A new Approach to Unification of Potential Fields Using GLT Model

Branko Novakovic, Dario Novakovic, and Alen Novakovic
FSB - University of Zagreb, Luciceva 5, P. O. B. 509,
10000 Zagreb, CROATIA
Fax. (+385) 1 61 68 354, E-mail: branko.novakovic@fsb.hr

Abstract

Because a scalar potential of a gravitational field has a unit of a specific energy (J/kg), the problem of unification of potential fields has been transformed into the problem of unification of specific potential energies of a particle in a multi-potential field. Consequently, the parameters α and α' of GLT model become the functions of a unified specific potential energy in a multi-potential field. Since all items like field tensors and Klein, Gordon and Fock equation are functions of parameters α and α', these items can be applied to multi-potential fields. Thus, a field tensor of a unified specific potential energy of a particle in central symmetric electromagnetic and gravitational fields in vacuum is derived. Finally, it has been shown that a momentum equation of photons will remain unchanged even if the photons may have the mass.

Keywords: Scalar and Vector Potentials of Gravitational Field, Unification of Specific Potential Energies, General Covariant Energy Equation, General Klein, Gordon and Fock Equation.

1 Introduction

There have been many attempts to create a unified theory of all fundamental laws of physics [1, 2, 3 ..]. The main problem is to put together General Relativity and Quantum Mechanics into one self consistent theory. The most popular candidate for the unified theory is Superstring Theory, in which all particles are just different vibration modes of very small loops of string. It is expected that a unified theory exists at the Planck scale, where all forces of nature are unified and quantum gravity is significant. If a unified theory can be constructed at all, then the first step should be a unification of specific potential energies of a particle in multi-potential fields. This paper presents one of the possible ways to create it. Starting with the new General Lorentz Transformation model (GLT-model in [4,5]) a general covariant energy equation and general momentum and Klein, Gordon and Fock (KGF) equations have been derived as a functions of two parameters α and α'. If the particle is in a multi-potential field, then parameters α and α' are functions of unified specific potential energies of that field. Thus, the point is to find out these functions. It has been done firstly for gravitational and electromagnetic fields and than has been generalized to a multi-potential field. In that case the generalized covariant energy equation and momentum and KGF equations are valid for a multi-potential field. Thus, the covariant energy equation of a particle in a gravitational field is obtained from the generalized one, after substitution of the identified parameters α and α' from the well-known Einstein equation of a gravitational field [5]. Comparing the covariant energy equation of a particle in a gravitational field with a related one of an electromagnetic field, we derived, on the natural way, scalar and vector potentials (i.e. a four-potential vector) of a gravitational field. Since a scalar potential of a gravitational field has a unit of a specific energy (J/kg), the problem of unification of potential fields has been transformed into the problem of unification of specific potential energies of a particle in a multi-potential field. This was very important step of our investigation, because it makes an ability to unification of specific potential energies of a multi-potential field, and gives more symmetry between gravitational and electromagnetic fields. This symmetry makes the possibility to create a new form of tensor of a gravitational field, Maxwell like equations of a gravitational field and, what is very important, an application of usual quantisation methods to a gravitational field (let say a canonical quantisation of an electromagnetic field [6]). In the case of the Newton limitation ($v << c$), this model is reduced to the well-known Newton model of a gravitational field, what we expected that should be.

The second aim of our investigation was to create procedures of unification of specific potential energies of a particle in a multi-potential field. This paper approach is based on the fact that parameters α and α' are functions of a state of a specific potential energy of a particle in each space-time point. Generally, a particle can be in a multi-potential field with n-different potentials. Here we have the problem of unification of all specific potential energies of a particle in that multi-potential field. This problem has been solved by an introduction of so called a specific potential energy four-vector of a certain multi-potential field. These specific potential energies have been defined in the form that enables to add all specific potential energies of a particle in a multi-potential field. The next step was to connect these specific potential energies with parameters α and α' through the corresponding mathematical relation. Since the all items like field tensors, field equations, energy equations and so on, can be described as functions of parameters α and α', these items become the functions of unified specific potential energies of a particle in multi-potential field. As an example, the field tensor of unified specific potential energy of a particle in central symmetric gravitational and electromagnetic fields has been derived.

In order to connect this approach with the Quantum Mechanics it has been derived a general energy-momentum equation, a general frequency - wave length relation and a general quantum Klein, Gordon and Fock (KGF) relativistic equation [7,8], as functions of parameters α and α'. These items have been derived by employing the general non-diagonal line element of GLT model given in [5]. Thus, the mentioned items are valid for a particle in a multi-potential field, after including the parameters α and α' as functions of a unified specific potential energy of that field. Since the general non-diagonal line element can be the starting point for derivation of a related wave function of a particle in a multi-potential field (including gravity), it seams that a general line element could be the candidate for the basic element of superstring theory.

A special attention has been devoted to the momentum equation and energy-momentum equation for particles with velocity equal to the light velocity in vacuum (i.e. v = c). It has been shown that momentum and energy-momentum equations of photons and gravitons will remain unchanged, even if the photons and the gravitons may have the mass ($m_o \neq 0$). It opens the possibility that photons and gravitons may really have the mass.

2 Derivation of General Covariant Energy Equation

The parameters α and α′ in GLT-model [4,5] define observation signal velocities in systems O and O′, where the system O′ is moving relatively to the system O with an arbitrary velocity v, along an arbitrary radius vector r. Thus, one can define that an observation signal (which is a bearer of information) has the velocity αc in system O, and α′c in system O′, where c is a constant reference signal velocity (let say c is a speed of light in vacuum). Generally, the parameters α and α′ are functions of the space-time coordinates:

$$\alpha = f(ct,x,y,z), \qquad \alpha' = f'(ct',x',y',z') \tag{1}$$

In the GLT-model, it has been employed (for the convenience) the parameter δ, where δ = 1, if an observer signal is emitted from the origin of the system O, and δ = -1 if an observation signal is emitted from the origin of the system O′.
Thus, the covariant energy equation can be derived from a general covariant metric tensor of the full form of the GLT-model [4, 5]:

$$[g_{ij}] = \begin{bmatrix} -1 & bx & by & bz \\ bx & 1 & 0 & 0 \\ by & 0 & 1 & 0 \\ bz & 0 & 0 & 1 \end{bmatrix}, \tag{2}$$

where the elements b_x, b_y and b_z are given by the equations:

$$bx = \frac{-\delta(\alpha-\alpha')_x}{2\sqrt{\alpha\alpha'}}, \quad by = \frac{-\delta(\alpha-\alpha')_y}{2\sqrt{\alpha\alpha'}}, \quad bz = \frac{-\delta(\alpha-\alpha')_z}{2\sqrt{\alpha\alpha'}}. \tag{3}$$

Now, the general form of the covariant energy equation E_c , can be derived by the procedures:

$$\begin{gathered} P_0 = g_{0j}\,P^j\,, \quad j = 0,1,2,3, \quad P_0 = -P^0 + b_x P^1 + b_y P^2 + b_z P^3, \\ P^0 = H\,m_0\sqrt{\alpha\alpha'}\,c, \quad P^i = H\,m_0\,v^i, \quad i = 1,2,3. \quad E_c = -P_0\sqrt{\alpha\alpha'}\,c\,, \\ E_c = H\,m_o\left[\alpha\alpha' c^2 + \frac{\delta(\alpha-\alpha')c\cdot\upsilon}{2}\right], \end{gathered} \tag{4}$$

where m_o is a particle rest mass, v is a particle velocity, P^i , i = 0, 1, 2, 3, are contravariannt momentum components, P_0 is a zero-component of a covariant momentum and parameter H is given by the equation:

$$H = 1 / \left[1 - \frac{\upsilon^2}{\alpha\alpha' c^2} + \frac{\delta(\alpha - \alpha')c \cdot \upsilon}{\alpha\alpha' c^2} \right]^{1/2} . \qquad (5)$$

It is easy to see, that in the case $\alpha = \alpha' = 1$ (there is no existence of any external potential field), the covariant energy equation (4) is transformed into the well-known Einstein's energy equation E:

$$E_c = E = H\, m_0\, c^2, \qquad H = \gamma = 1 / \left(1 - \frac{\upsilon^2}{c^2} \right)^{1/2} . \qquad (6)$$

3 Derivation of Four-Potential Vector of Gravitational Field

The observation signal velocity depends on the state of the specific potential energy in the field in which it is propagated. Thus, the parameters α and α' satisfy the Einstein's field equations of a gravitational field if they have the form [5]:

$$\delta = 1, \quad \alpha = 1 + \frac{\phi}{c^2} = 1 - \frac{GM}{rc^2}, \quad \alpha' = 1, \quad \delta = -1, \quad \alpha = 1, \quad \alpha' = 1 + \frac{\phi}{c^2} = 1 - \frac{GM}{rc^2}, \quad (7)$$

where $\phi = -GM / r$ is a scalar potential of a central symmetric gravitational field. On the other hand, the parameter α and α' satisfy the covariant energy equation (4) if they have the form:

$$\delta = 1, \quad \alpha = 1 - \frac{qV}{m_0 c^2}, \qquad \alpha' = 1, \quad \delta = -1, \qquad \alpha = 1, \qquad \alpha' = 1 - \frac{qV}{m_0 c^2}, (8)$$

where $V = q_s / r$ is a scalar potential of a central symmetric electromagnetic field and q and q_s are a particle and a potential source charges, respectively. Applying the parameters α and α' from (7) to the E_c equation in (4) we obtain the covariant energy equation of a particle in a central symmetric gravitational field:

$$E_c = H \left[m_0\, c^2 - \frac{m_0 GM}{r} - \frac{m_0 GM \cdot \upsilon}{2rc} \right] . \qquad (9)$$

In the case of a free fall motion in a central symmetric gravitational field we have $\upsilon = \upsilon_{ff} = -GM / rc$, and (9) is transformed into the form:

$$E_c = H \left[m_0\, c^2 - \frac{m_0 GM}{r} + \frac{m_0}{2} \left(\frac{GM}{rc} \right)^2 \right] . \qquad (10)$$

From (10) one can recognize a particle rest mass energy ($m_o c^2$), a gravitational potential energy (- $m_o GM / r$), and a gravitational kinetic energy ($m_o(GM / r\, c)^2 / 2$). On the other hand, applying parameters α and α' from (8) to the E_c equation in (4) we obtain the covariant energy equation of a particle in a central symmetric electromagnetic field:

$$E_c = H\left[m_0 c^2 - qV - \frac{qV \cdot \upsilon}{2c} \right]. \tag{11}$$

In the case of a free fall motion in a central symmetric electromagnetic field we have a free fall velocity $\upsilon = \upsilon_{ff} = -qV / m_0 c$, and (11) is transformed into the equation:

$$E_c = H\left[m_0 c^2 - qV + \frac{m_0 (qV / m_0 c)^2}{2} \right]. \tag{12}$$

From (12) one can recognize a particle rest mass energy ($m_o c^2$), an electric potential energy (- qV), and an electric kinetic energy ($m_o(qV/m_o c)^2 / 2$). Further, from the equation (11) one can recognize a scalar potential, $A^0{}_e$, and a vector potential, A_e of a central symmetric electromagnetic field:

$$A_e^0 = V, \quad A_e = \frac{V \cdot \upsilon}{c} = \frac{A_e^0 \cdot \upsilon}{c}, \quad A_e^1 = \frac{V_x \upsilon_x}{c}, \quad A_e^2 = \frac{V_y \upsilon_y}{c}, \quad A_e^3 = \frac{V_z \upsilon_z}{c}, \tag{13}$$
$$A_e = \left[A_e^1, A_e^2, A_e^3 \right] \qquad \underline{A}_e = \left[A_e^0, A_e^1, A_e^2, A_e^3 \right]$$

where $\underline{A}_e$ is a four-vector of electromagnetic potentials.

An analogous approach can be employed in a central symmetric gravitational field. Thus, from the equation (9) we can recognize a scalar potential, $A^0{}_g$, and a vector potential, A_g , of a central symmetric gravitational field:

$$A_g^0 = \phi = \frac{-GM}{r}, \quad A_g = \frac{\phi \cdot \upsilon}{c} = \frac{A_g^0 \cdot \upsilon}{c}, \quad A_g^1 = \frac{\phi_x \upsilon_x}{c}, \quad A_g^2 = \frac{\phi_y \upsilon_y}{c}, \quad A_g^3 = \frac{\phi_z \upsilon_z}{c}, \tag{14}$$
$$A_g = \left[A_g^1, A_g^2, A_g^3 \right] \qquad \underline{A}_g = \left[A_g^0, A_g^1, A_g^2, A_g^3 \right]$$

where $\underline{A}_g$ is a four-vector of a gravitational potentials. So, an unknown vector potential of a central symmetric gravitational field has been derived on the natural way.

4 Derivation of a Unified Specific Potential Energy of Particle in a Multi-Potential Field

Now, the covariant energy equation of a particle in an electromagnetic field can be described as a function of the scalar and vector potentials:

$$E_c = H\left[m_0 c + qA_e^0 + \frac{qA_e^0 \cdot \upsilon}{2c} \right], \qquad E_c = H\left[m_0 c + qA_e^0 + \frac{qA_e}{2} \right].. \tag{15}$$

where both q and $A^0{}_e$ can have a positive or negative sign. On the same way, the covariant energy equation of a particle in a gravitational field can also be the function of the corresponding scalar and vector potentials:

$$E_c = H\left[\, m_0c^2 + m_0A_g^0 + \frac{m_0A_g^0 \cdot \upsilon}{2c} \,\right], \qquad E_c = H\left[\, m_0c^2 + m_0A_g^0 + \frac{m_0A_g}{2} \,\right]. \quad (16)$$

Meanwhile, the problem is in the unification of electric and gravitational potentials, when a particle is in electromagnetic and gravitational fields at the same time. We know that two potentials can be added, but they must have the same dimension. Therefore, direct unification of electric and gravitational potentials can not be done, because of different dimensions. Since the scalar potential of a gravitational field has a unit of specific energy (J/kg), the problem of unification of potential fields has been transformed into the problem of unification of specific potential energies of a particle in a multi-potential field. Accordingly, one can unify specific potential energies of a particle in electromagnetic and gravitational fields. Generally, a particle can be in a multi-potential field with n-potentials. Here we have the problem of unification of all specific potential energies of a particle in a multi-potential field. This problem can be solved by introducing **a constant parameter η_i**, $i = 1, 2, .., n$, in the form that satisfies the following dimensional relation:

$$\dim\left(\eta_i\, A_i^0\right) = J / kg, \quad i = 1,2,...,n, \qquad (17)$$

where J means the energetic unit joule. The term $\eta_i A_i^0$ we called a specific potential energy of a particle in an i-th potential source. Since, all pairs $\eta_i A_i^0$ have the same dimension (17), they can be added in order to derive a unified scalar specific potential energy A^0:

$$A^0 = \sum_i \eta_i\, A_i^0, \qquad i = 1,2,...,n, \qquad (18)$$

where both η_i and A_i^0 can be positive or negative terms. Consequently, a unified three-vector of a specific potential energy, A, has the form:

$$A = \frac{A^0 \cdot \upsilon}{c} = \left(\sum_i \eta_i A_i^0 \right) \cdot \frac{\upsilon}{c} = \sum_i \left(\eta_i A_i^0 \cdot \frac{\upsilon}{c} \right). \qquad (19)$$

The corresponding unified four-vector of the specific potential energy, $\underline{A}$, is described by the relation:

$$\underline{A} = \left[A^0, A^1, A^2, A^3 \right] = \left[A^0, A^x, A^y, A^z \right]. \qquad (20)$$

This approach of unification of scalar and vector specific potential energies of a particle in a multi-potential field, can be applied to the unification of scalar and vector specific potential energies of a particle in electromagnetic and gravitational fields. It is easy to see that the term $\eta_g A^0_g$ of a central symmetric gravitation field satisfies the dimensional relation (17) if the parameter $\eta_g = 1$, and $A^0_g = - GM / r_g$. On the other hand, the term $\eta_e A^0_e$ of a central symmetric electromagnetic field satisfies the dimensional relation (17) if the parameter $\eta_e = \mp (q / m_0) = \mp G_e = \text{const.}$, and $A^0_e = q_s / r_e$, where the sign of η_e is negative or positive if q is negative or positive charge of a particle with rest mass m_0, and q_s is positive or negative charge a potential source. Here both q and q_s are in units $(kgm)^{1/2}$m/s. Thus, the specific potential energies $\eta_g A_g^0$ and $\eta_e A_e^0$ are given by the forms:

$$\eta_g A_g^0 = -\frac{GM}{r_g}, \qquad \eta_e A_e^0 = \mp \frac{G_e q_s}{r_e}, \tag{21}$$

The unified specific potential energy of a particle in central symmetric electromagnetic and central symmetric gravitational fields is given by the relations (18) and (21):

$$A^0 = \eta_g A_g^0 + \eta_e A_e^0 = \frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}. \tag{22}$$

where G_e is a constant ratio of a particle charge and mass, r_g is a radius of a particle position in central symmetric gravitational source and r_e is a radius of a particle position in central symmetric electric source. The related unified three-vector of the specific potential energy of a particle in central symmetric gravitational and central symmetric electromagnetic fields can be obtained by using (19) and (21):

$$A = -\frac{GM}{r_g c} \cdot \upsilon \mp \frac{G_e q_s}{r_e c} \cdot \upsilon = \left(\frac{-GM}{r_g c} \mp \frac{G_e q_s}{r_e c} \right) \cdot \upsilon. \tag{23}$$

Since the velocities of the free fall motions in central symmetric gravitational and in central symmetric electromagnetic fields are given by the relations:

$$\upsilon_{ffg} = -\frac{GM}{r_g c}, \qquad \upsilon_{ffe} = \mp \frac{G_e q_s}{r_e c}, \tag{24}$$

one can conclude from (23) that the magnitude of the three-vector of the unified specific potential energy is, in fact, a scalar product of a unified free fall velocity and a particle velocity in a certain multi-potential field. Following the previous considerations one can, generally, define the content of the parameters α and α′ for a particle in a multi-potential field :

$$\delta = 1, \quad \alpha = 1 + \frac{A^0}{c^2} = 1 + \sum_i \eta_i A_i^0 / c^2, \quad \alpha' = 1, \quad \delta = -1, \quad \alpha = 1, \quad \alpha' = 1 + \sum_i \eta_i A_i^0 / c^2, \tag{25}$$

where both η_i and A_i^0 , i = 1, 2, ..., n, can be positive or negative terms. It is ease to see that all terms ($\eta_i\ A_i^0 / c^2$) are non-dimensional terms. Applying α and α′ from (25) to the E_c equation in (4) we obtain (in both cases δ = 1, and δ = -1), the general covariant energy equation of a particle in multi-potential field:

$$E_c = Hm_0\left[\left(1+\frac{1}{c^2}\sum_i \eta_i A_i^0\right)c^2+\frac{\left(\sum_i \eta_i A_i^0\right)\cdot \upsilon}{2c}\right],\ E_c = Hm_0\left[\left(1+\frac{A^0}{c^2}\right)c^2+\frac{A^0\cdot\upsilon}{2c}\right],$$

$$E_c = Hm_0\left[\left(1+\frac{A^0}{c^2}\right)c^2+\frac{A}{2}\right],\qquad E_c = H\left[m_0c^2+m_0A^0+\frac{m_0A}{2}\right]. \tag{26}$$

On the other hand, if we know a potential energy of a particle in the i-th potential field, U_i, then parameters α and α' in a multi-potential field have the form:

$$\begin{aligned}&\delta=1,\quad \alpha=1+\sum_i\frac{U_i}{m_0c^2},\qquad \alpha'=1,\\ &\delta=-1,\quad \alpha=1,\qquad \alpha'=1+\sum_i\frac{U_i}{m_0c^2},\quad i=1,2,\ldots n.\end{aligned} \tag{27}$$

In that case unified scalar and vector specific potential energies of a particle in a multi-potential field can be calculate by the relations:

$$A^0=\sum_i\frac{U_i}{m_0},\qquad A=\left(\sum_i\frac{U_i}{m_0}\right)\cdot\frac{\upsilon}{c}, \tag{28}$$

where U_i / m_0 means a specific potential energy of the particle in the i-th potential field. If a particle is in electromagnetic and gravitational central symmetric potential fields then parameters α and α' are calculated by employing the relations (22):

$$\begin{aligned}&\delta=1,\quad \alpha=1-\frac{GM}{r_gc^2}\mp\frac{G_eq_s}{r_ec^2},\quad \alpha'=1,\\ &\delta=-1,\quad \alpha=1,\quad \alpha'=1-\frac{GM}{r_gc^2}\mp\frac{G_eq_s}{r_ec^2}.\end{aligned} \tag{29}$$

Thus, for a particle with negative or positive charge q, with rest mass m_0, moving in a two-potential electromagnetic and gravitational central symmetric field with a velocity v, the covariant energy can be calculated by the equations (4) and (29), or (22) and (26):

$$E_c = H\left[m_0c^2-\frac{m_0GM}{r_g}\mp\frac{qq_s}{r_e}-\frac{m_0}{2}\frac{GM}{r_gc}\cdot\upsilon\mp\frac{qq_s}{2r_ec}\cdot\upsilon\right], \tag{30}$$

where parameter H has to be calculated by employing A^0 and A from (22) and (23), respectively, and the equation :

$$H=1/\left[1-\frac{\upsilon^2}{c^2+A^0}+\frac{A}{c^2+A^0}\right]. \tag{31}$$

Remarks. One can choose parameter $\eta_e = \mp G_e = \mp(qq_s / m_o m_s) = \text{const.}$, and $A^0_e = m_s / r_e$, where m_s is a mass with charge q_s. In that case $\dim(G) = \dim(G_e) = m^3/kgs^2$. If $m_s = M$, and $r_e = r_g = r$, then a unified specific potential energy of a particle in electromagnetic and gravitational fields can be calculated by the relation:

$$A^0 = \eta_g A^0_g + \eta_e A^0_e = (-G \mp G_e)\frac{M}{r} = G_u \frac{M}{r}, \tag{32}$$

where G_u is a unified constant for related electromagnetic and gravitational fields.

The second remark comes from the covariant energy equation of a particle in multi-potential field (26). Compare this equation with the E_c equation in (4), one can derive the following relations:

$$\begin{aligned} \alpha\alpha' &= 1 + \frac{1}{c^2}\sum_i \eta_i A^0_i, & \delta(\alpha - \alpha') &= \frac{1}{c^2}\sum_i \eta_i A^0_i, \\ \alpha\alpha' &= 1 + \frac{A^0}{c^2}, & \delta(\alpha - \alpha') &= \frac{A^0}{c^2}, \\ \alpha\alpha' &= 1 + \frac{A^0}{c^2}, & \delta(\alpha - \alpha')\, c \cdot \upsilon &= A\,. \end{aligned} \tag{33}$$

Since the elements $\alpha\alpha'$ and $\delta(\alpha - \alpha')$ are constituents of the items like a general line element, the Einstein field equations, the Maxwell equations, the general momentum equations, the general quantum relativist Klein, Gordon and Fock equation (KGF-eq.) and so on, it seams that the relations (33) can be employed for a unification of the Einstein Special and General Theory of Relativity and Quantum Mechanics. Thus, this approach maybe opens the possibility to create the so cold unified (final) theory in physics.

5 Derivation of a Field Tensor of Unified Specific Potential Energy of Particle in a Multi-Potential Field

In order to derive a field tensor of unified specific potential energy of a particle in a multi-potential field one can start with the unified scalar and vector specific potential energies given by (18) to (20). Following the procedures for derivation of a tensor of an electromagnetic field, the components of the field tensor of a unified specific potential energy of a particle in a multi-potential field can be calculate by using the equations:

$$\begin{aligned} &F^{0i} = -\left(\frac{\partial A^0}{\partial x^i} + \frac{\partial A^i}{\partial x^0}\right), \quad F^{i0} = -F^{0i}, \qquad i = 1,2,3, \\ &F^{ij} = \frac{\partial A^j}{\partial x^i} - \frac{\partial A^i}{\partial x^j}, \qquad i,j = 1,2,3, \qquad F^{00} = 0\,, \\ &x = \left[x^0, x^1, x^2, x^3\right] = \left[\sqrt{\alpha\alpha'}ct,\, x,\, y,\, z\right]. \end{aligned} \tag{34}$$

As the result of that calculation we obtain the field tensor of a unified specific potential energy of a particle in a multi-potential field in the form :

$$F^{ij} = \begin{bmatrix} 0 & F^{01} & F^{02} & F^{03} \\ -F^{01} & 0 & F^{12} & -F^{31} \\ -F^{02} & -F^{12} & 0 & F^{23} \\ -F^{03} & F^{31} & -F^{23} & 0 \end{bmatrix}. \tag{35}$$

It is easy to see that the dimension of F^{ij} is given by the relation:

$$\dim\left[F^{ij}\right] = \frac{J}{kgm} = \frac{N}{kg} = \frac{m}{s^2}, \tag{36}$$

where N means Newton (i.e. the basic unit of a force). Thus, the field tensor of a unified specific potential energy of a particle in a multi-potential field is the tensor of accelerations. As we expected, this tensor is anti-symmetric, and has the trace equal to zero. The corresponding unified acceleration field is described by the relations:

$$F^0 = \left[F^{01}, F^{02}, F^{03}\right], \qquad F^1 = \left[F^{23}, F^{31}, F^{12}\right], \tag{37}$$

where F^0 is a three-vector related to the time-space coordinates, while F^1 is a three-vector related to the space-space coordinates. Generally, following the analogy with an electromagnetic field, the three-vectors F^0 and F^1 of a unified acceleration field in a vacuum, can be calculated by the equations:

$$F^0 = -\, grad\, A^0 - \frac{\partial A}{\partial\sqrt{\alpha\alpha'}ct}, \qquad F^1 = rot\, A\ , \tag{38}$$

where F^0 and F^1 are given by (37), and A^0 and A are presented by (18) and (19), respectively. Since the field tensor of a unified specific potential energy of a particle in multi-potential field is given by (34) and (35), one can derive the Maxwell's like equations of a that field in a vacuum in the tensor form, following the related relations for an electromagnetic field in a vacuum:

$$\begin{gathered} F^{\mu\nu}{}_{,\nu} = 0\ , \qquad F_{\mu\nu} = g_{\mu\alpha} g_{\nu\beta} F^{\alpha\beta}, \\ F_{\mu\nu;\lambda} + F_{\nu\lambda;\mu} + F_{\lambda\mu;\nu} = 0, \quad \mu,\nu,\lambda = 0,1,2,3, \end{gathered} \tag{39}$$

where $F_{\mu\nu}$ and $F^{\mu\nu}$ are covariant and contravariant components of a field tensor given by (34) and (35), and $g_{\mu\nu}$ is a metric tensor obtained by substitution parameters α and α' from (25) into the equations (2) and (3).

Now, one can use the previous procedure for derivation of the field tensor of unified specific potential energy of a particle in central symmetric electromagnetic and gravitational fields in vacuum. For this purpose we shall start with the calculation of the elements of that tensor, employing (22), (23) and (34). Thus, the three-vector F^0 related to the time-space coordinates is given by the relations:

$$F^{01} = -\frac{\partial}{\partial x}\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right) - \frac{\partial}{\partial\sqrt{\alpha\alpha'}ct}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_x \frac{\upsilon^x}{c}\right],$$

$$F^{02} = -\frac{\partial}{\partial y}\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right) - \frac{\partial}{\partial\sqrt{\alpha\alpha'}ct}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_y \frac{\upsilon^y}{c}\right], \quad (40)$$

$$F^{03} = -\frac{\partial}{\partial z}\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right) - \frac{\partial}{\partial\sqrt{\alpha\alpha'}ct}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_z \frac{\upsilon^z}{c}\right].$$

The components of the three-vector F^1 related to the space-space coordinates are given by the equations:

$$F^{23} = \frac{\partial}{\partial y}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_z \frac{\upsilon^z}{c}\right] - \frac{\partial}{\partial z}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_y \frac{\upsilon^y}{c}\right],$$

$$F^{31} = \frac{\partial}{\partial z}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_x \frac{\upsilon^x}{c}\right] - \frac{\partial}{\partial x}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_z \frac{\upsilon^z}{c}\right], \quad (41)$$

$$F^{12} = \frac{\partial}{\partial x}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_y \frac{\upsilon^y}{c}\right] - \frac{\partial}{\partial y}\left[\left(\frac{-GM}{r_g} \mp \frac{G_e q_s}{r_e}\right)_x \frac{\upsilon^x}{c}\right].$$

In order to calculate the tensor components $F^{\mu\nu}$ by employing (40) and (41) one has to know for the each concrete case the relations:

$$r_e = f_e(x, y, z)\,, \qquad r_g = f_g(x, y, z)\,, \qquad (42)$$

related to an electromagnetic potential source, and to a gravitational potential source, respectively. Now, one can calculate the related Maxwell like equations in vacuum by using (39), (40) and (41).

Remarks. In the case where a gravitational potential source does not exist, one can put the parameter $M = 0$ in the equations (40) and (41). For that case one can cut down the parameter G_e in the equations (40) and (41), and the equations (39) are transformed into the well-known Maxwell equations of a central symmetric electromagnetic field in vacuum. On the other hand, in the case where an electric potential source does not exist, we can put the parameter $q_s = 0$ in the equations (40) and (41). For that case the equations (39) are transformed into the Maxwell like equations for a central symmetric gravitational field in vacuum. The four - force vector on a particle in a multi-potential field and the corresponding differential equations of a particle motion in that field will be presented in the next paper.

6 Derivation of General Momentum Equation and Klein, Gordon and Fock Equation

In order to derive the general momentum and Klein, Gordon and Fock (KGF) equations, one can start with the general form of the line element, described in [5] by the relation:

$$-ds^2 = \alpha\alpha' c^2\left(1-\frac{\upsilon^2}{\alpha\alpha' c^2}+\frac{\delta(\alpha-\alpha')\upsilon}{\alpha\alpha' c}\right)dt^2 . \tag{43}$$

Applying the multiplication of the equation (43) by $H^2 m_o^2$, where m_o is a particle rest mass, and dividing by dt^2, the equation (43) can be transformed into the form:

$$-H^2 m_0^2\left(\frac{ds}{dt}\right)^2 = H^2 m_0^2 \alpha\alpha' c^2 - H^2 m_0^2 \upsilon^2 + H^2 m_0^2 \delta(\alpha-\alpha')\, c\upsilon . \tag{44}$$

On the other hand, general forms of the contravariant energy E^c and the momentums P^s, P^0, P and P^g can be described by the following relations:

$$\left(\frac{ds}{dt}\right)^2 = \left(\upsilon^s\right)^2 = \frac{-\alpha\alpha' c^2}{H^2}, \qquad \left(P^s\right)^2 = \left(Hm_0\,\upsilon^s\right)^2, \qquad E^c = Hm_0\alpha\alpha' c^2,$$
$$\left(P^0\right)^2 = \left(Hm_0\sqrt{\alpha\alpha'}c\right)^2 = \frac{(E^c)^2}{\alpha\alpha' c^2}, \qquad (P)^2 = (Hm_0\upsilon)^2, \qquad P^g = Hm_0\delta\,(\alpha-\alpha')c , \tag{45}$$

where P is a three-momentum vector. Substituting (45) into the equation (44) we obtain the general momentum equation and the general energy-momentum equation as follows:

$$\left(P^0\right)^2 - (P)^2 + PP^g = m_0^2\alpha\alpha' c^2, \qquad \frac{(E^c)^2}{\alpha\alpha' c^2} - (P)^2 + PP^g = m_0^2\alpha\alpha' c^2. \tag{46}$$

After introducing a new composed momentum $P^k = P - P^g$ the previous equations can be transformed into the form:

$$\left(P^0\right)^2 - PP^k = m_0^2\alpha\alpha' c^2, \qquad \frac{(E^c)^2}{\alpha\alpha' c^2} - PP^k = m_0^2\alpha\alpha' c^2. \tag{47}$$

In the case $\alpha = \alpha' = 1$, the equations (47) are transformed into the well-known relations in the Special Relativity:

$$\left(P^0\right)^2 - P^2 = m_0^2 c^2, \qquad \frac{E^2}{c^2} - P^2 = m_0^2 c^2, \tag{48}$$

where $E^c = E = Hm_oc^2$, and H should be calculated by employing $\alpha = \alpha' = 1$. On the other hand the covariant energy equation E_c is given by (4) and (5). Since the covariant momentum of zero components of a four-momentum vector $P_0 = -E_c/\sqrt{\alpha\alpha'}c$, from (4) and (45) one can derive the following relation:

$$P_0^2 = \frac{E_c^2}{\alpha\alpha' c^2} = \left(P^0\right)^2 + PP^g + P^2 b^2, \qquad b = \frac{-\delta(\alpha-\alpha')}{2\sqrt{\alpha\alpha'}}. \tag{49}$$

Taking into account (49) and (46) it has been derived the general momentum equation and the general energy-momentum equation in the form:

$$(P_0)^2 - P^2(1+b^2) = m_0^2\alpha\alpha' c^2. \qquad \frac{E_c^2}{\alpha\alpha' c^2} - P^2(1+b^2) = m_0^2\alpha\alpha' c^2. \tag{50}$$

In the weak potential fields the parameter $b^2 \approx 0$, and the equations (50) are transformed into the forms:

$$(P_0)^2 - P^2 = m_0^2\alpha\alpha' c^2, \qquad \frac{E_c^2}{\alpha\alpha' c^2} - P^2 = m_0\alpha\alpha' c^2. \tag{51}$$

In order to derive a general relation between frequency and wave length, one can start with the equation (50). Following the well-known de Broglie relations between frequency and energy as well as between momentum and wave length, one can derive a general de Broglie relations in the form:

$$E_c = h\nu, \qquad P(1+b^2)^{1/2} = h/\lambda., \tag{52}$$

where E_c is a covariant particle energy (4), P is a contravariant particle three-momentum (45), b is parameter given by (49), h is Planck constant, while ν and λ are a frequency and a wave length. In the case $\alpha = \alpha' = 1$, we have b = 0, and the equation (52) is transformed into the well-known de Broglie relations in Quantum Mechanics: $E = h\nu$, $P = h/\lambda$. Substituting (52) into the second equation in (50) we obtain a general frequency-wave length relation in the form:

$$\frac{\nu^2}{\alpha\alpha' c^2} - \frac{1}{\lambda^2} = \frac{m_0^2\alpha\alpha' c^2}{h^2}. \tag{53}$$

Now, one can define a general rest mass frequency:

$$\nu_0 = \frac{E_0}{h} = \frac{m_0\alpha\alpha' c^2}{h}. \tag{54}$$

Applying (54) to the equation (53) we obtain a general relation between frequency and wave length in the form:

$$\frac{\nu^2}{\alpha\alpha' c^2} - \frac{1}{\lambda^2} = \frac{\nu_0^2}{\alpha\alpha' c^2}. \tag{55}$$

where λ and ν_0 are given by the equations (52) and (54), respectively. In the case $\alpha = \alpha' = 1$, (there is no any potential field), the equation (55) is transformed into the well-known relation between frequency and wave length in Quantum Mechanics.

In order to derive the general quantum relativist Klein, Gordon and Fock equation, one can start with the general relation between frequency and wave length (55). Applying the substitutions:

$$\nu_0^2 = \frac{m_0^2(\alpha\alpha')^2 c^4}{h^2}, \qquad \hbar = \frac{h}{2\pi}, \tag{56}$$

to the equation (55) and using the four-vectors concept :

$$X \to 0\left(\sqrt{\alpha\alpha'}ct, x, y, z\right) = \{x^i\}, \quad X' \to 0'\left(\sqrt{\alpha\alpha'}ct', x', y', z'\right) = \{x'^i\}, \quad i = 0,1,2,3, \qquad (57)$$

where X has the component in the frame 0, while X' has the components in the frame 0′, we obtain the general relativistic Klein, Gordon and Fock equation, following the usual procedures:

$$\frac{-\partial^2\psi_{(x)}}{\partial\left(\sqrt{\alpha\alpha'}ct\right)^2} + \frac{\partial^2\psi_{(x)}}{\partial x^2} + \frac{\partial^2\psi_{(x)}}{\partial y^2} + \frac{\partial^2\psi_{(x)}}{\partial z^2} = \frac{m_0^2\alpha\alpha' c^2}{\hbar^2}\psi_{(x)}. \qquad (58)$$

Now, one can define a new space-time Laplacian $\nabla_g^2 = \Delta_g$ (or the Dalembertian operator $\Box$):

$$\Box = \nabla_g^2 = \Delta_g = \frac{\partial^2}{\partial X^2} = \left[\frac{-\partial^2}{\partial\left(\sqrt{\alpha\alpha'}ct\right)^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]. \qquad (59)$$

By application of the operator (59), the equation (58) is transformed into the operator form of the general quantum relativist Klein, Gordon and Fock equation:

$$\Box\psi = \nabla_g^2\psi_{(x)} = \frac{m_0^2\alpha\alpha' c^2}{\hbar^2}\psi_{(x)} = \chi^2\psi_{(x)}, \qquad \chi = \frac{m_0\sqrt{\alpha\alpha'}c}{\hbar}, \qquad (60)$$

where the parameters α and α' include a unified specific potential energy of a particle in multi-potential field through the relations (25) and (27). In the case $\alpha = \alpha' = 1$ (i.e. if any external potential field does not present), the equation (60) is transformed into the well known quantum relativist Klein, Gordon and Fock equation in Quantum Mechanics:

$$\Box\psi = \nabla_g^2\psi_{(x)} = \frac{m_0^2 c^2}{\hbar^2}\psi_{(x)} = \chi_1^2\psi_{(x)}, \qquad \chi_1 = m_0 c/\hbar , \qquad (61)$$

what we expected that should be. If a particle is in electromagnetic and gravitational central symmetric potential fields, then the parameters α and α' include a unified specific potential energy of a particle in that two-potential field, given by (29). In that case the general quantum relativist Klein, Gordon and Fock equation (60), with parameters α and α' from (29), is valid for central symmetric electromagnetic and gravitational potential fields.

7 About the Possibility that Photons may Have a Mass

Let start with the combination of the equations (44) and (45) from where we obtain the relation :

$$m_0^2\alpha\alpha' c^2 - m_0^2\upsilon^2 + m_0^2\delta[\alpha - \alpha']c \cdot \upsilon = m_0^2\alpha\alpha' c^2 / H^2. \qquad (62)$$

This relation can be transformed into a new momentum equation:

$$\left(\underline{P}^{0}\right)^{2}-\underline{P}^{2}+\underline{P}\underline{P}^{g}=m_{0}^{2}\alpha\alpha' c^{2}/H^{2}, \tag{63}$$

where the presented moments do not contain parameter H:

$$\underline{P}^{0}=m_{0}\sqrt{\alpha\alpha'}c, \qquad \underline{P}=m_{0}\upsilon, \qquad \underline{P}^{g}=m_{0}\delta(\alpha-\alpha')c. \tag{64}$$

Since the parameter H is given by the relation (5), the momentum equation (63) can be transformed into the form:

$$\left(\underline{P}^{0}\right)^{2}-\underline{P}^{2}+\underline{P}\underline{P}^{g}=m_{0}^{2}c^{2}\left(\alpha\alpha'-\frac{\upsilon^{2}}{c^{2}}+\frac{\delta(\alpha-\alpha')c\cdot\upsilon}{c^{2}}\right). \tag{65}$$

In the region without any potential field, parameters α and α' satisfied the equation $\alpha = \alpha' = 1$, and $\underline{P}^{0}=m_{0}c$, $\underline{P}=m_{0}\upsilon$, and $\underline{P}^{g}=0$. Thus, for that case, we have the momentum equation and the energy-momentum equation as follows:

$$\left(\underline{P}^{0}\right)^{2}-\underline{P}^{2}=m_{0}^{2}c^{2}\left(1-\frac{\upsilon^{2}}{c^{2}}\right), \qquad \frac{E^{2}}{c^{2}}-\underline{P}^{2}=m_{0}^{2}c^{2}\left(1-\frac{\upsilon^{2}}{c^{2}}\right), \tag{66}$$

where E is Einstein's rest mass energy, $E = m_o c^2$, and $\underline{P}=P=m_{0}\upsilon$.

Now, generally, for the particle with a velocity v = c in the region without any potential field, or in the potential field that do not interact with the particle (a photon in an electromagnetic field and a graviton in a gravitational field), the momentum equation and the energy-momentum equation have the form:

$$\left(\underline{P}^{0}\right)^{2}-\underline{P}^{2}=0, \qquad \frac{E^{2}}{c^{2}}-\underline{P}^{2}=0. \tag{67}$$

This result is a consequence of putting v = c in (66). Of course, the same result we can obtain if we put $m_o = 0$ on the right side of the same equations. But in that case, the left side of the equations (66) is also equal to zero, because of $\underline{P}^0 = \underline{P} = 0$, for $m_o = 0$, and we have a trivial solution. It seams that for particles with a velocity v = c we have illusion that $m_o = 0$, but it probably does not happen. In spite of the possibility that photons and gravitons may have a mass, the equations (67) will remain unchanged.

Remarks. It has to be noticed that the equations (67) are obtained from (66) under assumption that v = c. We did not use the assumption $m_o = 0$. This result can be interpreted in the sense that even the photons and the gravitons, may have the mass $(m_0 \neq 0)$, the equations (67) are remaining unchanged. This gives to us the idea that photons and gravitons may really have a mass $(m_0 \neq 0)$. The second interpretation could be: a) one can define the particle rest mass moment $P_{rm} = m_o c\left(1-\upsilon^{2}/c^{2}\right)^{1/2}$, b) when a particle has a velocity v = 0, then it has an accumulated rest mass moment $P_{rm} = m_o c$, and c) when a particle has a velocity v = c the rest mass moment vanishes, $P_{rm} = 0$, but a particle rest mass remains unchanged ($m_o \neq 0$).

8 Conclusion

Scalar and vector potentials (i.e. a four-potential vector) of a gravitational field have been derived on the natural way from a general covariant energy equation. Since a scalar potential of a gravitational field has a unit of a specific energy (J/kg), the problem of unification of potential fields has been transformed into the problem of unification of specific potential energies of a particle in a multi-potential field. This problem has been solved by an introduction of so called a four-vector of specific potential energy of a particle in multi-potential field. As the result of this approach the parameters α and α' of GLT model become the *functions* of a unified specific potential energy of a particle in a multi-potential field. Since all general items like field tensors, Maxwell like equations, energy-momentum equation and Klein, Gordon and Fock equation are functions of parameters α and α', the mentioned items can be applied to multi-potential field. As an example, a field tensor of a unified specific potential energy of a particle in central symmetric electromagnetic and gravitational fields in vacuum has been derived. Finally, it has been shown that a momentum equation of photons and gravitons will remain unchanged, even if the photons and the gravitons may have the mass ($m_o \neq 0$).

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