Digital Image Enlargement Based on Kernel Component Estimation

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Abstract A new approach for enlarging digital images is proposed. In existing approaches, the assumed reducing operators must be suitable ones for the methods, which means that desirable results are not obtained in other situations. Therefore, an enlargement scheme that can appropriately take reducing operators into account is needed. In this paper, we propose a new enlargement method that can be used for any reducing operators based on the framework of image restoration problems and estimation of the component that belongs to the kernel space of the reducing operator by using statistical properties of natural images.
Keywords : enlargement of digital images, image restoration, kernel space, the Laplace distribution, ℓ_1-norm.

1 Introduction

Enlargement of digital images is widely used in many fields of digital image processing. However in general, this problem is difficult, since it involves the estimation of the lost information that the unknown enlarged (detailed) image may have. A spline based interpolation technique is usually adopted for this problem. Nearest neighbor, bi-linear, and cubic convolution interpolations [2] are representative ones, which correspond to those by spline functions of degree 0, 1, and 3, respectively. In interpolation scheme, it is assumed that given image is generated by simple down-sampling from the unknown detailed image. However, this assumption is not rational, since reducing processes that yield given images are generally accompanied with a kind of low-pass filtering effect. On the other hand, methods of enlarging images by estimating unknown Laplacian components are proposed ([1],[7]). However, in these methods, the assumed reducing processes must be also suitable ones for the methods. Thus, desirable results are not obtained by these methods when the actual reducing processes that generate the given images are not assumed ones.

In this paper, we propose a new method of enlarging digital images. Main idea of the method is adopting the framework of digital image restoration problems and kernel space component estimation according to statistical properties of natural images. Some numerical examples are also presented to verify the efficacy of the proposed method.

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2 Image Reducing and Enlargement Model Based on Image Restoration Framework

Applying the framework of linear image restoration problems [6], the image reducing process is modeled as follows:

$$\boldsymbol{g} = A\boldsymbol{f}, \quad \boldsymbol{f} \in \mathbf{R}^m, \ \boldsymbol{g} \in \mathbf{R}^n, \tag{1}$$

where $\mathbf{R}^m$ and $\mathbf{R}^n$ denote m-dimensional and n-dimensional real metric vector spaces called the space of enlarged images and that of observed images respectively, and, $\boldsymbol{f}$ and $\boldsymbol{g}$ denote an unknown enlarged (detailed) image and an observed image, respectively. $A : \mathbf{R}^m \to \mathbf{R}^n$ denotes a reducing operator. In general, n is much smaller than m. The aim of enlargement is to estimate $\boldsymbol{f}$ as precisely as possible. Limiting enlarging operators to linear ones, the minimum norm least-squares solution $\hat{\boldsymbol{f}}_M$ is given as

$$\hat{\boldsymbol{f}}_M = A^+\boldsymbol{g}, \tag{2}$$

where A^+ denotes the Moore-Penrose generalized inverse matrix [5] of A. As is well known, $\hat{\boldsymbol{f}}_M$ is the orthogonal projection of $\boldsymbol{f}$ to $\mathcal{R}(A')$, the range of the transposed matrix of A. Therefore, the component lost in the reducing process is not recovered in $\hat{\boldsymbol{f}}_M$. By the way, the general solution of eq.1 is written as

$$\hat{\boldsymbol{f}} = A^+\boldsymbol{g} + (I_m - A^+A)\boldsymbol{w} \tag{3}$$

where I_m and $\boldsymbol{w}$ denote the m-dimensional identity matrix and an m-dimensional arbitrary vector. The second term of eq.3 belongs to $\mathcal{N}(A)$, the kernel space of A, and it corresponds to the component lost in the reducing process. Therefore, the main problem of enlarging digital images in this framework is reduced to the optimal estimation of the vector $\boldsymbol{w}$.

3 Statistical Properties of Natural Images

In general, the Laplace distribution is usually assumed for differential images [4]. Let L be the operator that makes differential images by calculating the difference between intensities of neighboring pixels. For instance, L can be written as following $m \times m$ matrix for m-dimensional vector representation of images.

$$L = \begin{bmatrix} 0.5 & -0.5 & 0 & \cdots & \cdots & 0 \\ 0 & 0.5 & -0.5 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0.5 & -0.5 \\ -0.5 & 0 & 0 & \cdots & 0 & 0.5 \end{bmatrix}. \tag{4}$$

The joint probability density function (pdf) of $L\boldsymbol{f}$ is written as

$$p(L\boldsymbol{f}) = \prod_{i=1}^{m} \left(\frac{\beta}{2} \exp\left(-\beta |\boldsymbol{e}_i' L\boldsymbol{f}|\right) \right), \tag{5}$$

with the *i.i.d.* assumption, where β and $\boldsymbol{e}_i$ denote the parameter of the Laplace distribution and the i-th canonical basis, respectively.

As mentioned in the previous section, images are represented as elements of $\mathbf{R}^m$. However, the set consisting of all possible natural images is existing in some biased subset of $\mathbf{R}^m$. Therefore, the statistical property described above is one of natures that characterize the set of images. By the way, the correlation function of natural images is usually modeled as

$$R(x, y) = C \exp\left(-\alpha\sqrt{x^2 + y^2}\right), \tag{6}$$

where x and y denote horizontal and vertical distance between two pixels and α and C denote parameters that depend on the set of images [4]. From this relation, it is suggested that between not only neighboring pixels but also distanced pixels have similar statistical properties with neighboring pixels. Therefore, we investigate the statistical properties of $L^k\boldsymbol{f}$. Two sample images (256×256 pixels, 256 gray scales) are shown in figures 1 and 2 and their histograms of $L^k\boldsymbol{f}$ $(k = 1, 3, 5, 10)$ are shown in figures 3 and 4. From these result, it can be assumed that $L^k\boldsymbol{f}$ $(k \in N)$ are also Laplace distributed random vectors.

4 Digital Image Enlargement Based on Kernel Component Estimation

As described in Section 2, we write the candidates for the enlarged image as follows:

$$\hat{\boldsymbol{f}} = A^+\boldsymbol{g} + (I_m - A^+A)\boldsymbol{w}, \tag{7}$$

with the parameter vector $\boldsymbol{w}$. Based on the knowledge described in the previous section,

$$L^k\hat{\boldsymbol{f}} = L^kA^+\boldsymbol{g} + L^k(I_m - A^+A)\boldsymbol{w} \tag{8}$$

should be Laplace distributed random vectors and the joint pdf of $L^k\boldsymbol{f}$ $(k = 1, \ldots, q)$ is written as

$$\begin{aligned} & p(L\boldsymbol{f}, \ldots, L^q\boldsymbol{f}) \\ &= \prod_{k=1}^{q} p(L^kA^+\boldsymbol{g} + L^k(I_m - A^+A)\boldsymbol{w}) \\ &= \prod_{k=1}^{q}\prod_{i=1}^{m} \left(\frac{\beta}{2} \exp\left(-\beta|\boldsymbol{e}_i'(L^kA^+\boldsymbol{g} + L^k(I_m - A^+A)\boldsymbol{w})|\right) \right), \end{aligned} \tag{9}$$

Fig. 1: Sample image #1.

Fig. 2: Sample image #2.

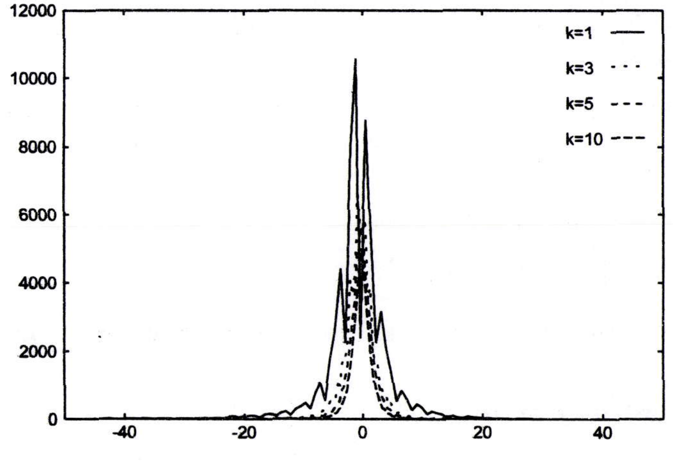

Fig. 3: Histogram of differential image of the sample image #1.

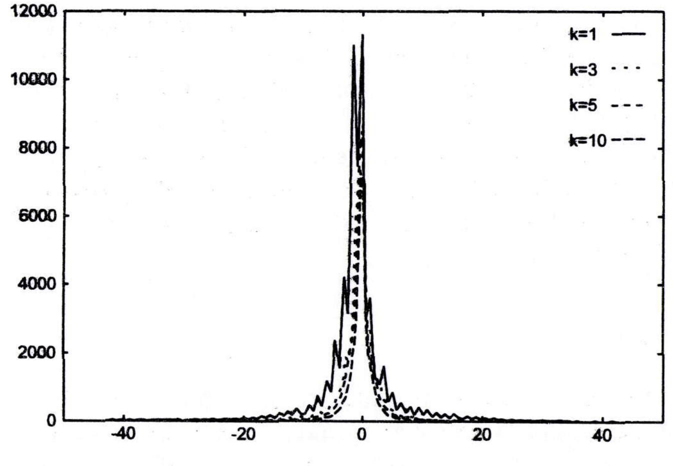

Fig. 4: Histogram of differential image of the sample image #2.

with the assumption that $L^k \boldsymbol{f}$ are *i.i.d.* Therefore, the maximum likelihood estimator of eq.9 must minimize the criterion

$$J(\boldsymbol{w}) = \sum_{k=1}^{q} \sum_{i=1}^{m} |\boldsymbol{e}_i'(L^k A^+ \boldsymbol{g} + L^k(I_m - A^+ A)\boldsymbol{w})|, \tag{10}$$

that is the ℓ_1-norm of $((L\boldsymbol{f})', (L^2\boldsymbol{f})', \ldots, (L^q\boldsymbol{f})')'$. Finally, substituting

$$\boldsymbol{w}_{opt} = \mathrm{argmin}_{\boldsymbol{w}} J(\boldsymbol{w}) \tag{11}$$

to eq.7 yields the optimal enlarged image

$$\hat{\boldsymbol{f}}_{opt} = A^+\boldsymbol{g} + (I_m - A^+ A)\boldsymbol{w}_{opt}. \tag{12}$$

The minimizer of $J(\boldsymbol{w})$ can be calculated by linear programming technique [3].

5 Numerical Experiments

In this section, we show some numerical examples in order to verify the efficacy of the proposed method. We investigate the restoration performance for the sample images shown in section 3. As the assumed reducing operator, we use simple down-sampling(SD) and down-sampling with averaging(DA). These operators reduce the size of images to 1/4 both horizontally and vertically. We adopt cubic convolution interpolator(CCI) [2] as the competitor, since it is well known as a good approximator of Shannon's ideal interpolater and its tendency for the change of assumed reducing operator is similar to that of existing methods. Reduced images and enlarged images are shown in figures 5$\sim$ 16. Correspondence table of these figures are shown in Table 1. Table 2 presents the SNR of each enlarged images.

Although the results of the proposed method is slightly inferior to that of CCI in the case of SD operator, that is assumed to be suitable one for CCI, the proposed method outperforms the competitor in the case of DA operator in terms of SNR, which means the information of reducing operators are effectively used in the proposed method. On the other hand, marked differences are not recognized in terms of the evaluation by the human eyes.

6 Conclusion

In this paper, we proposed a new method of digital image enlargement using the framework of image restoration problems and kernel space component estimation based on statistical properties of natural images. A kind of efficacy of the proposed method is also confirmed by some numerical experiments. Improvement of the proposed method using properties of human eyes and another information about natural images are one of future works.

Table 1: Correspondence table of figures.

	SD		DA	
Sample image	#1	#2	#1	#2
Reduced image	Fig.5	Fig.6	Fig.7	Fig.8
Enlarged image(CCI)	Fig.9	Fig.10	Fig.11	Fig.12
Enlarged image(Proposed)	Fig.13	Fig.14	Fig.15	Fig.16

Table 2: SNR(dB) of enlarged images.

	SD		DA	
Sample image	#1	#2	#1	#2
Enlarged image(CCI)	17.50	19.79	15.37	17.85
Enlarged image(Proposed)	16.30	19.11	18.68	21.32

Fig. 5: Reduced image #1 (SD)

Fig. 6: Reduced image #1 (DA)

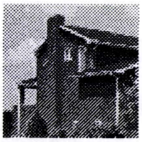

Fig. 7: Reduced image #2 (SD)

Fig. 8: Reduced image #2 (DA)

Fig. 9: Enlarged image #1 (SD/CCI)

Fig. 10: Enlarged image #1 (DA/CCI)

Fig. 11: Enlarged image #2 (SD/CCI)

Fig. 12: Enlarged image #2 (DA/CCI)

Fig. 13: Enlarged image #1 (SD/Proposed)

Fig. 14: Enlarged image #1 (DA/Proposed)

Fig. 15: Enlarged image #2 (SD/Proposed)

Fig. 16: Enlarged image #2 (DA/Proposed)

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