Power of the Nets in Algorithmic

Jean Alphonse Doucet
Haute Ecole Rennequin Sualem ,
Département Ingénieurs Industriels
Province de Liège
Quai Gloesener 6 B-4020 Liège
Fax: 04 / 344 64 11 Email: cerisil @ cci.be.

Abstract
From the observation of the intrinsic properties of the nets, it seems advantageous to introduce their use in a lot of additional various domains to elaborate and understand easily the interactive dynamical structures. Nodes and edges, net components, are well suited to display the influences or exchanges between objects, topics and departments. Consequently they bring a synoptic help for the development of regulated systems and for increasing the efficiency of the data banks.The flow driving between sender nodes and receiver ones permits to detect a strict functional parallelism between algorithms and nets.The polar configuration of nets is also useful for displaying the nodal functional hierarchy
Keywords: Net Topology, Hypertext, Multimodulation, Phasors, Flow Distribution

1 Introduction

The objective of this communication is to highlight the various operational advantages and potentialities of nets in a few domains.Therefore we shall survey their basic properties which are related to the multivector structures, the **"Z"** transport functions or the flow channeling from the nodal causalities to the outcomes through the edges.
Nets are propagating tools suited for displaying every evolution.The introduction of the Sign and Dirac's Pulse operators for the systematic net browsing allows to bring a didactic help for the dynamic interactive structures, for the iterative reasonings and for data sorting. Besides, the net insertion into phasor topology enhances the duality between the radial connections carrying the various components of the central input and the tangential ones pointing out the correlations between these components.

2 Presentation of the Structure of Nets

Components of Nets:
- Nodes (n°): play the role of potential or qualitative variables, they may be considered as covariante components

- Edges or Connections (c): play the role of quantitative or interaction variables, they may be considered as contravariante components.
- Correlation between Flows and Edges: there is an intense isomorphism between flow and edge. Indeed an edge is essential to let stream a flow between the pair of its end points; and vice versa each flow needs an edge for its driving.

2.1 Geometric Definition of a Net

A net is composed by a set of intercorrelated nodes what needs an intrinsic pair of complementary components, essential for building any net (Fig. 1).The numerical display of a net is a bivector elaborated from the association of the nodal vector (first vector) with the edge vector (second vector). We may transcribe this vector concept as follows:[(n°);(c)],where the first parenthesis (n°) contents the node vector and the second one (c) the edge vector.

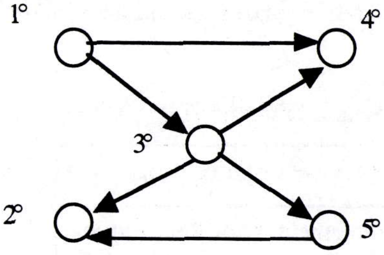

Fig.1: Geometric Characteristics of a Directional Net

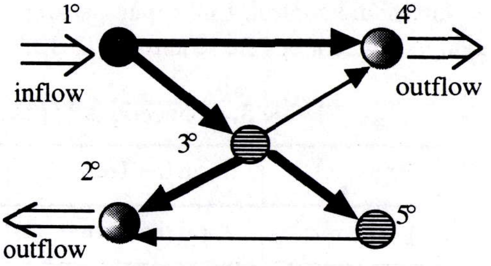

Fig.2: Topologic Charcteristics of a Directional Net

The development of the bivector notation is given in the Table 1

Table 1: Vector Composition of the Geometry of the Net of (Fig.1)

Nodes vector	1°	2°	3°	4°	5°	
Edges vector	(1 > 3)	(1 > 4)	(3 > 2)	(3 > 4)	(3 > 5)	(5 > 2)

2.2 Topologic Configuration of a Net

The topologic sight allows to survey the flows propagation and to discover the various roles of nodes and edges in relation with these migrating behaviours (Fig. 2).
The various functions of nodes and edges required for the management of the flows are displaid in the tables 2 & 3. (Wai-Kai Chen 1990; Stagg & El-Abiad 1968)

Table 2: Nodal Types and Related Functions

Roots	Flows Entries	Ingates
Leafs	Flows Outstreams	Outgates
Intermediate Nodes Transient Nodes	Through Nodes	Operators or Modulators

After the choice of a root node which usually corresponds to the ingate of the inflow, we choose the branches to join this root to the total residual set of the nodes (Table 2). The branche (Br) set forms a tree, which is a subnet connecting the total nodal set of the initial net, but without any loop. Usually the branches, transport the essential nodal interactions. Consequently the branches may be the progessive forward axes of the net. The links (Lk) are the complementary edge set of the tree and form the cotree. Each link brings an additional loop in the net (Table 3) and transports feedback effects necessary to the internodal control. Links may perform the iterative processes and also incursive behaviours where loops are required (Fig. 12).

Table 3: Connections Types and Related Functions

Branches	form the Tree	Progressive or Direct Edges
Links	form the Cotree	Complementary or Rear Edges

Table 4: Vector Composition of the Topology of the Net of (Fig.1)

Root or Inflow Node:	1°
Transient or Through Nodes:	3° 5°
Leaves or Outflow Nodes:	2° 4°
Branches of the Tree:	(1 > 3) (1 > 4) (3 > 2) (3 > 5)
Links of the Cotree :	(3 > 4) (5 > 2)

3 Numerical Coding of Net

The geometric configuration of a net can be easily described by the Incidence (nodes - edges) Matrix noted[n°A c], where each column corresponds to a node (n°) and each row to an edge (c).

Table 5: Incidence (Nodes -Edges) Matrix [n°A c] of the Net of the (Fig. 2)

	Root 1°	Node 2°	Node 3°	Node 4°	Node 5°
Branch (1>3)	1		-1		
Branch (1>4)	1			-1	
Branch (3>2)		-1	1		
Branch (3>5)			1		-1
Link (3>4)			1	-1	
Link (5>2)		-1			1

The dimension of this [A] matrix is [{n° {c], where {n points out the nodes number and {c the edges number. The components of this matrix may be as follows: $a_{lk} = 1$ if there is a connection from (l°) towards (k°) , or $a_{lj} = -1$ if there is a connection from (j°) towards (l°), or $a_{ln} = 0$ if there is no connection between (l°) and (n°) (Table 5). This matrix contents the whole information necessary for the design of the associated net. Composition of this [A] matrix: each row contains the numerical values (1 & -1) and 0 values for the residual components.These are similar to the value space of the sign operator.

4 Presentation of the sg() and δ() operators

They form the basic dual operators pair for any comparison as presented in Table 6

Table 6: Presentation of the Working of the **sg()** & **δ()** Operators

sg(A-B) = +1 if A > B	**sg**(A-B) = 0 if A = B	**sg**(A-B) = -1 if A < B
δ(A-B) = +1 if A = B	**δ**(A-B) = 0 if A ≠ B	

We underline that sg(A -B) and δ(A -B) are essential for any sorting and reasoning. Therefore we assert the 1 Thesis. (Table 13)

5 Logical Display of the Net Structure

For pointing out a directed edge between a pair of points (a° & b°), it is possible to use a topological version of the sign operator which is here noted like **Sg(a° > b°)** with a capital. This topological odd operator is very convenient to detect any directional edge connecting a pair of nodes (Table 7) and links net architecture with sorting processes.This gives nets a lot of supplementary applications in sorting the topics of any development.

5.1 The Sg(>) Nodal Matrix of a Net

This last **[Sg(>)]** Matrix (Table 8) contents the same information as the Incidence (nodes - edges) Matrix [n°A c̲], but its elaboration is deduced from the nodal connections. It may be considered as more causal, because related to the nodal influences and exchanges, what is the logical motivation of the net conception. This **[Sg(l°>k°)]** Matrix is always a skew symmetric matrix with the dimension of $[\{n°]^2$, where{n° is the nodal number.The diagonal components **Sg(l°>l°)** have the value 0, if there is no nodal self reactivity expressed by loops with a single edge (l° -> l°) whose origine and arrival are the same node. Due to the skew symmetry with the diagonal cancellation, we may condense the information delivered by the **[Sg(l°>k°)]** into a triangular form **Tr[Sg(l°>k°)]** with only $(1/2)[\ (\{n°)^2 - \{n]]$ components; what allows to save a few of memory space.

Table 7: Presentation of the Meaning of the Topological **Sg(>)**

Sg(a° > b°) = +1 if there exists an edge from a° node to b° node
Sg(a° > b°) = 0 if there is no edge between a° node and b° node
Sg(a° > b°) = -1 if there exists an edge from b° node to a° node

We may choose the upper or the lower triangular part of this **Tr[Sg(l°>k°)]**. See the 2 Thesis. (Table 13).

Table 8 : Matrix **Sg(<u>l° > k°</u>)**: displays the Nodal Directed Connections

	Root 1°	Node 2°	Node 3°	Node 4°	Node 5°
Root 1°	▨		1	1	
Node 2°		▨	-1		-1
Node 3°	-1	1	▨	1	1
Node 4°	-1		-1	▨	
Node 5°		1	-1		▨

5.2 Nets and associated Spaces

It is often advantageous to consider a net like a finite dotted space where the countable points correspond to the nodes and the selected mobilities channels to the edges. Therefore each net produces a space suitable for pointing out the algorithm working and accordingly acts like a didactical key in a large set of interactive domains.

Table 9: Inventory of Characteristics or Functions supported by Nodes and Edges

Nodes	Edges
Location	Connection
Element	Channel
Potential variable	Propagation Axis
Kernel	Influence & Exchange
Subsystem or Subnet	Internodal convolution (Z Op.)
Operator	Internodal Differentiator or Gradient

5.3 Functions of Nodes and Edges

Table 9 gives the essential functions of nodes and edges what causes the various nets applications. The selection of nodes and edges is an importent step in the analysis of any system and constitutes the primitive net suited for explaining the following necessary transformations.

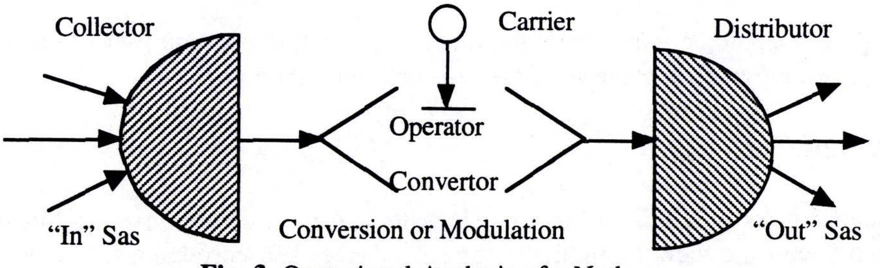

Fig. 3 Operational Analysis of a Node

Fig. 3 shows the different parts of an operational node. This displays the nodal merging capacity, because any set of operators or subnet can be inserted in the inside part .This nodal agglutinating power permits to condense any net over only 2 or 3 key nodes.This property is noted in the 3 Thesis (Table 13).
Fig.4 &Table 10 display the edge functions.

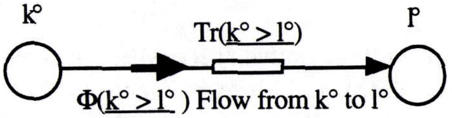

Fig.4: Operational Analysis of an Edge $(\underline{k^\circ > l^\circ})$

Table 10: Edge Characteristics

Transmittance:	$Tr(\underline{k^\circ > l^\circ})$
Flow:	$\Phi(\underline{k^\circ > l^\circ})$
Mobility:	$\lambda(\underline{k^\circ > l^\circ})$
Gradient:	$\Delta(\underline{k^\circ > l^\circ})$
Efficiency:	$\eta(\underline{k^\circ > l^\circ})$
Direction:	$Sg(\underline{k^\circ > l^\circ})$

6 Nets in Algorithmic

Nets carry a systematic help for conceiving and understanding the algorithms because an algorithm is a ramification of operations to convert a data set into a results set what requires moving quantities or flows. These operations need nets for their run along their state trajectories (Fig. 7). Therefore we assert the 4 Thesis (Table 13).

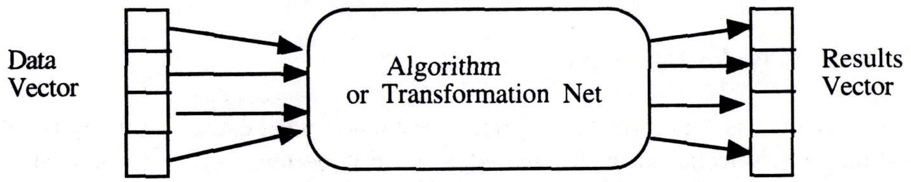

Fig. 5 : Isomorphism between Algorithm & Net

6. 1 Correlation between Net and Operational Graphs

Any Graph is a topologic form of any algorithm and from this assertion we can deduce that an operational graph explains the flow transformations through the net.

7 Logic and Nets

The essential tasks of Logic are to sort elements and to structure anarchic lots of components with the view to highlight correspondances and correlations. For these purposes it is necessary to use in cascade the operators sg(A-B) and δ (A-B); see (Fig 6).

The topological structures of the nets are scheduled and detected by the topological operator **Sg(a° > b°)**. Consequently net building is supplied by an analogous way to the elaboration of logical operations. Therefore nets sustain synoptically the logical correlations because they weave selected connections between nodal topics.

From these considerations we can deduce the similarity between the structures of logic and nets in (Figs.6 &7). See 5 Thesis (Table 13). Here we are able to underline the strategic function of the nets in the classifying procedures because the nets are working alike logical structures. Due to their common operator of discrimination sg(A-B), logic and nets can develop for each other an advantageous interaction as follows: nets bring into logic a synoptical tool and reciprocally logic adds in the nets the logical distribution of the "inflow" over the nodal set accordingly their common meanings.

Table 11: Functional Similarities between Modulation & Topic Identification

Modulation	Topic Identification
Carrier	Key Topic
Modulating Signal	Data Flow
Modulated Signal	Identified Info.with Key Topic

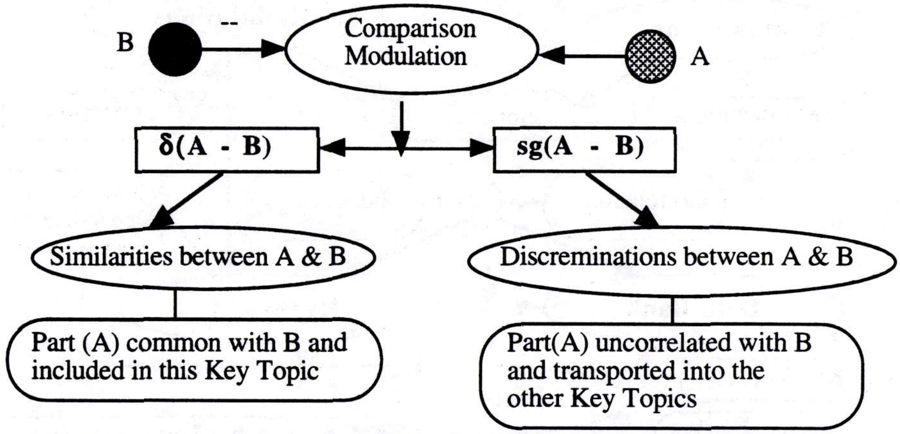

Fig.6 : Operational Structure of Comparison or of Extended Modulation

8 Hypertexts, Modulation Chains and Sorting Nets

Each recorded information has to be sorted and projected on a set of key topics (A_k) in order to form structured knowledge.To perform these analyzing tasks we elaborate a net where the edges channel the Data flows (**s**), for their identification, towards the key topic node (A_k) under the regulation of the sorting operator: sg(**s** -A_k) (Fig. 10). We remark that each partial identification of (**s**) with each (A_k) presents a similar funtional morphology with a special modulation, where the carrier is also A_k (Table 11). Indeed each hypertext seems similar to a multimodulation chain. See 6 Thesis (Table 13).

A modulating net may be expansed in the referential constituted by its modulators (Ak)

and appears like a topological vector serie where the basic elements are these carriers (Ak) with cofactors (**s**/Ak) results of the modulation of the incident signal (**s**) with each basic topic (Ak) and are allocated to each carrier node of the net of (Fig.8).The complementary part of (**s**) which contains no similarity nor analogy to the (Ak) set constitutes the residue of this modulating serie and is sent into the condensed complementary subnet embedded in the hexagonal node (Fig. 8).
From the introspection of (Fig. 8) we can write the topological expansion of its net:

$$\mathbf{s} = \Sigma_k \{(\mathbf{s}/Ak)\ \delta(\mathbf{s} - Ak)\}\ (\mathbf{1}k) + \{[(\mathbf{s}) - (\mathbf{s}/Ak)]\ \ sg(\mathbf{s} - Ak)\}\ {}^{c}(\mathbf{1}k) \qquad (1)$$

where $\delta(\mathbf{s} - Ak)$ is the identifier of (**s**) to (Ak) and $sg(\mathbf{s} - Ak)$ is the detector of the difference between (**s**) and (Ak).

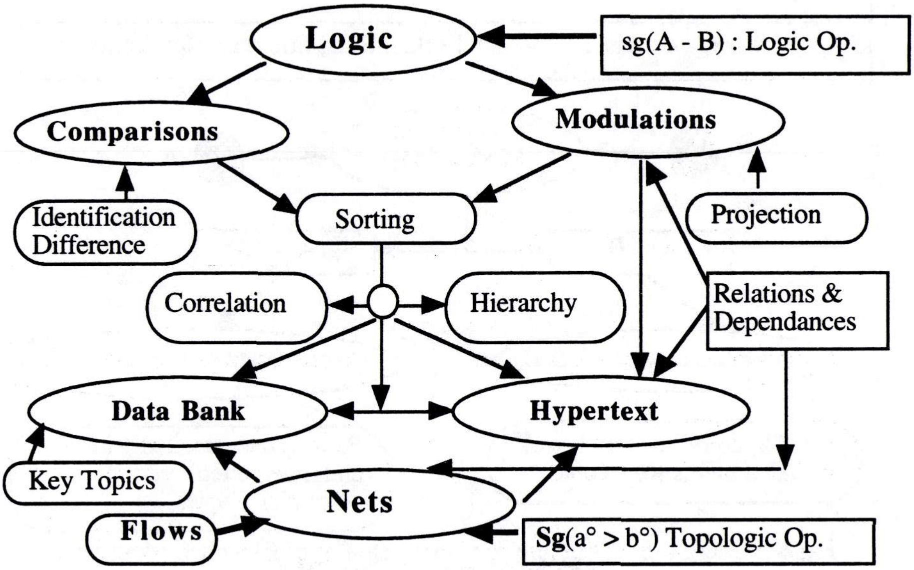

Fig.7: Complementary Interference between Nets Structures and Logical Laws

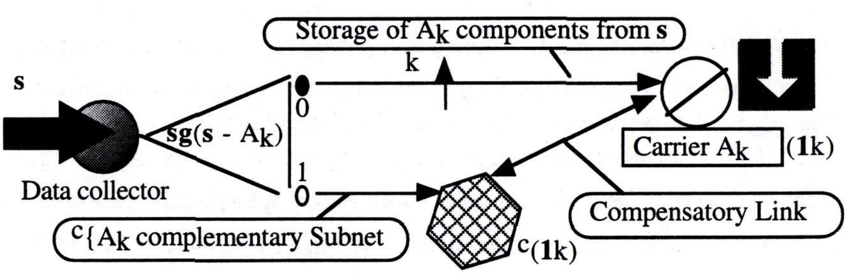

Fig.8: Analyzing Net with Basic Topics A_k

(1k) is the basic vector related to (Ak) and ^{c}(1k) the basic vector supporting the part of s complementary to (Ak). These basic vectors connect the carriers (Ak) & ^{c}(Ak) to a vector referential. This shrinking transformation gives topologic flexibility to the nets and shows the structural similarity between any multimodulation and any algebraic or vector expansion.

9 Polar Anamorphosis of Net

The polar configuration of a sorting net shows the progressive hierarchy in the distribution of the key topics over a set of circular horizons. From this polar geometry the discrimination between the different topics levels becomes clear. On the radial directions are underlined the dependances between main topics and their derivative pseudoharmonics. On each horizon, are distributed the explanatory topics of the same rank in the definition of the recorded Info at the ingate (1) (Fig. 11).Therefore the 7 Thesis (Table 13).

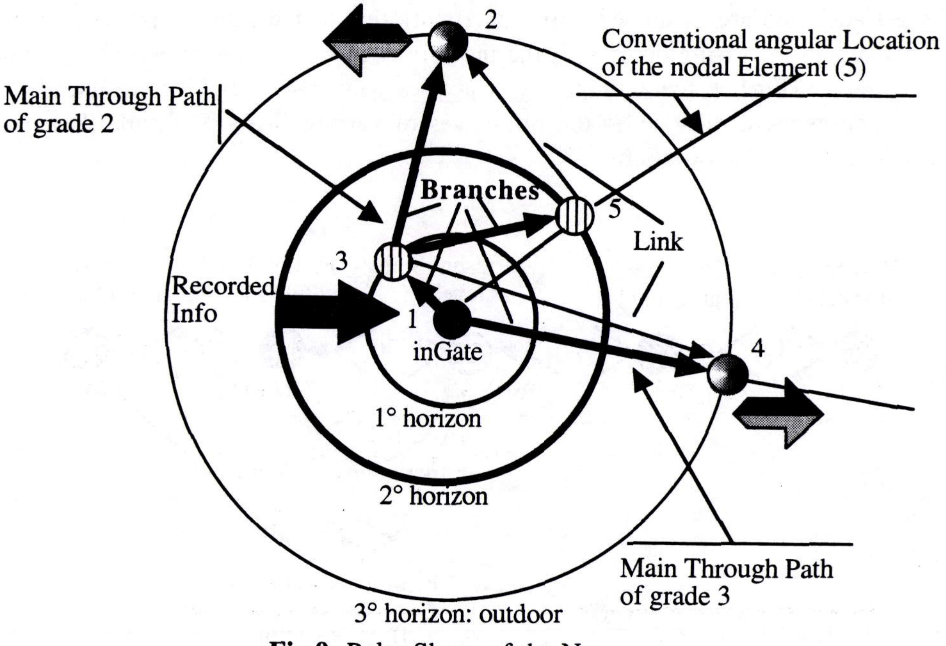

Fig.9: Polar Shape of the Net

10 Presentation of the Incursivity Processes by means of Nets

Incursivity has to use loops for driving the future influences (Dubois 1992; Doucet 1998). In (Fig.10) we present an anticipatory algorithm with a double grade of future retroaction, from the states at the time levels +1 & +2. We observe the crucial role of the future links for improving the stability of the anticipatory behaviour. To sum up see 8 Thesis (Table 13).
The equation (2) is deduced from the net of (Fig.10) and it shows the dependances of the signal **s(0)** at the present state, on one side from the past states at the levels (-2) & (-1) and on the other side from the future states at the levels (+2) & (+1).
From (Fig. 12) we can retrieve the evolutionary law of **s**:

$$\mathbf{s(0)} = s(-2)[{}^{-2}\mathbf{T}_{-1}] + s(-1)\,[{}^{-1}\mathbf{T}_0] + s(1)\,[{}^{1}\mathbf{R}_0] + s(2)\,[{}^{2}\mathbf{R}_0] \qquad (2)$$

where $[{}^{k}\mathbf{T}_{l}]$**:** progressive changing operator of s between the past time k and the following one l.

$[{}^{p}\mathbf{R}_0]$: feedback from the future time p to the present 0.

These translators are evaluated with uncertainty due to the future hazards and they bring loops in the evolution of **s**. If the incursivity process is incorporated in a polar configuration (Fig11), the anticipatory structures gain more flexibility to modify the present adjustment of **s** under the influences of various drifts of future. These are supported by angular variations.

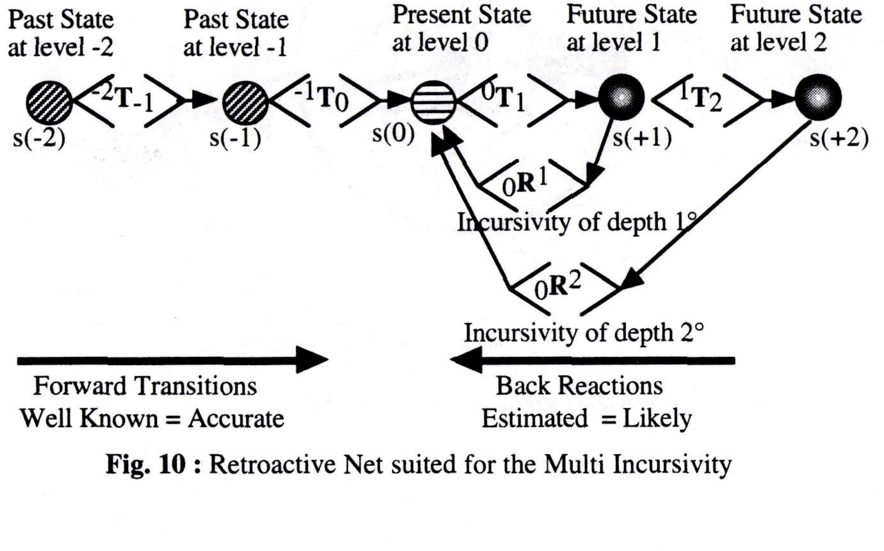

Fig. 10 : Retroactive Net suited for the Multi Incursivity

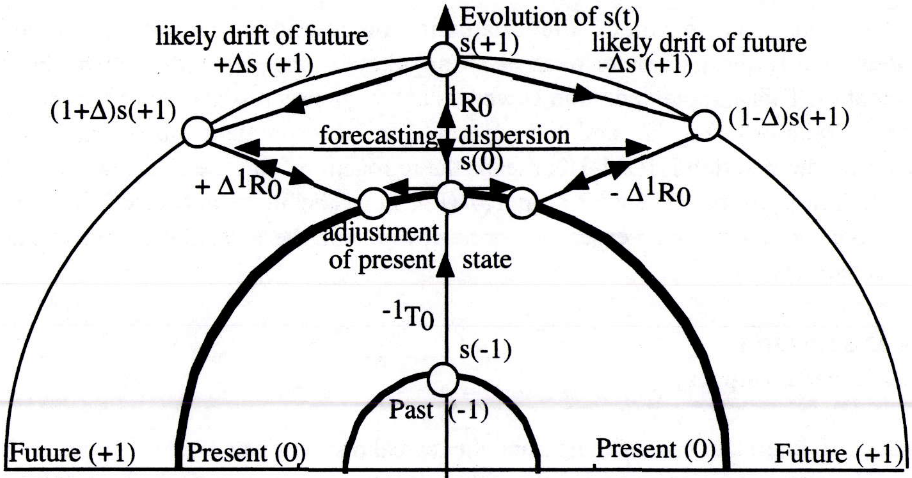

Fig.11 Adjustment of the present State due to likely Drift of Future State

The use of the angular variations brings additional flexibility for analyzing the evolutionary behaviours. It is also an easy suggestive solution for introspecting the multiparameter incursivity because we can attribute for each parameter,vector component, an angular area (Fig. 12).

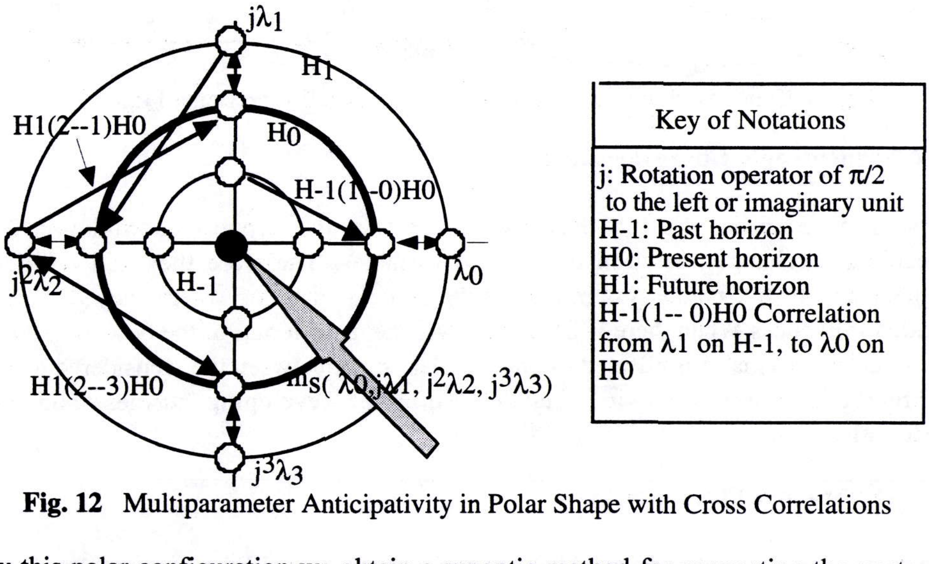

Fig. 12 Multiparameter Anticipativity in Polar Shape with Cross Correlations

By this polar configuration we obtain a synoptic method for supporting the vector or multiparameter anticipativity. In (Figs. 11 & 12) the double arrows in opposition condense the retroloops between present and future states (Fig. 13). Like this we obtain a bidirectional edge instead of a twoonodal loop and it allows a spreading reduction. Besides we may introduce the imaginary or rotation operators j^p to position systematically the parameter axes at an angular distance of $p(\pi/2)$ from the initial orientation.This complex notation is well suited for the representation of the parameter crosscorrelations (Fig.12) and for generating a correspondent mnemonic code. For example: the notation H1(2-1)H0: means the retroaction from the future value of $\lambda 2$ on H1, to the present value of $\lambda 1$ on H0 (Fig.12). The introduction of the complex notations produces more ease and more power for the systematic analysis of any intricated nets.

11 Discussion

11.1 General Impact

Strategical interlacing between nets and the logical operator pair **sg()** & **δ()** gives key for their dynamic conception and for highlighting the space quantification of the flows through the edges. In sciences and technology nets can help for understanding and simulation (Fig.13 & Table 12). Progresses in the knowledge development were implicitely stimulated by means of generalized multimodulation nets; see 9 Thesis (Table 13). The crucial purposes of this communication are collected under 9 theses, related to their particuliar development part, in the Table 13. This one acts as the summary window of this presentation.

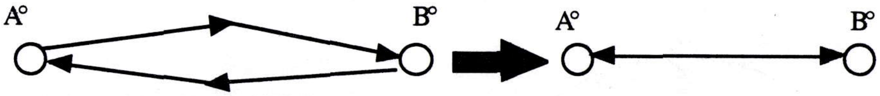

Fig.13 Condensation of a Twoonodal Loop into a Bidirectional Edge

11.2 Neguentropic Characteristics

Nets produce space structure because they select and support coordination and directional exchanges between their nodal variables. Therefore they add a kinetic neguentropy to the nodal system and to the associated space where they drive the decided influences.When there is hesitation over the discovering of the best solution of an intricated enigma, it is advised to draw a relation net between the considered topics to prepare a convivial solving method, useful for developing business and task specificities.

11.3 Algebra and Flexibility

The signal expansion over selected key topics in association with nodal mergings allows a flexible knowledge analysis yielding shape variations.This fact gives complementary algebraic properties to the nets.

11.4 Prospectives

Nets may act as efficient tools for developing further synoptical modelisation of any intricated function. Like this, they should stimulate search undertakings.

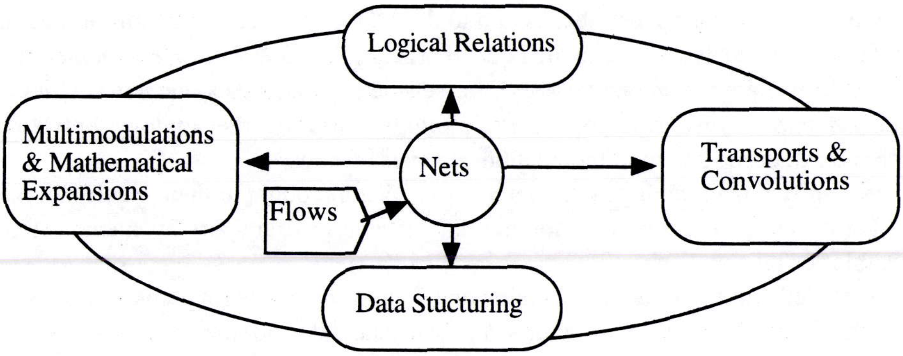

Fig. 14: Domains embedded by the Nets

Table 12 : Main Types of Nets with the Specificities of their Components

Nets Functions	**Edges**	**Nodes**	
		In Gates	Out Gates
Power Transport	Way, Canal	Producer	Consumer
Communication	Channel, Waves	Sender	Receiver
Algorithmic	Op. Path	Argument	Result
Hypertext	Sorting Bifurcations	Data Collector	Key Topic
Process Control	Operational Decisions	Initial State	Final State

Table 13: Specific Proposals deduced from Nets Power

Part (4)	**1 Th.**	Ops. sg() & δ() can structure any reasoning & logical decision
Part (5.1)	**2 Th.**	Topological **Sg(>)** builds the nets and reduces their data
Part (5.3)	**3 Th.**	Nodal merging condenses nets to put forward the key topics
Part (6)	**4 Th.**	Algorithms are transformation nets
Part (7)	**5 Th.**	Structural analogy between nets and logical laws
Part (8)	**6 Th.**	Isomorphism between hypertext, multimodulation chain & net
Part (9)	**7 Th.**	Polar configuration of nets displays the stages of hypertexts
Part (10)	**8 Th.**	Net is a synoptical tool for anticipatory systems
Part (11)	**9 Th.**	Nets are stimulators of any social and economic progress

12 Conclusion

Nets are powerful tools to describe and understand a large range of tasks and reasonings in technology as in algorithmics because they manage and drive flows which can explain the evolutionary systems. Here was shown the suitability of nets to configure the logical relations, to elaborate the data banks and to describe the structures of multimodulations and knowledge analysis. Due to the nodal merging (Figs. 3 & 8), we have underlined the net flexibility, advantageous to stress the selected nodal topics, carrier vectors of the algebraic expansions. Due to this morphology variation the analogy between any algebraic expansion and multimodulation becomes obvious.

Consequently it is possible to obtain a net classification ought to their specific domains:

- "Hard Net": in technology for matter and energy transport by means of pipes, cables or tubes.
- "Soft Net" : in information collecting and sorting for developing data banks.
- "Algo Nets" in algorithmic for driving the idea and developing specific softwares.
- "Socio Nets" in any structured group for developping synergy among their members.

Nets are synoptical efficient designs for structuring a lot of various domains and they supply a common synoptical support for algebra, geometry, logic, mutimodulation and structured knowledges. Net insertion in the phasors gives flexibility and accuracy for describing the retroloops and crosscorrelations of any anticipatory behaviour.

References

Stagg & El-Abiad. (1968) Computer Methods in Power System Analysis. Mc Graw-Hill.

Wai-Kai Chen (1990) Theory of Nets. John Wiley & Sons (New York).

Karnopp D. & Margolis D. & Rosenberg.(1990) System Dynamic. John Wiley (N.- Y.).

Dubois D. M. & Resconi G.(1992) Hyperincursivity: a New Mathematical Theory. Presses Universitaires de Liège (1992).

Robert J. & Robichaud L. & Boisvert M. (1962) Graphes de Fluence. Presses Université Laval & Dunod.

Doucet J.A. (1992) Présentation des Graphes Opérationnels et de leur Utilisation en Electromagnétisme. Société Royale Belge des Electriciens. Revue E. n° 2 - 92 (p.45 to 50).

Doucet J.A. (1998) Incursivity Operational Display. CASYS, International Journal of Computing Anticipatory Systems, vol. 1, pp. 249-257.

Doucet J.A. (2000) Phasors and Didactic Knowledges Mapping. CASYS, International Journal of Computing Anticipatory Systems, vol. 6, pp. 277-295.