Choosing the Parameter of Regularized MP-Inverses by Modified SIC in Image Restoration Problems

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Abstract Image restoration is a typical ill-posed problem. Regularization technic represented by a regularized MP-Inverse filter (RMPIF) is widely used to deal with the ill-posedness. In order to derive the best performance of the filter, the parameter, which controls the regularizability, should be appropriately chosen.

In this paper, we present a new criterion for parameter choosing based on modifying the subspace information criterion which is first proposed for model selection of supervised learning problems. Some numerical examples are also shown to verify the efficacy of the proposed criterion.
Keywords: image restoration, regularization parameter, subspace information criterion, squared error, unbiased estimator

1 Introduction

Restoration of images degraded by some linear transformations and observation noise is one of most important issues in the field of image processing. So far, many restoration filters have been proposed. By the way, image restoration is a typical ill-posed problem. Regularization technic (Tikhonov et al., 1977) represented by regularized MP-inverse filter, is widely used to deal with the ill-posedness. In order to derive the best performance of regularization filters, the parameter, which controls the regularizability, should be appropriately chosen. Therefore, choosing such a parameter is one of most important topics in image restoration problems.

In (Thompson et al., 1991), four criteria of choosing the parameter are introduced:

(A) Minimization of Total Predicted Mean Squared Error Choice (TPMSE)

(B) Generalized Cross Validatory Choice (GCV)

(C) χ^2 Choice (CHI)

(D) Equivalent Degrees of Freedom Choice (EDF)

Although the validity of these criteria has been shown by the results of numerical experiments (Thompson et al., 1991), relation between the criteria and the squared error, which is generally adopted to evaluate the restoration performance, is not theoretically substantiated.

Sugiyama and Ogawa proposed Subspace Information Criterion (SIC) (Sugiyama et al., 2001) for model selection of supervised learning problems and showed that SIC is an unbiased estimator of the expected squared error between the unknown model function and an estimated one. They also applied this criterion to noise reduction of images, i.e., restoration of images only contaminated by additive noise (Sugiyama et al., 2000). However it can not be directly applied to general restoration problems.

In this paper, we present a modified version of SIC for general restoration problems and show that it is an unbiased estimator of the expected squared error between a restored image and the projection of an original image onto the estimable linear subspace. Some numerical examples are also shown to verify the efficacy of the proposed criterion.

2 Mathematical Preliminaries

Definitions of the notations used in this paper are shown below.

$\mathbf{R}^m$	:	the m-dimensional real metric vector space
$\mathbf{R}^{r\times c}$	:	the set consisting of all $r \times c$ real matrices
I_n	:	the identity matrix of degree n
A'	:	the transpose matrix of A
A^+	:	the Moore-Penrose generalized inverse of A (Rao et al., 1971)
$\mathrm{tr}\{A\}$	:	the trace of A
$\|A\|$	:	the operator norm of A
$\|\boldsymbol{x}\|$	:	the Euclidean norm of a vector $\boldsymbol{x}$
$\mathcal{N}(A)$	:	the null space of A
$\mathcal{R}(A)$	:	the range of A
P_S	:	an orthogonal projector onto a linear subspace S
$S^\perp$	:	the orthogonal complement of a linear subspace S
$E_{\boldsymbol{v}}$	:	the expectation over a random vector $\boldsymbol{v}$

3 Formulation of Image Restoration Problems

Degradation and restoration are modeled as follows (Stark, 1987):

$$\boldsymbol{g} = A\boldsymbol{f} + \boldsymbol{n}, \quad (1)$$

$$\hat{\boldsymbol{f}} = B\boldsymbol{g}, \quad \boldsymbol{f}, \hat{\boldsymbol{f}} \in \mathbf{R}^m, \quad \boldsymbol{g}, \boldsymbol{n} \in \mathbf{R}^n, \quad (2)$$

where $\mathbf{R}^m$ and $\mathbf{R}^n$ are called the space of original images and that of observed images, respectively, and $\boldsymbol{f}$, $\hat{\boldsymbol{f}}$, $\boldsymbol{g}$, and $\boldsymbol{n}$ denote an unknown original image, a restored image, a degraded image, and zero-mean additive noise (which may be colored), respectively. Let Q be the correlation operator of $\boldsymbol{n}$ and defined as $Q = E_{\boldsymbol{n}}[\boldsymbol{n}\boldsymbol{n}']$. $A : \mathbf{R}^m \to \mathbf{R}^n$ and $B : \mathbf{R}^n \to \mathbf{R}^m$ denote a degradation operator and a restoration filter, respectively. In this paper, we assume that A and B are linear. Therefore, they are elements of $\mathbf{R}^{n \times m}$ and $\mathbf{R}^{m \times n}$, respectively.

In this paper, we adopt a regularized MP-Inverse filter (RMPIF) for a regularization filter. RMPIF is defined as the operator B by which the following criterion is minimized.

$$J = ||A\hat{\boldsymbol{f}} - \boldsymbol{g}||^2 + \lambda||\hat{\boldsymbol{f}}||^2. \tag{3}$$

Here, $0 < \lambda \leq 1$ denotes a real parameter. Such a operator can be uniquely written as

$$B_{RMPIF} = (A'A + \lambda I_m)^{-1}A', \tag{4}$$

as well known.

4 Existing Criteria for Parameter Choosing

In this section, we again enumerate the existing criteria for choosing the parameter of image restoration filters (Thompson et al., 1991) and summarize their properties.

(A) Total Predicted Mean Squared Error (TPMSE)

The TPMSE criterion is defined as follows:

$$C_{TPMSE}(\lambda) = ||(I_n - AB(\lambda))A\boldsymbol{f}||^2 + \mathrm{tr}\{AB(\lambda)QB'(\lambda)A'\}, \tag{5}$$

and let λ_{TPMSE} be the minimizer of $C_{TPMSE}(\lambda)$.

TPMSE indirectly evaluates the fidelity of a restored image in the space of degraded images. However, as pointed out in (Ogawa, 1988), indirect evaluation does not always correspond to direct evaluation, especially in view of noise suppression.

(B) Generalized Cross Validatory Choice (GCV)

The GCV criterion is defined as follows:

$$C_{GCV}(\lambda) = \frac{||(I_n - AB(\lambda))\boldsymbol{g}||^2}{[\mathrm{tr}\{I_n - AB(\lambda)\}]^2}, \tag{6}$$

and let λ_{GCV} be the minimizer of $C_{GCV}(\lambda)$.

GCV has the advantage of not requiring variance of observation noise; however restored images with the parameter based on this criterion may be too sharp or noisy.

(C) χ^2 Choice (CHI)

The CHI criterion is defined as follows:

$$C_{CHI}(\lambda) = |\ ||(I_n - AB(\lambda))\boldsymbol{g}||^2 - \mathrm{tr}\{Q\}\ |, \tag{7}$$

and let λ_{CHI} be the minimizer of $C_{CHI}(\lambda)$.

CHI also evaluates the fidelity in the space of observed images indirectly.

(D) Equivalent Degrees of Freedom Choice (EDF)

The EDF criterion is defined as follows:

$$C_{EDF}(\lambda) = |\ ||(I_n - AB(\lambda))\boldsymbol{g}||^2 - \frac{\mathrm{tr}\{Q\}\mathrm{tr}\{I_n - AB(\lambda)\}}{n}\ |, \tag{8}$$

and let λ_{EDF} is the minimizer of $C_{EDF}(\lambda)$.

EDF is a modified version of CHI and has the same problem of indirect evaluation as do TPMSE and CHI.

5 Subspace Information Criterion for Choosing the Parameter of Image Restoration Filters

Subspace Information Criterion(SIC) was first proposed for model selection of supervised learning problems by Sugiyama and Ogawa (Sugiyama et al., 2001). SIC is an unbiased estimator of the expected squared error between the unknown model function and the estimated one. They also applied it to noise reduction of images, i.e., restoration of images only contaminated by additive noise (Sugiyama et al., 2000). This criterion is written as

$$SIC(\lambda) = ||(B(\lambda) - I_m)\boldsymbol{g}||^2 + 2\mathrm{tr}\{B(\lambda)Q\} - \mathrm{tr}\{Q\}, \tag{9}$$

as shown in (Sugiyama et al., 2000). However, eq.9 can not be applied to general restoration problems, that is, the case of $A \neq I_m$. In this section we present a modified version of SIC that can be applied to general cases.

The expected squared error can be decomposed into two terms:

$$E_{\boldsymbol{n}}||\hat{\boldsymbol{f}} - \boldsymbol{f}||^2 = E_{\boldsymbol{n}}||\hat{\boldsymbol{f}} - E_{\boldsymbol{n}}\hat{\boldsymbol{f}}||^2 + ||E_{\boldsymbol{n}}\hat{\boldsymbol{f}} - \boldsymbol{f}||^2, \tag{10}$$

as shown in (Sugiyama et al., 2000). The first term of eq.10 is the variance component and the second term is that of the bias. It is easy to show that

$$E_{\boldsymbol{n}}||\hat{\boldsymbol{f}} - E_{\boldsymbol{n}}\hat{\boldsymbol{f}}||^2 = E_{\boldsymbol{n}}||B\boldsymbol{n}||^2 = \mathrm{tr}\{BQB'\}, \tag{11}$$

as described in (Sugiyama et al., 2000).

Let A_N^- be a minimum norm generalized inverse (Rao et al., 1971) of A, i.e., $(A_N^- A)$ is an orthogonal projector onto $\mathcal{R}(A')$. The bias component can be written as follows with the assumption of $\mathcal{R}(A') = \mathcal{R}(B)$, which is an ordinary condition in image restoration problems:

$$
\begin{aligned}
&||E\boldsymbol{n}\hat{\boldsymbol{f}} - \boldsymbol{f}||^2 \\
&= ||\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}||^2 - ||\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}||^2 + ||E\boldsymbol{n}\hat{\boldsymbol{f}} - \boldsymbol{f} - E\boldsymbol{n} A_N^- \boldsymbol{n}||^2 \\
&= ||(B - A_N^-)\boldsymbol{g}||^2 \\
&\quad -||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) + (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad +||E\boldsymbol{n}\hat{\boldsymbol{f}} - E\boldsymbol{n}(\boldsymbol{f} + A_N^- \boldsymbol{n})||^2 \\
&= ||(B - A_N^-)\boldsymbol{g}||^2 - ||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad -||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad +2(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}))'(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})) \\
&\quad +||E\boldsymbol{n}(\hat{\boldsymbol{f}} - \boldsymbol{f} - A_N^- \boldsymbol{n})||^2 \\
&= ||(B - A_N^-)\boldsymbol{g}||^2 - ||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad -||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad +2(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}))'(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})) \\
&\quad +||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- A\boldsymbol{f} - (I_m - A_N^- A)\boldsymbol{f} - A_N^- \boldsymbol{n})||^2 \\
&= ||(B - A_N^-)\boldsymbol{g}||^2 - ||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad -||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&\quad +2(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}))'(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})) \\
&\quad +||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (I_m - A_N^- A)\boldsymbol{f}||^2 \\
&= ||(B - A_N^-)\boldsymbol{g}||^2 \\
&\quad +2(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}))'(E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})) \\
&\quad -||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 + ||(I_m - A_N^- A)\boldsymbol{f}||^2. \qquad (12)
\end{aligned}
$$

Let T_i $(i = 1, 2, 3, 4)$ be the i-th term of eq.12. Therefore,

$$||E\boldsymbol{n}\hat{\boldsymbol{f}} - \boldsymbol{f}||^2 = T_1 + T_2 + T_3 + T_4.$$

The expectation over $\boldsymbol{n}$ of T_2 $(E\boldsymbol{n}T_2)$ disappears and that of T_3 becomes

$$
\begin{aligned}
E\boldsymbol{n}T_3 &= -E\boldsymbol{n}||E\boldsymbol{n}(\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g}) - (\hat{\boldsymbol{f}} - A_N^- \boldsymbol{g})||^2 \\
&= -E\boldsymbol{n}||E\boldsymbol{n}(B - A_N^-)(A\boldsymbol{f} + \boldsymbol{n}) - (B - A_N^-)(A\boldsymbol{f} + \boldsymbol{n})||^2 \\
&= -E\boldsymbol{n}||(B - A_N^-)\boldsymbol{n}||^2 \\
&= -\mathrm{tr}\{(B - A_N^-)Q(B - A_N^-)'\} \\
&= -\mathrm{tr}\{BQB'\} + 2\mathrm{tr}\{BQA_N^{-\prime}\} - \mathrm{tr}\{A_N^- Q A_N^{-\prime}\}.
\end{aligned}
$$

Replacing $||E\boldsymbol{n}\hat{\boldsymbol{f}} - \boldsymbol{f}||^2$ with

$$
\begin{aligned}
&T_1 + E\boldsymbol{n}(T_2 + T_3) + T_4 \\
&= ||(B - A_N^-)\boldsymbol{g}||^2 - \mathrm{tr}\{BQB'\} + 2\mathrm{tr}\{BQA_N^{-\prime}\} \\
&\quad -\mathrm{tr}\{A_N^- Q A_N^{-\prime}\} + ||(I_m - A_N^- A)\boldsymbol{f}||^2 \qquad (13)
\end{aligned}
$$

in eq.10 and substituting the result of eq.11 to eq.10 yield a modified version of SIC,

$$
\begin{aligned}
&MSIC(\lambda) \\
&= \|(B(\lambda) - A_N^-)\boldsymbol{g}\|^2 + 2\mathrm{tr}\{B(\lambda)QA_N^{-\prime}\} - \mathrm{tr}\{A_N^-QA_N^{-\prime}\} \\
&\quad + \|(I_m - A_N^-A)\boldsymbol{f}\|^2.
\end{aligned}
$$

Here we show a theorem about unbiasedness of MSIC.

Theorem 1

$$E_{\boldsymbol{n}}(MSIC(\lambda)) = E_{\boldsymbol{n}}\|\hat{\boldsymbol{f}} - \boldsymbol{f}\|^2. \tag{14}$$

Proof

$$
\begin{aligned}
&E_{\boldsymbol{n}}(MSIC(\lambda)) \\
&= E_{\boldsymbol{n}}\|(B(\lambda) - A_N^-)\boldsymbol{g}\|^2 + 2\mathrm{tr}\{B(\lambda)QA_N^{-\prime}\} - \mathrm{tr}\{A_N^-QA_N^{-\prime}\} \\
&\quad + \|(I_m - A_N^-A)\boldsymbol{f}\|^2 \\
&= E_{\boldsymbol{n}}\|(B(\lambda) - A_N^-)(A\boldsymbol{f} + \boldsymbol{n})\|^2 + 2\mathrm{tr}\{B(\lambda)QA_N^{-\prime}\} \\
&\quad - \mathrm{tr}\{A_N^-QA_N^{-\prime}\} + \|(I_m - A_N^-A)\boldsymbol{f}\|^2 \\
&= \|(B(\lambda) - A_N^-)A\boldsymbol{f}\|^2 + E_{\boldsymbol{n}}\|(B(\lambda) - A_N^-)\boldsymbol{n}\|^2 \\
&\quad + 2E_{\boldsymbol{n}}((B(\lambda) - A_N^-)A\boldsymbol{f})'((B(\lambda) - A_N^-)\boldsymbol{n}) \\
&\quad + 2\mathrm{tr}\{B(\lambda)QA_N^{-\prime}\} - \mathrm{tr}\{A_N^-QA_N^{-\prime}\} + \|(I_m - A_N^-A)\boldsymbol{f}\|^2 \\
&= \|(B(\lambda) - A_N^-)A\boldsymbol{f}\|^2 + \|(I_m - A_N^-A)\boldsymbol{f}\|^2 \\
&\quad + \mathrm{tr}\{(B(\lambda) - A_N^-)Q(B - A_N^-)'\} + 2\mathrm{tr}\{B(\lambda)QA_N^{-\prime}\} \\
&\quad - \mathrm{tr}\{A_N^-QA_N^{-\prime}\} \\
&= \|B(\lambda)A\boldsymbol{f} - \boldsymbol{f}\|^2 + \mathrm{tr}\{B(\lambda)QB(\lambda)'\} \\
&= \|E_{\boldsymbol{n}}\hat{\boldsymbol{f}} - \boldsymbol{f}\|^2 + E_{\boldsymbol{n}}\|\hat{\boldsymbol{f}} - E_{\boldsymbol{n}}\hat{\boldsymbol{f}}\|^2 = E_{\boldsymbol{n}}\|\hat{\boldsymbol{f}} - \boldsymbol{f}\|^2. \qquad \text{Q.E.D.}
\end{aligned}
$$

From this theorem, it is shown that $MSIC(\lambda)$ is an unbiased estimator of the expected squared error between the unknown original image and a restored image. Therefore the parameter of a restoration filter B that minimizes $MSIC(\lambda)$ is a candidate for a good one. However, $MSIC(\lambda)$ includes an unknown component $\boldsymbol{f}$, so we define an omitted version of $MSIC(\lambda)$ which does not include unknown components as follows:

$$
\begin{aligned}
&OMSIC(\lambda) \\
&= \|(B(\lambda) - A_N^-)\boldsymbol{g}\|^2 + 2\mathrm{tr}\{B(\lambda)QA_N^{-\prime}\} - \mathrm{tr}\{A_N^-QA_N^{-\prime}\}.
\end{aligned} \tag{15}
$$

The minimizer of $OMSIC(\lambda)$ also minimizes $MSIC(\lambda)$, since the omitted term $\|(I - A_N^-A)\boldsymbol{f}\|^2$ is constant, and it is obvious that OMSIC is an unbiased estimator of $E_{\boldsymbol{n}}\|\hat{\boldsymbol{f}} - P_{\mathcal{R}(A')}\boldsymbol{f}\|^2$, when $\mathcal{R}(B) = \mathcal{R}(A')$. Hereafter, we write the minimizer of $OMSIC(\lambda)$ as λ_{OMSIC}.

6 Numerical Examples

In this section, some numerical examples are shown to verify the efficacy of OMSIC choice. We evaluate the squared error of restored images by RMPIF based on λ_{TPMSE}, λ_{CV}, λ_{CHI}, λ_{EDF}, and λ_{OMSIC}. The original image $\boldsymbol{f}$ is used for the first term in TPMSE to derive the best performance of TPMSE choice and A^+ for A_N^- in OMSIC.

6.1 Colored Noise Case

We investigated the performance of OMSIC with colored additive noise whose correlation operator is known. As an original image and a degradation operator, LENA(128 $\times$ 128 pixels, 256 gray levels) and the vertically averaging operator over 32 pixels were used. A vertically colored random vector $\sim N(0, \sigma^2 Q)$ with $\sigma = 1.0,\ 2.0,\ 3.0,\ 4.0$, and

$$Q = \begin{bmatrix} 0.5 & 0.25 & 0 & \cdots & 0 & 0.25 \\ 0.25 & 0.5 & 0.25 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 0.25 & 0.5 & 0.25 \\ 0.25 & 0 & \cdots & 0 & 0.25 & 0.5 \end{bmatrix}.$$

were used for $\boldsymbol{n}$.

Table 1 shows parameters chosen by five criteria and SNR of restored images based on these parameters. The field MSE is based on the squared error between the actual original image and restored ones. Therefore λ in an MSE field is the parameter which derive the best performance of RMPIF.

Table 1: Chosen parameters and SNR(dB) of restored images to the original image (colored noise case).

		σ			
		1.0	2.0	3.0	4.0
MSE	λ	0.0015	0.0043	0.0074	0.0108
	SNR	14.56	13.84	13.36	13.00
TPMSE	λ	0.0081	0.0141	0.0194	0.0243
	SNR	13.88	13.32	12.95	12.67
GCV	λ	0.0008	0.0015	0.0021	0.0025
	SNR	14.48	13.37	11.43	11.52
CHI	λ	0.0027	0.0059	0.0091	0.0124
	SNR	14.47	13.80	13.34	12.99
EDF	λ	0.0016	0.0039	0.0064	0.0089
	SNR	14.56	13.84	13.35	12.98
OMSIC	λ	0.0015	0.0043	0.0074	0.0108
	SNR	14.56	13.84	13.36	13.00

Figure 1 shows the original image, and Figs.2 and 3 show a degraded image and a restored image based on $\lambda = 0.0074$ by which MSE and OMSIC are minimized in the case of $\sigma = 3.0$.

Figure 1 : An original image (LENA).

Figure 2 : A degraded image ($\sigma = 3.0$).

Figure 3 : A restored image by RMPIF with $\lambda = 0.0074$ by which MSE and OMSIC are minimized ($\sigma = 3.0$).

6.2 White Noise Case

Next, we investigated the performance of OMSIC with known white additive noise. As an original image and a degradation operator, HOUSE(128 $\times$ 128 pixels, 256 gray levels) and the vertically averaging operator over 7 pixels were used, and a normal random vector $\sim N(0, \sigma^2 I_n)$ with $\sigma = 1.0,\ 2.0,\ 3.0,\ 4.0$ were used for $\boldsymbol{n}$.

Table 2 shows the same result with the colored noise case.

Table 2: Chosen parameters and SNR(dB) of restored images to the original image (known white noise case).

		σ			
		1.0	2.0	3.0	4.0
MSE	λ	0.0050	0.0141	0.0233	0.0316
	SNR	25.67	23.06	21.60	20.57
TPMSE	λ	0.0217	0.0398	0.0558	0.0708
	SNR	23.78	21.61	20.24	19.18
GCV	λ	0.0011	0.0026	0.0042	0.0058
	SNR	24.28	20.72	18.67	17.22
CHI	λ	0.0056	0.0111	0.0164	0.0216
	SNR	25.66	22.99	21.41	20.30
EDF	λ	0.0016	0.0041	0.0068	0.0096
	SNR	24.83	21.63	19.81	18.57
OMSIC	λ	0.0048	0.0137	0.0226	0.0309
	SNR	25.67	23.06	21.60	20.56

Figure 4 shows the original image, and Figs.5, 6, and 7 show a degraded image, a restored image based on $\lambda = 0.0233$ by which MSE are minimized, and one based on $\lambda = 0.0226$ by which OMSIC are minimized in the case of $\sigma = 3.0$.

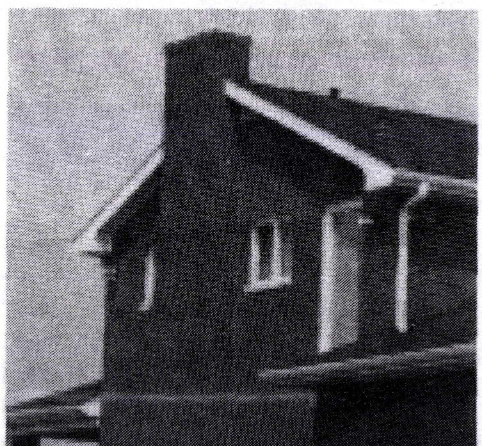

Figure 4 : An original image (HOUSE).

Figure 5 : A degraded image ($\sigma = 3.0$).

Figure 6 : A restored image by RMPIF with $\lambda = 0.0233$ by which MSE and are minimized ($\sigma = 3.0$).

Figure 7 : A restored image by RMPIF with $\lambda = 0.0226$ by which OMSIC are minimized ($\sigma = 3.0$).

The two numerical examples shown above confirm that OMSIC yields better parameters compared with existing criteria.

7 Conclusions

In this paper, as a criterion for choosing a parameter of image restoration filters, we presented a modified version of SIC (OMSIC) that can be applied to general restoration problems. We showed that OMSIC is an unbiased estimator of the expected squared error between a restored image and the projection of an original image onto the estimable linear subspace $\mathcal{R}(A')$. Numerical examples confirmed that the parameter chosen by the proposed criterion is better than those chosen using existing criteria.

Variance analysis of OMSIC and applying to cases of noises with unknown properties are topics for future works.

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