_n(\cdot)$ and $\tilde{R}_n(\cdot)$ of discrete events A and B occurrence respectfully;

2) the total duration of events A and B in respect of any time interval under consideration;

3) taking into account continuous random variables and total events duration.

However, in practice the specified conditions are very often not kept, hence the description of processes with use of the Poisson distribution law leads to the significant errors appearance. For elimination of the specified errors the generalized mathematical GANCO law is developed and applied, that allows to describe the processes mentioned above. With application of some mathematical methods (differential equations, convolutions etc.) the analytical formulas such as

$$R_n(t, a, b, t, T, n, \tau) = \overline{R}_n(\cdot) + \tilde{R}_n(\cdot)$$

are obtained i.e. differentiated GANCO law in the tabulated form (by analogy to periodic D.I.Mendeleev table) where on a vertical there are rigidity factors values (according to the E.S.Ventzel's definition), namely parameter $a = (M\tau_A)^2 / D\tau_A$, (for a that varies from $a = 1$ to $a \to \infty$), and across there are rigidity factors values $b = (M\tau_B)^2 / D\tau_B$, (from that varies from $b = 1$ to $b \to \infty$), describing related "densities" in GANCO law, and hence its view and properties depending on concrete values a and b for three conditions:

$$1)\ M\tau_A < M\tau_B, \qquad 2)\ M\tau_A = M\tau_B \qquad \text{and} \qquad 3)\ M\tau_A > M\tau_B$$

(in connection with the limited opportunities of calculations of the maximal value $a = b = 2\times10^9$).

4 Results Discussion

The analysis shows, that at the parameters $a = 1$ and $\tau_A = 0$, the GANCO law describes the Poisson distributions law, and at the parameter $a \to \infty$ (as well as $b \to \infty$) the GANCO law shows a determined process.

Thus, the GANCO law is more general and allows to normalize transition of casual processes into determined processes and back. The obtained formulas allow to find the seven complex analytical parameters determined by the common expression such as

$$\varphi_i(t,\ H \le H^*,\ V \ge V^*,\ P \le P^*),\ i = 1\ldots7,$$

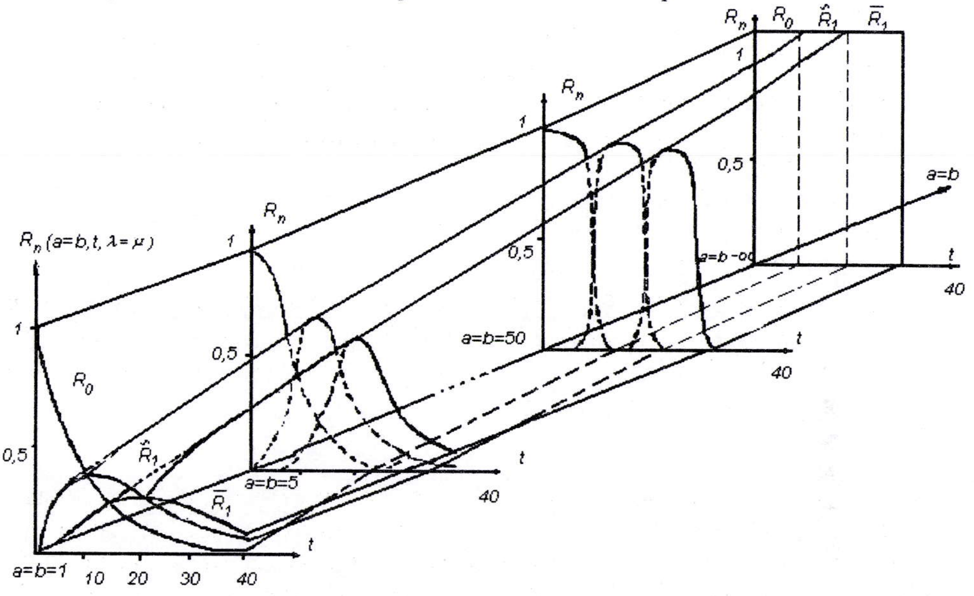
that represents considered requirements on reliability H, productivity V, rhythm P and their combinations in order to find rational optimum ratio between the given parameters for the accuracy assurance (or metrological quality) of the subject under investigation. It corresponds to the probability, with which the performance of any combination from three requirements n_{guar}, τ_{guar} and T_{guar} is guaranteed, that allows carrying out the analysis of the various combinations of random variables distribution processes. The basic analytical parameters of the GANCO law mentioned above and their expressions and characteristics analysis were given in paper [1], [2] and [3]. This paper contains only some common related definitions. The figure (see below) graphically presents the general GANCO law view.

Proposed by the authors the formalistic approach to the values metrological measurement is based on a number of analytical models and takes into account various combinations of the probabilistic events, distribution laws of which are in a wide range, from Poisson law with casual parameters to distribution laws with strictly determined parameters that are subject to the metrological inspection. It is to note, that the given approach is also characterized by universality, which is proved by that it allows the measured modelling (including such numerical characteristics of random variables as a dispersion, asymmetry, excess etc.) in a wide time range of measurement (from second shares to many years).

The essence of the approach based on the GANCO law consists in that all probable combinations of the probabilistic events appearance (discrete and continuous), with reference to the parameters described above, are modelled. Simulation of events on the basis of the formal models characterizes real processes that take place in systems of measurement of the various content metrological parameters.

Originality and novelty of these mathematical models opens to researchers an opportunity for more exact analysis of the parameters to be measured and of measurement processes prediction. The GANCO law, being modelling basis, represents multiple-parametric discrete-continuous parameters' distribution law and is considered as a generalization of well-known distribution laws (Poisson law, Gauss law, the binomial and geometrical laws, etc.).

Figure: The GANCO law representation for certain parameter values



For instance, in order to formalize the task of the measurements accuracy maintenance it is necessary to answer the following question: what is necessary to know for an authentic analysis and prediction of the probabilistic processes under research?

To answer this question, original mathematical models (formulas) are used to describe the parameters distribution laws, including measurements accuracy parameters that have obviously more difficult distributions "laws".

The given approach contains a synthesis of such distribution law that would take into account simultaneously three conditions as follows: 1) probability of discrete events occurrence; 2) each event duration; 3) total events duration (as continuous random values) in any time interval under consideration.

5 Conclusions

Thus, the given paper presents opportunities for **the multiple-parameter discrete-continuous distribution law of random variables, i.e. the GANCO law**, which, in comparison with well-known laws (elementary, normal, binominal, geometrical etc.) is their system generalization. Reliability of the given approach is confirmed by the mathematically correct formulas and their proof, and also on the basis of the analysis of the static data big volumes (by the real systems functioning results) with usage of the special random-number generator.

On the basis of the GANCO law many analytical and simulation open models have been developed providing an opportunity of adaptation to various processes and objects to be analysed, as well as the software for simulation and calculations, calculation data bank having the volume of many thousand realizations, that allows to carry out researches of various objects (technical, economic, ecological, social and others) with the specified accuracy.

So, application of the GANCO law allows carrying out the analysis and synthesis of quality parameters (on the example of the accuracy) for various nature objects with the help of **7 complex analytical parameters** and **15 integrative parameters** for the modelling, including requirements on

- accuracy $\Psi(t, \varepsilon_{guar})$,
- reliability $\Psi(t, H_{guar})$,
- productivity $\Psi(t, V_{guar})$ and
- rhythm $\Psi(t, P_{guar})$

and their combinations, to find rational optimum ratio between the given parameters for the maintenance of required level of metrological quality of the subject under investigation. The common expression of the integrative requirements is as follows:

$$\Psi_k (t, \varepsilon_{guar}, H_{guar}, V_{guar}, P_{guar}), k = 1 \ldots 15.$$

The approach, proposed in the given paper, was used for research of a number of systems (communication systems, flexible industrial systems in mechanical engineering, data transmission systems, statistical data processing systems), in metrological reference complexes, and also for the cancer diseases prediction and diagnostics, for the prediction of the highly skilled sportsmen special readiness of and in other areas.

Practical examples of the procedure modelling relating measurements of parameters in metrological accuracy assurance tasks, at the level to 18 significant figures after a point, are given. Examples are used in national standards development.

References

1. Golovin A. (1983) Complex Evaluation Method for The Reliability Influence to the Metrological Characteristics of the Information and Measuring Systems - Theoretical and Practical Limits of Measurement Accuracy - Proceedings of the IMEKO Symposium. Budapest, May 10-12, 1983, pp. 47-57.

2. Zhagora N. and Golovin A. (1999) Scientific Basis for the Stability and Quality of the Metrological Assurance System Functioning in the Republic of Belarus - Metrology and Machine Engineering, volume 2. Minsk: Belarusian State Institute of Metrology Publishers, pp. 3-50 (in Russian)

3. Zhagora N., Astafyeva L. and Golovin A. (2003) Stability Estimation and Forecasting of Metrological Complexes Functioning. - Minsk: Belarusian State Institute of Metrology Publishers, 253 pp. (in Russian)