Sheaf Cohomology of Conscious Entity

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Abstract
Awareness of a conscious entity can exist without elements; therefore, the general notion of an object of a category is employed. One of the characterization of understanding is: for a given local information (awareness) there exists a global information whose restriction is the given information. For such mental activities, category and sheaf theories are employed to formulate consciousness. We will show that the cohomology (more general precohomology) object, a subquotient object, better represents the essence of a conscious entity than an object itself. We will also give a definition of an observation to formulate the collapse of the wave and the wave property.
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1 Introduction to Category and Sheaf

In our earlier work, [10], we introduced the notion of *conscious universe* $\hat{T}$ (note that in [10] we used $\underline{\boldsymbol{U}}$ for the conscious universe) as a category of presheaves on the category associated with a topological space. More precisely, $\hat{T}$ is the category of contravariant functors from the category $\boldsymbol{T}$ associated with a topological space **T** to a product category $\prod_{\alpha\in\Gamma} C_\alpha$ of categories where Γ is an index set. The category $\boldsymbol{T}$ is said to be the *generalized time category* (or *generalized time space*) when the real line **R** is embeddable in $\boldsymbol{T}$. Such a contravariant functor P in $\hat{T}$ is said to be a presheaf defined on $\boldsymbol{T}$ with value in $\prod_{\alpha\in\Gamma} C_\alpha$. Namely,

$$\hat{T} = \left(\prod_{\alpha\in\Gamma} C_\alpha\right)^{T^{opp}} \tag{1}$$

To be more explicit, for an object V in $\boldsymbol{T}$, i.e., an open set V of **T**, and for an object P in $\hat{T}$. We have $P(V) = (P_\alpha(V))$, $\alpha\in\Gamma$ where each $P_\alpha(V)$ is an object of C_α. Recall that a *conscious entity* is a presheaf P in $\hat{T}$ where $\{C_\alpha, \alpha\in\Gamma\}$ represents the totality of mental and physical categories of conscious entities. Further note that some of the categories in the product category are discrete categories with structures, i.e., categories

with no morphisms (namely, identity morphisms only) but with specifically given structures in those categories (see Part 3). A functor F between discrete categories is an assignment of objects. That is, for an identity morphism 1_X of object X, we have $F(1_X) = 1_{FX}$, by regarding $X = 1_X$. An embedded real line **R** in **T** corresponds to *time*. In the program which will be described in Part 3, it may be important to consider **R** as associated with each P. Namely, **R** should be written as R_P. Then, for objects P and Q in $\hat{T}$, there is an isomorphism from R_P to R_Q, Let i be an embedding from **R** to **T**. Then i induces a functor from the category of presheaves on $\boldsymbol{T}$ to the category of presheaves on **R** denoted as i^{-1}. (See [4] for operations among sheaves.) That is, for P in $\hat{T}$, $i^{-1}(P)$ is a presheaf on **R**, namely, the restriction of P on **R**. One often writes $i^{-1}(P)$ as $P|_R$. There are different types of consciousness in the usual sense. The first is *awareness*, which is in this sheaf theoretic definition of consciousness, $P(V) = (P_\alpha(V))$, $\alpha \in \Gamma$ in the category $\prod_{\alpha\in\Gamma} C_\alpha$. In Zen philosophy, one begins with the concept of being here and now. Then one reaches the stage of having no thoughts so that each component of $P(V \cap R)$ in each category C_α is a trivial (final) object. We will return to this topic in Part 3. (As an elemental introduction to Zen, one may read [17].) The second type of consciousness is *attention*. When one has a thought on a certain topic, it is the component $P_\alpha(V)$, the image of the projection from $P(V)$ in $\prod_{\alpha\in\Gamma} C_\alpha$ to a particular category C_α where the thought occurs.

Now we should answer the following natural questions for this sheaf and category formulation of consciousness.

1.1 "Why Category?"

Cognitive awareness has been considered to have clear existence, as Rene Descartes indicated *thinking implies existing*. However, for a conscious entity P, a certain component $P_\alpha(V)$ of the awareness $P(V)$, for a generalized time period V, need not consist of elements. That is, it is just an object in the category C_α without elements. Hence, the general notion of an object of a category is needed. When there are elements in an object, they are said to be *thoughts*. For two objects P and Q in $\hat{T}$, namely, two conscious entities, the communication from P to Q in a category C is a correspondence from P to Q. For the sake of simplicity, we did not index P and Q, namely, we regard P and Q in the category C as the C-components of P and Q in $\hat{T}$. That is, for U and U' in the generalized time category $\boldsymbol{T}$, the information $P(U)$ for the generalized time U is communicated to $Q(U')$ over U' by a morphism $P(U) \longrightarrow Q(U')$ in the category C. This type of communication is said to be a *horizontal communication* in [10]. When $U=U'$, such a morphism from $P(U)$ to $Q(U)$ is a *natural transformation* in the usual sense from functor P to functor Q. In particular, the identity map $\rho_U^U: P(U) \longrightarrow P(U)$ in C is the *self-awareness* of a conscious entity P in category C. Rene Descartes said, "I think. Therefore, I am."

Descartes would say in the above sheaf theoretic sense "I am aware, i.e., $\rho_U^U : P(U) \longrightarrow P(U)$. Therefore, I am." A *vertical communication* is an information flow from P to P. Namely, for an object U in $\boldsymbol{T}$, a vertical communication within P is an assignment from category C_α to category C_β defined by $I_\beta^\alpha : P_\alpha(U) \longrightarrow P_\beta(U)$. For example, when a conscious entity P studies a certain mathematical field C_α to understand another field C_β, this vertical communication $I_\beta^\alpha : P_\alpha(U) \longrightarrow P_\beta(U)$ is an interpretation of the information that P has in the category C_α as the information in the category C_β. Then, as shown in [10], for a horizontal communication of information in C_α, I_β^α induces a horizontal communication in C_β. (See [14], [2], [16] for category theory, where [16] treats categorical sheaf theory as well.)

1.2 "Why Sheaf?"

Especially in the study of algebraic geometry and complex analytic geometry, sheaf theory and sheaf cohomology theory have been used to connect local properties to global properties. As is described in [10], in this formulation, sheaf theoretic restriction map ρ_U^V from $P(V)$ to $P(U)$ is interpreted as an understanding (perceiving) morphism in a category. Namely, if a section s in $P(U)$, which is called a *thought*, is obtained as $s = \rho_U^V(s')$, where s' is a section of $P(V)$, then section s' is said to be an understanding of section s. We also say that s is *understandable (or perceivable)* if such a V exists satisfying $s = \rho_U^V(s')$, and ρ_U^V is also said to be a *perception morphism.* When there does not exist such a $V \neq U$, s is said to be *preunderstandable.* We will consider this idea in Part 3. As in music or literature, when only a few notes or words are shown, such information is not understandable until enough information is obtained by extending a generalized time period. This corresponds to a covering in sheaf theory. (See Part 3 for a more precise formulation.) One can also formulate the notion of a unique understanding and a misunderstanding of a thought in terms of sheaf language. An *extension (or understandable) problem* : for a given thought s in $P(U)$ whether there exists a thought s' in $P(V)$ so that ρ_U^V may map s' onto s, or not (that is, whether ρ_U^V is epimorphic or not) is an important question. When such an s' exists, s' is said to be an extension of thought s. Note that in sheaf theory, if for any open sets $U \subset V$, $F(V) \xrightarrow{\rho_U^V} F(U)$ is always epimorphic (surjective), then F is said to be a flabby sheaf. It is a simple exercise to rephrase such a notion as a unique extension of s in terms of consciousness terminology. When it is impossible to extend s beyond $P(V)$, then s' is said to be a *terminal thought* of s. Thus, brain functions from local information to global information correspond to realization of the local information as the restriction of the global information in the above sheaf theoretic sense. Two initial motivations for using sheaves for conscious entities are the following. At the very moment when one notices (or discovers) something after some effort, one usually recognizes the fact of discovering before knowing what it is. This type of recognition

corresponds to the map ρ_U^V. A reason why $P(U)$ rather than $P_t = \varinjlim_{t \in U} P(U)$ is considered is that one needs a generalized time period U rather than the exact moment to have awareness.

For applications to physics, the most realistic model for a conscious entity in our sheaf theoretic formulation is the following. Let Ω_ξ be the set of all objects (open sets) in $\boldsymbol{T}$ containing $\xi \in \mathbf{R}$. For P in $\hat{T}$, the object in the product category $\prod_{\alpha \in \Gamma} C_\alpha$

$$P(\Omega_\xi) = \{P(V) | V \in \Omega_\xi\} \qquad (*)$$

indicates the totality of P's awareness at time $\overset{\xi}{\varsigma} \in \mathbf{R}$. Or, for P to exist at $\overset{\xi}{\varsigma} \in \mathbf{R}$ is to assign an object $P(\Omega_\xi)$ of $\prod_{\alpha \in \Gamma} C_\alpha$. See [1], [6]. [11], or [4] for sheaf theory and sheaf cohomology which will be needed in Part 2.

2 Cohomologies, Precohomologies, and Limits

In part 1, a horizontal communication is a morphism between two conscious entities $P(U)$ and $Q(U')$ in a category C. In general, let us consider a sequence in C:

$$---\xrightarrow{\gamma} P(U) \xrightarrow{\delta} Q(U') \xrightarrow{\varphi} R(U'') \xrightarrow{\eta} --- \qquad (2)$$

such that this sequence forms a cochain complex. Namely, any consecutive composition of morphisms in (2) is trivial. In terms of conscious entities, the composite of any consecutive communication is trivial. Then the cohomology at $Q(U')$, denoted by $H^*(-- \to Q(U') \to --)$, is defined as the *subquotient*

$$Ker\varphi \,/\, Im\delta \quad . \qquad (3)$$

Let us consider special cases of the above sequence next. In the case where there is only one conscious entity Q, i.e., the above sequence becomes

$$---\longrightarrow 0 \longrightarrow Q(U) \longrightarrow 0 \longrightarrow --- \quad . \qquad (4)$$

Then the cohomology at $Q(U)$ is $Q(U)$ itself. That is, the subobject of $Q(U)$ which has no influence on anyone is the whole $Q(U)$, and no one influences Q. Namely, the subquotient $Ker\varphi_U / Im\delta_U$, the cohomology at $Q(U)$, is $Q(U)$ itself. Next, consider the case where there are only two conscious entities involved. That is, the above sequence becomes

$$---\longrightarrow 0 \longrightarrow P(U) \xrightarrow{\delta} Q(U) \longrightarrow 0 \longrightarrow --- \qquad (5)$$

Then the cohomology at $Q(U')$ is the quotient $Q(U')/Im\,\delta_U$. That is, the cohomology at $Q(U')$ is the quotient object obtained by regarding the influence or information $Q(U')$ receives from $P(U)$ as the trivial part of $Q(U')$. On the other hand, the cohomology at $P(U)$ is the subobject $Ker\,\delta$. In this case, there is no influence from anyone, and the "core" or "private" conscious part is what P does not share with anyone. As one can observe from these special cases, the cohomology at a conscious entity approximates the core and private consciousness of the entity. When one meditates, (without communication with anyone, namely, *Ker*-part, and closes eyes and listens to nothing, namely, not influenced by anyone, namely, *modulo Im*-part), the cohomology represents the real identity of a conscious entity. However, this is merely the first step in Zen meditation. Some of the goals in meditation will be formulated in Part 3.

In the study of consciousness, it is too strong to assume that sequence (2) always forms a cochain complex. Namely, the influence of influence will not be lost in general. One needs a stronger invariant than cohomology for a sequence which need not be a cochain complex. Such an invariant should coincide with the notion of cohomology when
the sequence happens to be a cochain complex. From a sequence, which is a not necessarily a cochain complex

$$---\xrightarrow{\gamma} P(U) \xrightarrow{\delta} Q(U') \xrightarrow{\varphi} R(U'') \xrightarrow{\eta} ---$$

like (2), we construct the following sequence:

$$---\xrightarrow{\delta^*} Q(U')/Im(\delta\circ\gamma) \xrightarrow{\varphi^*} R(U'')/Im(\varphi\circ\delta) \xrightarrow{\eta^*} --- \ . \qquad (6)$$

One can confirm that sequence (6) becomes a cochain complex. Then we define the *precohomology* at $Q(U')$ as the cohomology of the cochain complex (6), i.e.,

$$Ker\varphi^*/Im\delta^* \ . \qquad \text{(Pre. 3)}$$

We write the precohomology as $Ph^*(--\rightarrow Q(U')\rightarrow --)$. There is a dual definition for constructing a cochain complex. See [7] for this construction, the self-duality theorem, and related properties of precohomology.

The basic yoga for considering cohomology (or precohomology) is that the true nature of a conscious entity in a complex of network of communication and influence in a society is the cohomological object, i.e., the subquotient not the object itself. That is, one should consider the *derived category of conscious entities*. See [6], [4] for the theory of derived category.

Remark Let $Co^+(\hat{U})$ be the category of complexes of conscious entities and let $P^\bullet$ be an object of $Co^+(\hat{U})$. Then such an object $P^\bullet$ is considered as a community or society of conscious entities. We will study the hypercohomology of $P^\bullet$ and spectral sequences associated with a generalized time period and $P^\bullet$ in a forthcoming paper [8].

Next we will consider the notions of inverse limit and direct limit in the context of consciousness. One will notice that the inverse limit of a conscious entity is coherency of conscious entity. Let P be an object of $\hat{T} = \left(\prod_{\alpha \in \Gamma} C_\alpha \right)^{T^{opp}}$. That is, P is a conscious entity. Then, for V in $\boldsymbol{T}$, $P(V)$ is an object of $\prod_{\alpha \in \Gamma} C_\alpha$. Namely, $P(V)$ can be expressed as $P(V) = (P_\alpha(V))_{\alpha \in \Gamma} \in \prod_{\alpha \in \Gamma} C_\alpha$.

Conversely, a family of presheaves $P_\alpha : T^{opp} \longrightarrow C_\alpha, \alpha \in \Gamma$, determines a presheaf $P : T^{opp} \longrightarrow \prod_{\alpha \in \Gamma} C_\alpha$. That is, we have $\hat{T} = \left(\prod_{\alpha \in \Gamma} C_\alpha \right)^{T^{opp}} = \prod_{\alpha \in \Gamma} (C_\alpha{}^{T^{opp}})$.

From part 1, we have the vertical communication $I_\beta^\alpha : P_\alpha(U) \longrightarrow P_\beta(U)$ within the conscious entity P. This communication I_β^α is a typical brain function of the conscious entity P. Then I_β^α induces $\overline{I_\beta^\alpha} : C_\alpha{}^{T^{opp}} \longrightarrow C_\beta{}^{T^{opp}}$ such that $\overline{I_\beta^\alpha}(P_\alpha) = I_\beta^\alpha \circ P_\alpha$. Consequently, we obtain

$$- - - - \longrightarrow C_\alpha{}^{T^{opp}} \xrightarrow{\overline{I_\beta^\alpha}} C_\beta{}^{T^{opp}} \xrightarrow{\overline{I_\gamma^\beta}} C_\gamma{}^{T^{opp}} \longrightarrow - - - \quad . \tag{7}$$

Then, define an inverse limit of conscious entities as

$$\varprojlim_{\alpha \in \Gamma' \subset \Gamma} C_\alpha{}^{T^{opp}} = \{ (P_\alpha) \in \prod_{\alpha, \beta \in \Gamma} C_\alpha{}^{T^{opp}} : \overline{I_\beta^\alpha}(P_\alpha) = P_\beta, \alpha, \beta \in \Gamma' \} \quad . \tag{8}$$

That is, the inverse limit $\varprojlim_{\alpha \in \Gamma' \subset \Gamma} C_\alpha{}^{T^{opp}}$ is a subcategory of the conscious universe $\hat{T} = \left(\prod_{\alpha \in \Gamma} C_\alpha \right)^{T^{opp}} = \prod_{\alpha \in \Gamma} (C_\alpha{}^{T^{opp}})$. Let $\pi_\alpha : \varprojlim_{\alpha \in \Gamma' \subset \Gamma} C_\alpha{}^{T^{opp}} \longrightarrow C_\alpha{}^{T^{opp}}$ be the natural projection to satisfy the universal mapping property. One can also prove $\varprojlim_{\alpha \in \Gamma' \subset \Gamma} C_\alpha{}^{T^{opp}} = (\varprojlim_{\alpha \in \Gamma' \subset \Gamma} C_\alpha)^{T^{opp}}$. From the definition (8) of the inverse limit, the inverse limit is a collection of vertically well communicated conscious entities. The inverse limit $\varprojlim_{\alpha \in \Gamma' \subset \Gamma} C_\alpha{}^{T^{opp}}$ is said to be the collection of coherent or comprehensive conscious entities.

On the other hand, the meaning of the corresponding dual limit of the inverse limit, i.e., the direct limit $\underset{\alpha \in \Gamma^{opp}}{lim} C_\alpha^{T^{opp}}$ may be read as the collection of *final awareness of conscious entities*.

Next, let us consider a direct limit. Intuitively speaking, we make a generalized time period small. For the sequence in a category C as in (2)

$$- - - \xrightarrow{\gamma} P(U) \xrightarrow{\delta} Q(U') \xrightarrow{\varphi} R(U'') \xrightarrow{\eta} - - - , \tag{9}$$

first take the inverse limit in the category C

$$\underset{\leftarrow}{lim}(-- \to P(U) \to Q(U') \to --) . \tag{10}$$

Note that the above inverse limit is the usual inverse limit within a category. In terms of consciousness, the limit (10) may be said to be the *collective consciousness* (or the *conscious tie*) of conscious entities P, Q, R, ---. Next, take the direct limit over generalized periods U, U', U'', --- simultaneously, then we have

$$\underset{T \to}{\lim}(\underset{\leftarrow}{lim}(-- \to P(U) \to Q(U') \to --)) , \tag{11}$$

which is called the *germs of collective consciousness* of P, Q, ---. We will also need

$$\underset{\leftarrow}{lim}(\underset{T \to}{\lim}(-- \to P(U) \to Q(U) \to --)) \tag{11'}$$

for later study.

3 Program

The goal of this section is to build a sheaf theoretic ontology which is consistent with physics. We defined the conscious universe is the category of presheaves, i. e, $\hat{T} = \left(\prod_{\alpha \in \Gamma} C_\alpha \right)^{T^{opp}}$. A conscious entity, i.e., a presheaf in $\hat{T}$, is said to have *thinking ability* or *coherent understanding ability* if the presheaf is a sheaf. See [10] for details. The totality of conscious entities with thinking or coherent understanding ability is the subcategory of $\hat{T}$ which may be said to be the ***conscious topos***, denoted as $\tilde{T}$. Note that the topos $\tilde{T}$ is absolute in the following sense. The composition of the functors

$$\tilde{T} \xrightarrow{Hom(-,-)} \hat{\tilde{T}} \xrightarrow{assoc.sheaf} \tilde{\tilde{T}} \tag{12}$$

is an equivalence of categories, where $Hom_{\tilde{T}}(-,-):\tilde{T}\longrightarrow\hat{\tilde{T}}$ is defined by $P\longrightarrow Hom_{\tilde{T}}(-,P)$. Namely we have $\tilde{\tilde{T}}\approx\tilde{T}$.

The index set Γ may be divided into several parts. The first part of Γ is used for physical world categories. We will use integers as indices for physical categories: C_j, $j = 0, 1, 2, \text{---} \in\Gamma$ where C_0 is the generalized time category $\boldsymbol{T}$ itself, C_1 is the micro world, and C_2 is the macro world. We consider that C_1 and C_2 are discrete categories with structures. In this formulation, the physical existence, i.e., the object in C_2, of a conscious entity like a human being is only a "slice" (or a "foam" as in Zen) of the product category $\prod_{\alpha\in\Gamma}C_\alpha$. For example, non-organic matter M without cognitive functions like non-living things in the usual sense can be considered as a presheaf M such that the trivial components of $M(U)$ are in cognitive categories. For example, even if $M(U)$ appears at two different locations in any distance apart in C_2, as long as it is an entity M, communication of information between the locations should be simultaneous. Namely, an object in the product category $\prod_{\alpha\in\Gamma}C_\alpha$, therefore whose component in each category C_α, is the image of the functor from C_0 to $\prod_{\alpha\in\Gamma}C_\alpha$.

Let P and Q be conscious entities and let U and V be generalized time periods in $\boldsymbol{T}$. Note that for P and Q whose C_2-components of $P(U)$ and $Q(U)$ are non-trivial, and for a morphism from U to V, there are no morphisms from $P(V)$ to $P(U)$ and from $P(U)$ to $Q(U)$ in C_2. This is because category C_2 is discrete. This means that there is no communication in C_2. However, in a cognitive category, there can exist a morphism from $P(U)$ to $Q(U)$. For a conscious entity P and a generalized time period U, the components in these categories of $P(U)$ are the P's awareness in those categories.

Our approach to ontology may begin with the following definition in terms of sheaf-category theory.

3.1 *Definition of Existence.* An E exists if and only if there exists a presheaf E in the conscious universe $\hat{T}$ such that if E is an object in a cognitive category C, then E is *isomorphic* to the C-component of $E(U)$ for a generalized time period U, and if E is an object in a discrete category e.g., C_1 and C_2, then E *equals* the corresponding component of $E(U)$. We say that E exists *purely non-cognitively* (purely physically) when the only non-trivial components of the associated presheaf are in the physical categories like in C_1 and C_2. We also say that E exists *purely cognitively* when the associated presheaf E has non-trivial components only in cognitive categories.

Notice that the above definition of existence is most general in the sense that it gives the meaning of the notion "TO EXIST." Note that a conscious entity like a human being exists physically and cognitively. For example, an electron itself exists purely non-cognitively.

3.2 *Definition of Observation.* Let P be an object (conscious entity) of $\hat{T}$ and let m be an object of $\hat{T}$. Then P *observes* m in C_2 over a generalized time period V if there

exists a category C such that there exists a morphism from $m(V)$ to $P(V)$ in the category C. Notice that in order for P to be able to observe m, $P(V)$ needs to be a non-trivial object in C, i.e., P needs to be non-trivially aware. Namely, V needs to be "large enough" for P to be 'aware' to receive information from another object through a morphism in C. That is, for a smaller $U \subset V$ in V, there may not exist a morphism from $m(V)$ to $P(V)$. More generally, for P and Q in $\hat{T}$, we define: P *observes* Q in a category C if there exists a morphism in C from $Q(V)$ to $P(V)$. Recall that this definition coincides with the definition of the communication (or influence) from $Q(V)$ to $P(V)$. In Zen, one sometimes says that Nature is inside one's Heart. The precise description of this phrase should be as follows. When a conscious entity like human being P observes m in C_2, the above definition of observation means that in a category C there is a morphism f from $m(V)$ to $P(V)$. The observation of the object $m(V)$ by $P(V)$ is the image $f(m(V))$ 'inside' the object $P(V)$. Namely, in Zen this means: When one opens one's eyes to see scenery, i.e., the C_2-component of $m(V)$, one is seeing the image $f(m(V))$ in $P(V)$ of his mind in the category C (kokoro in Japanese).

Next, we would like to apply the concept of a covering of an open set to consciousness. Let V be a generalized time period of T. Consider a covering of V $V = \bigcup V_j$. For a conscious entity P, in a non-discrete cognitive category C, we have a morphism called the restriction map from $P(V)$ to $P(V_j)$. Let m be an elementary particle, e.g., electron, (which is regarded as an object of $\hat{T}$). Then in the discrete category C_1, we have unobserved objects $m(V_j)$ for those V_j. That is, the location of m in C_1 cannot be determined. However, when m is observed by a conscious entity P, for some V there exists a morphism from $m(V)$ to $P(V)$. Then the location of m in C_1 can be determined for this specified V. (See the above definition of observation.) In the following diagram

$$\begin{array}{ccc} m(V) & \longrightarrow & P(V) \\ \downarrow & & \downarrow \\ m(V_i) & & P(V_i) \end{array} \qquad (13)$$

a morphism from $m(V_j)$ to $P(V_i)$ may not exist. For this specified V, the uniquely determined object $m(V)$ exists, and such a morphism from $m(V)$ to $P(V)$ is induced. Namely, our sheaf theoretic formulation provides the quantum properties, i.e., *collapse of the wave* and *the wave property*.

3.3 Remark For a covering of a generalized time period V, i.e., $V = \bigcup_i V_i$, consider the following diagram (13').

$$\begin{array}{ccc} m(V) & & P(V) \\ \downarrow & & \downarrow \\ m(V_i) & \longrightarrow & P(V_i) \\ \downarrow & & \downarrow \\ m(V_i \cap V_j) & \rightarrow & P(V_i \cap V_j) \end{array} \qquad (13')$$

If for all i and j in the indexed set, the observations s_i during the generalized time periods V_i of m coincide with the observations s_{ij} during the intersections $V_i \cap V_j$, then since P is a sheaf, there exists s in $P(V)$ such that the restriction of s to each V_i coincides with s_i, i.e., $s = \rho^V_{V_i}(s_i)$. This indicates that the global observation of m in $\hat{T}$ by a conscious entity P in $\tilde{T}$ can be obtained by the local observation data of m by P.

3.4 *Definition of Wave State* Let m be an object of $\hat{T}$, e.g., electron. Then the wave state of m is defined by the collection $\{m(V)\}$ where unspecified (undetermined) generalized time periods V belong to Ω_ξ. Simbolically, the wave state of m in $\hat{T}$ at $\xi \in \mathbf{R}$ is defined as $m(\Omega_\xi)$. When m in $\hat{T}$ is observed by a conscious entity P in $\tilde{T}$, the unbiguity of the choice of V in Ω_ξ is collapsed, namely, spesfiying V in Ω_ξ.

As an application of this dependency on C_0 as a domain of a functor, i.e., consequently, the simultaneity in C_2, we will give a sheaf theoretic interpretation of the Einstein-Podolsky-Rosen paradox. Here is a sketch of our formulation. A full length paper will appear in [9]. Consider one state comprised of two particles e.g., two electrons (e, e') with opposite spins. Let us denote the presheaf associated with the pair by *(e, e')*. Since our focus is on category C_2, for a generalized time period V, we consider the C_2-component of *(e, e')(V) = (e(V), e'(V))*. Namely, a state is determined by a generalized time period V. When one measures (observes) the spin of one particle, i.e., by specifying a generalized time period V' in Ω_ξ, one will know what the spin of the other is simultaneously since a state is totally determined by the time period V': *(e, e')(V') = (e(V'), e'(V'))*. See the above definition 3.2 of observation and definition 3.4 of wave state.

During meditation, it is ideal for one to think nothing. Then, as we mentioned earlier, the cohomological object, i.e., the subquotient, is important. In deeper meditation, it may be said that to make all the components of $P(U)$ final (and initial) objects in categories is even more important. When one thinks nothing, each object in each category C_α, $\alpha \neq 1,2,---$, is a trivial object. Then the cohomology is isomorphic to the original object, i.e., the trivial object. In Zen, "It is the oneness with the wholeness," might mean that to a final object in each category there exists a morphism (communication) from every object. Then the self-awareness map $\rho^U_U : P(U) \longrightarrow P(U)$ is a trivial morphism where $P(U)$ is a final object in a category. A fractal-like self-similarity equation appears when one formulates this in terms of a sheaf category setting. Among $\{C_\alpha\}_{\alpha \in \Gamma}$, let C_ω be the conscious universe $\hat{T}$ itself.

Let P be a conscious entity in $\hat{T}$ and let V be a generalized time period in $\boldsymbol{T}$ as before. Then the ω-component of $P(V)$ is a conscious entity in $\hat{T}$. Namely, $P_\omega(V)$ is an object of $\hat{T}$. Hence, it does make sense to evaluate at a generalized time V'. That is, one can consider $(P_\omega(V))(V')$, which is an object of $\prod_{\alpha\in\Gamma} C_\alpha$. Then by considering its components in C_0 and/or C_ω repeatedly, one can obtain various ***self-similarity*** equations

$$P^l(V'^m) \tag{14}$$

where l, $m = 1, 2, 3, \cdots$, and the subscripts 0 and ω are omitted in (14). According to the equation (14), when one says "I," one would not know of which level of "I" one is speaking. Let us consider a special sequence in a cognitive category C as follows:

$$\cdots \to P(U) \xrightarrow{\rho_U^U = id_{P(U)}} P(U) \xrightarrow{\rho_U^U = id_{P(U)}} P(U) \to \cdots . \tag{15}$$

Then the usual inverse limit (in the sense that the limit is taken in one category) of this sequence (15): $\varprojlim P(U)$ is an object in C. This inverse limit indicates the high self-awareness of P in C. Equation (14) and the inverse limit $\varprojlim P(U)$ of (15) both indicate the ambiguity of the notion of "Self." The inverse limit of (15) corresponds to one of the fundamental introductory questions in Zen: *Who is that I who asks who I am* ?.

It is not clear what the direct limit $\varinjlim P(U)$ of the sequence (14) means in terms of consciousness.

4 Conclusion

We capture the awareness of an entity P as the image of a contravariant functor (i.e., a presheaf) from a generalized time category $\boldsymbol{T}$ into a product category of categories. More precisely, we have equation (*): $P(\Omega_\xi) = \{P(V) | V \in \Omega_\xi\}$ in $\prod_{\alpha\in\Gamma} C_\alpha$. An auto-communication within the entity P, including e.g., understanding and self-awareness, is a morphism from $P(V)$ to $P(V')$, where V and V' are objects of $\boldsymbol{T}$. Communication (information exchange) between two entities P and Q is a natural transformation between them. We build a scheme in terms of sheaf theory and categorical notions so that the interactions among entities with mind (conscious entity in the usual sense) and entities without mind (matter in the usual sense) can provide ontologically consistent precise formulations of an observation by a conscious entity, hence the collapse of the wave in quantum mechanics. The notions of limits and cohomological objects provide formulations for higher mental activities of conscious entities.

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