Geometrical Invariants Approach to Recognition the Structures in Time Series and Abstract Maps

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Abstract
The problem of recognition of geometrical objects in different types of data and signals is considered. The main tool in the proposed methodology is a new definition of a cluster based on the invariants of some transformations of space. The invariant properties are closely connected to the symmetry groups of objects. As an illustration, the classical symmetries of space such as continuous groups (Lie *transformations*) are considered. The particular case of spike recognition in neurophysiology is described in details. Preliminary investigations show the high potential power of method. The further prospects of the proposed method are discussed including the problem of perception and models of mentality.
Keywords: signal processing, symmetry, invariant, spike, clustering.

1 Introduction

The problem of recognition of abrupt changes and intrinsic structures is very important in the recent developments of data processing, pattern recognition and modelling methods. The recognition of structural elements in continuous time series data has a particular interest in these studies. The examples are cycles in economics, spikes in neurophysiology (Aksenova et al., 2003), pharmaceutical fingerprints in analytical chemistry (Tetko et al, 1999), textures in pattern recognition. These studies consider the structural properties of these objects as geometrical ones. The application of geometrical approach to recognition implies a new definition of a cluster of similar objects. The current study covers the problem of recognition of objects that have the form of mappings of a segment onto a manifold. The proposed method is based on symmetries analysis (Makarenko, 2001; Makarenko et al., 1999). The applications to spikes recognition in neurophysiology are provided.

2 Abstract statement of the problem

Let us consider the following function of class k:

$$\varphi : [a;b] \to M \quad (1)$$

where $[a;b] \subset R$, and M is some manifold of class k. We shall designate the set of such functions Φ. Therefore, functions of this kind (1) describe geometrical objects in M.

As is shown in (Kelley, 1965), manifolds are metrizable. Let us designate one of possible distances

$$d_M : M \times M \to [0;+\infty). \quad (2)$$

Similarly, we may introduce a distance function between geometrical objects in M:

$$d : \Phi \times \Phi \to [0;+\infty) \quad (3)$$

$$d(\varphi_1, \varphi_2) \mapsto \int_a^b d_M(\varphi_1(x), \varphi_2(x)) dx \quad (4)$$

Consider an arbitrary transformation T that acts on Φ:

$$T : \Phi \to \Phi. \quad (5)$$

Then we can define clusters $K_T(\varepsilon)$ of geometrical objects in M:

$$K_T : (0;+\infty) \to 2^{\Phi}, \quad (6)$$

where 2^{Φ} is a set of subsets in Φ, and

$$K_T(\varepsilon) = \{\varphi : d(T\varphi, \varphi) < \varepsilon\}. \quad (7)$$

Suppose that $A \subset 2^{\Phi}$ consists of several clusters $K_{T_i}(\varepsilon_i)$, $i = 1..n$. Then the problem of classification lies in finding T_i if A is given.

3 Geometrical objects described by an ordinary differential equation

Suppose we have n ordinary differential equations of order m:

$$\begin{cases} \frac{dy^{(0)}}{dt} = y^{(1)} \\ \frac{dy^{(1)}}{dt} = y^{(2)} \\ \dots \\ \frac{dy^{(m-1)}}{dt} = f_i\left(y^{(0)}, y^{(1)}, \dots, y^{(m-1)}\right) \end{cases}, \tag{8}$$

where $i = 1..n$, and f is a bounded function defined on some area $D \subset R^m$. Then all functions $y(t)$ that are solutions of (7) induce a set of geometrical objects on R^m. In this particular case (1) takes the form

$$\varphi : A_\varphi \subset R \to D, \tag{9}$$

where

$$\varphi : t \mapsto \left(y^{(0)}(t), y^{(1)}(t), \dots, y^{(m-1)}(t)\right)^T \tag{10}$$

and φ is a function of class 1. Then for an arbitrary point $p \in D$ exists the only function $y_p(t)$, for which $\left(y_p^{(0)}(0), y_p^{(1)}(0), \dots, y_p^{(m-1)}(0)\right)^T = p$. Let us define such mapping X of D into R^m, that

$$X : p \mapsto \left(y_p^{(1)}(0), y_p^{(2)}(0), \dots, y_p^{(m)}(0)\right)^T. \tag{11}$$

It is a vector field on D, induced by equation (8) (Warner, 1983). Similarly, we can define a local one-parametric group G that acts on D:

$$g_\tau : p \mapsto \left(y_p^{(0)}(\tau), y_p^{(1)}(\tau), \dots, y_p^{(m-1)}(\tau)\right)^T. \tag{12}$$

Consider the action of G on set Φ of geometrical objects represented by functions (9):

$$g_\tau : \Phi \to \Phi \tag{13}$$

$$g_\tau(\varphi)(t) = g_\tau(\varphi(t - \tau)). \tag{14}$$

Let us introduce set of functions

$$Q_\tau : \Phi \to [0;+\infty), \tag{15}$$

that act as

$$Q_\tau : \varphi \mapsto d(g_\tau(\varphi), \varphi) \tag{16}$$

It is obvious that $Q_\tau(\varphi) = 0$ for any admissible τ if and only if φ is a solution of (8). If we find the numerical representation of the vector field, we shall have a simple criterion of evaluation of adjustment the geometrical object to the given differential equations (8).

4 Particular example of methodology implementation

Here we illustrate the application of the above methodology for the spike recognition problem in neurophysiology (Abeles, 1992; Tetko, Villa, 1997). For obtaining numerical presentation of vector fields, we classified separate spikes and formed their spike classes. The algorithm includes three stages: spike detection from the noisy signal, calculation of distances between the phase trajectories of the detected spikes and, at last, forming clusters of spikes that hypothetically belong to the same neuron. More details of this illustration can be found in (Polyarush et al, 2003).

4.1 Spike detection from the noisy signal and derivative calculation methods

The first and second derivatives as well as the mean square of the latter were calculated for the analyzed signal. The time points when the second derivative exceeded 3σ were accepted as the centers of potential spikes. The duration of a spike was specified to be 2.5 ms.

It is shown in (Aksenova, Shelehova, 1994; Aksenova et al, 2003), the *i*-th derivative of function $x(t)$ can be estimated using integral operator

$$D_\alpha^i x(t) = \int_R \omega_\alpha^i (t-\tau) f(\tau) d\tau, \tag{17}$$

where ω_α^i is the *i*-th derivative of function ω_α, which satisfies the following conditions:

$$\omega_\alpha(t)=0, \text{ if } |t|>\alpha; \tag{18}$$

$$\int_R \omega_\alpha(t)dt = 1 \qquad (19)$$

and ω_α has i continuous derivatives.

$D^i_\alpha f(t)$ tends to $\frac{d^i}{dt^i}f(t)$ if $\alpha \to 0$. A fast algorithm of derivative estimation (Letevier, Weber, 2000) used piece-wise polynomial kernel functions as ω^1_α and ω^2_α:

$$\omega^1_\alpha(t) = \begin{cases} -\frac{24}{\alpha^3}t - \frac{24}{\alpha^2}, t \in [-\alpha, -\alpha/2] \\ \frac{24}{\alpha^3}t, t \in [-\alpha/2, \alpha/2] \\ -\frac{24}{\alpha^3}t + \frac{24}{\alpha^2}, t \in [\alpha/2, \alpha] \end{cases}, \qquad (20)$$

$$\omega^2_\alpha(t) = \begin{cases} \frac{c}{\alpha}t + 4c, t \in [-\alpha, -3\alpha/4] \\ -\frac{c}{\alpha}t - 2c, t \in [-3\alpha/4, -\alpha/4], \\ \frac{c}{\alpha}t, t \in [-\alpha/4, 0] \end{cases} \qquad (21)$$

where $c = \frac{1}{8\alpha^3}$. The kernel functions $\omega^1_\alpha(t)$ and $\omega^2_\alpha(t)$ are used to estimate the first and second derivative of the signal. The integral operator D^i_α acts also as a band-pass filter on a signal. Low and high cutoff frequencies for kernels (10) and (11) were found (Aksenova et al, 2003): $\omega_{low} \approx 0.88/\alpha$, $\omega_{high} \approx 4.76/\alpha$ for ω^1_α, and $\omega_{low} \approx 1.76/\alpha$, $\omega_{high} \approx 5.44/\alpha$ for ω^2_α. The choice of parameter α depends on the noise frequency and neuronal spike duration. The latter ranges from 0.5 to 4 ms, so in the present study the low cutoff frequency was set to 1 kHz.

4.2 Distance function for phase trajectories

Let $x(t)$, where $t \in [0; a]$, be an "ideal" spike (i.e., spike generated by a neuron before its form is disturbed by noise), $x^1(t)$ and $x^2(t)$ be the evaluation of its first and second derivatives respectively. Then $\bar{x}(t) = (x^1(t), x^2(t))^T$ describes the phase trajectory of that spike. For estimating the deviation of trajectory $\bar{y}(t)$ from $\bar{x}(t)$ we define such vector $\bar{n}(t)$, that $\bar{x}(t) + \bar{n}(t) = \bar{y}(\omega(t))$ for some $\omega(t)$, and $\bar{n}(t)$ is normal to $\bar{y}(\omega(t))$ (Gudzenko, 1962; Aksenova et al, 2001). Its norm can be calculated as

$$\left|n(x(\cdot),y(\cdot))(t)\right|^2 = \min_{t'\in[t-c;t+c]\cap[a;b]}\left(\left(x^1(t)-y^1(t')\right)^2+\left(x_2(t)-y_2(t')\right)^2\right) \tag{22}$$

We can introduce a difference function for phase trajectories of spikes $x(t)$ and $y(t)$ (Aksenova, Shelehova, 1994; Aksenova et al 2003).

$$d(x(\cdot),y(\cdot)) = \min\left(\tilde{d}(x(\cdot),y(\cdot)),\tilde{d}(y(\cdot),x(\cdot))\right) \tag{23}$$

where

$$\tilde{d}(x(\cdot),y(\cdot)) = \min \int_0^a w(t)\,n(x(\cdot),y(\cdot))(t)^2(t)dt\,, \tag{24}$$

and

$$w(t) = \begin{cases} \dfrac{t}{a_1}, t\in[0;a_1) \\ 1, [a_1;a_2] \\ \dfrac{a-t}{a-a_2}, (a_2;a] \end{cases} \tag{25}$$

is a weight function. In our implementation the parameters were as follows: a_1=0.5 ms, a_2=1 ms and a=2 ms. Then we define matrix $D=\{d_{i,j}\}$, where $d_{i,j}=d(x_i(\cdot),x_j(\cdot))$.

4.3 Numerical representation of vector field and integration on it

The vector field that characterizes the activity of a neuron can be described by phase trajectories of the most typical representatives of its cluster. In our implementation, we chose the center of the cluster of spikes corresponding to this neuron and two spikes that are closest to the center.

4.4 Modelling of neural activity

The proposed method was verified using data from our previous study (Aksenova et al, 2003). Inverse Fourier transform was used to generate 50 seconds trial of an artificial noise with evenly distributed phases and magnitude that represented $1/f$ noise signature. The sampling frequency was 44 kHz. The mean-square deviation of the noise equaled $\sigma^2 \approx 1.00957e+06$. 30000 templates corresponding to 10000 spikes of three different neurons were imposed additively on the noise. Spikes did not overlap. The duration of every spike equaled to 2.43 ms, the duration of the whole experiment was 1097 s.

The distance from the every template (vector field) to the potential spikes were calculated. If the distance from the template to the spike did not exceed the radius of the class, the spike was considered to belong to that class.

The calculated results (Table 1) indicate higher performance of the new method compared to the template matching in phase space of (Aksenova et al, 2003).

Table 1: Comparison of recognition methods.

'Uclassified' is the number of spikes of the given class that were not detected or were referred to another cluster.

'Misclassified' is the number of impulses that belong to another class or represent noise but were classified as spikes of the given cluster.

$$ErrorIndex = \sqrt{(unclassified)^2 + (misclassified)^2}$$

Tem plate	Symmetry group (current approach)			Template matching in phase space (Aksenova et al, 2003)		
	Unclassified	Misclassified	Error index	Unclassified	Misclassified	Error index
I	32	0	32	10	12	15.6
ii	103	120	158.1	122	165	205.2
Iii	94	22	96.5	165	76	181.7

5 Some possible applications and generalizations

The proposed methodology may have a great number of applications in different fields of science. For example, it can be applied to analysis of sound signals, such as speech, music and rhythms (Morgavi et al, 2001), electrocardiogram (Abarbanel, 1995). Another possible fields of application may constitute signals with bursts: sea hydroacoustics, seismic activity, bursts on the sun, objects recognition in astronomy and radio location, spectral analysis, chromatography (Tetko et al, 1999) and the decoding theory.

The proposed methodology can be also generalised. First of all we may consider not only problems of recognition of geometrical objects of type $\varphi:[a;b]\rightarrow M$, but also some projection maps to another space W, $P:\varphi(t)\rightarrow P_{\varphi}\in W$, where P_{φ} - geometrical object in W. This problem is of particular interest in studies on objects stimulus in perception (Rosenblatt, 1962). Some of the examples of such projection include the so-called attractor reconstruction procedure from time series (Abarbanel, 1995) and design of attractors (Feo, Hasler, 2003). In such cases, the interaction of object with observer in experiment is important. Our methodology potentially may be applicable to the pattern recognition of synchronization and clustering in oscillator arrays.

It is also possible to consider maps of type $\varphi : N \rightarrow M$, where N may be the space R^k, *k>1,* or some manifold. While the scope of the method stay the same the difference will be in spaces, transformations classes and criteria. Of course, it needs special investigations of necessary mathematical properties for each particular problem.

Let us describe some possible examples.

Example 1. Differential equations in Banah spaces with one-parametrical transformation group. The method will allow considering the partial differential equations (Henry, 1981). Of course, it requires the development of appropriate approximation theory.

Example 2. Multivalued maps and general transformations. In such case there is also a question on changeable transformation groups for equations with anticipation.

Example 3. It is known (Ovsiannikov, 1978) that the general differential equations may be represented as geometrical object in extended infinite-dimensional extended spaces with special (Lie group) transformations. The proposed methodology may be useful also for such objects too.

The generalization of the method can be also useful to analyse geometrical objects in three-dimensional space, in pattern recognition, morphogenesis and in analysis of art.

6 Conclusion

A new general definition of a cluster of geometrical objects was proposed. An important feature of this definition is an absence of strict bounds of the cluster. In our example the objects from the same cluster were generated by the same set of differential equations. The proposed methodology is very general and may be useful for many fields of investigations where the recognition of the bursts is important, including many aspects of anticipatory systems. Moreover we can suppose that the approach can help in anticipation and recognition of spikes in anticipatory systems.

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