Eccentric Anticipation: Fractal Geometry in Sensori-Motor Activity

Cornelius A. Steckner
FOCAS, 50996 Cologne Etruskerstr. 22
fax +49 221 2212 4030 - e-mail csteckner@hotmail.com

Abstract: "Eccentricity" is a term for a displacement, while "eccentric anticipation" is defined as the control of forthcoming contact across locomotive kinematic chains. The disturbing fact there is found in the driving vertebrate kinematics and the related locomotion technologies. To fit human performance, these all fall to specific spatiotemporal standards. Local self-similarity and a nonlinear feedback stabilization is binding the kinematic chain to the demands of a mechanism.
Keywords: vertebrate kinematics, locomotion, gravity, fractal geometry.

1 What is "eccentricity"?

Eccentricity, in the first instance, is a measure for the displacement of a focus from the center of the ellipse (Fig.1). The measure of displacement predetermines the properties of the ellipse. And this constraint forces the hand to follow the path. This interrelation F. Reuleaux adapted for his foundation of kinematics to become the rule of pairs, while the forced geometrical binding became his static "closed kinematic chain" or "mechanism". Another aspect of predetermination Reuleaux had in view, when he compared the path of a satellite around a planet (Reuleaux, 1875 [19], Fig.2) and a crank-driven regular wheel (Fig.3). This spatiotemporal predetermination by paired dislocation across the components in a wider sense is eccentricity.

The difference in the predetermined eccentric path of a planet and an eccentric wheel Reuleaux found in the spatiotemporal reference to a driving force. The crank-driven wheel unveils its constraints only when it is forced, then the axis of rotation AB determines all loci of R at a moment T (Fig.3) This forced determination is called "Zwangslaufmechanik", which is translated as "positively driven mechanics"). The original term refers to the initial conditions. The planet also has this forcing of its orbit at a moment (T), but these conditions do not allow the stop-and-go and the to-and-fro of a mechanism, though it is clear, that both of these initial path-driving conditions bind space and time by a specific but common geometry to have it stabilized in a quasi-periodicy. This potiential embedding in similar phase spaces is the reason to look out for fractal geometry at work. In case a planet is simulated by a rotary pendulum its quasi-periodicity generates a torus (Worg, 1993[20]). According to Reuleaux, a mechanism may model this behavior.

International Journal of Computing Anticipatory Systems, Volume 12, 2002
Edited by D. M. Dubois, CHAOS, Liège, Belgium, ISSN 1373-5411 ISBN 2-9600262-6-8

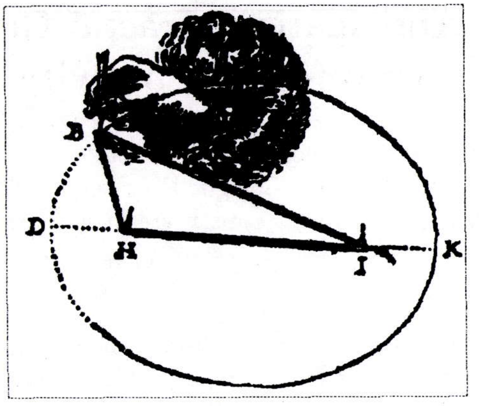

Fig. 1: Bifocal binding of the forced ellipse path (from R. Descartes, Geometry).

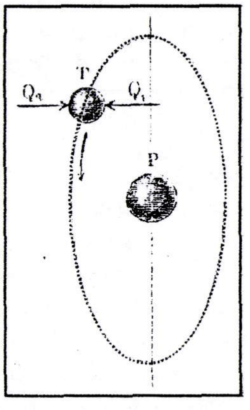

Fig. 2: Positive drive of a planet (Reuleaux, 1875, Fig. 2).

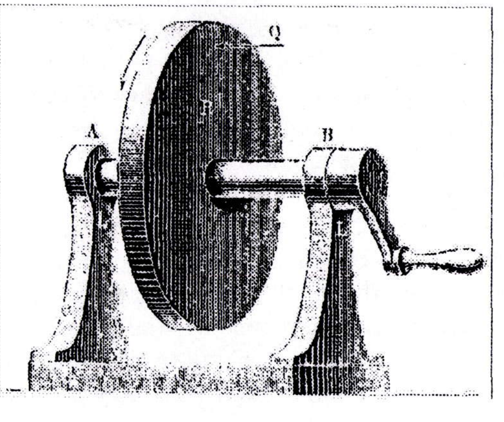

Fig. 3: Positive drive of a crank-driven wheel (Reuleaux, 1875, Fig. 3).

2 Time and the mechanism

Reuleaux implicitly distinguished planetary phase time $r(T)$ and predetermining permanent local time $r(t)$. His Cartesian frame of reference catched the spatiotemporal properties of any location in respect to the axis of rotation (1):

$$r(t) = (x(t), (x(t), z(t)). \tag{1}$$

The mechanical compound runs with equal time for all its locations. It makes no difference, to measure time by two different clocks t and t' for a specific location $(x(t), (x(t), z(t))(2)$:

$$t' = at + \Delta t \tag{2}$$

(where a is a synchronizing constant) or by the same two clocks at two different locations. Reuleaux instead supposes $Q = r(t)$ equivalent to $Q' = r'(t')$ for all locations of a mechanism (3):

$$t'(r') = t(r). \tag{3}$$

All the locations of the system at moment of move (T) or at standstill (t) equally have the same spatiotemporal properties. This is true for any additional element joining a closed kinematic cain. And it is true also for the spatiotemporal properties for any contact across the mechanics to any forthcoming place (4):

$$t(r) = t'(r') = t''(r'') = t^n(r^n). \tag{4}$$

The spatiotemporal kinematic acces across mechanical devices fits experience. The rotating wheel (Fig.3) on a surface becomes an unicycle (Fig.4). Any crank-driven rotation at $Q' = r'(t')$ will turn the axis and the wheel which subsequently will contact a surface at $Q^n = r^{n\prime}(t^n)$ with the same spatiotemporal properties.

But how is this spatiotemporal effect of mechanics related to the driving human force at the handle of the crank at a moment of move (T)? To generate the appropriate spatiotemporal effects, both the concept of driving and the mechanical effect have to be commensurable.

Our own experience reflects this spatiotemporal commensurability. In the moment we write on a paper (Fig.5), the hand only has contact with the pencil at $t(r)$, but the touch we feel is dislocated to the edge of the graphite at $t'(r')$ in contact with the surface of the paper at $t''(r'')$. This spatiotemporal phenomenon of edge touch across tools and other mechanical devices equivalently is a dislocation of a focus. Therefore the dislocation across kinematic extensions also is called "eccentricity".

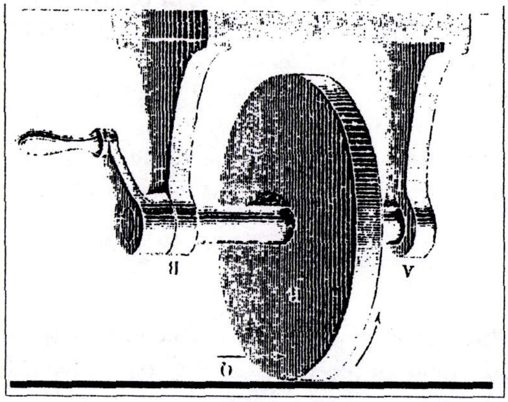

Fig. 4: The crank-driven wheel on the ground becomes an unicycle.

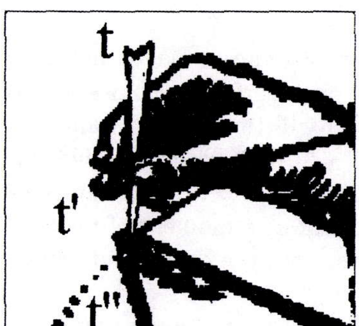

Fig. 5: Eccentricity is the spatiotemporal dislocation of actual touch at $t'(r')$ across the pencil to its edge at $t''(r'')$ with an effect on the surface at $t''\prime(r'')$.

3 Eccentric anticipation

The path driven forcing of kinematics takes to the anticipatory aspect of eccentricity. Whenever motion or locomotion by the vertebrate mechanics of an organism is performed, the moving edge of contact has to avoid self-destructive force-feedback even in its first step. This is true both for the vertebrate system itself and its eccentric use of mechanical attachments. Accordingly we have to suggest "eccentric anticipation", defined as a spatiotemporal dislocation across motor activity related kinematic extensions. These extensions are bridged in respect of force guiding - the incoming and outgoing nerve signals take time - and in respect of kinematical path generation - any motor force has to be released before its effect (Kornhuber , 1976 [11]) according to the just spatiotemporal order (Libet, 1979 [14]). The mechanical extensions of locomotion (stilts, bikes) and motion (tools) act in respect to the internal organization of vertebrate mechanics. So they may serve as a mirror of the related internal organization.

4 Spatiotemporal synchronization of bicycles

While the system generates is path by its own and extended mechanics, it has to know its spatiotemporal effects then and there, even across mechanical devices. Anticipation inverts the sequence. In this respect, inverted kinematics is looking-back model of an actually active predetermination of a contact to come at $t'(r')$ for $t''(r'')$. To get a surface (= grasp) or to pass on a surface (= locomotion) without destructive collision as the effect of spatiotemporal coordinate misfit in a predetermined future, there must be a constant synchronizing factor a for the guiding and the making. This refers to the first step and the permanent move. The initial step has not yet access to path related force-feedback, and move and locomotion as well have to be aware to predetermine forthcoming spatiotemporal events. In all cases the right spatiotemporal proportion must found permanently. This demands a spatiotemporal synchronization. Without such an spatiotemporal "escapement", neither the mental control nor the mechanics could keep its own spatiotemporal coordinates up-to-date. The system without a spatiotemporal self-control would loose its "realistic" contact with the environment on its generated mechanical path. The mechanical "in case", by its geometry and measure, presupposes a realistic result. In the moment of "awakening" as well as for perpetual locomotion we have to deal with "eccentric anticipation" to fit $t^n(r^n)$.

Spatiotemporal "isochronia" in normal human locomotion is generated by the legs. There the legs perform a sort of cycloidal pendulum to drive motion (Fig. 6). The picture refers to the effort of the two legs. The circle with the diameter a mirrors to the resultant driving force $2r$ which is adapted by the path of the bicycle pedal at Q. This device mirrors the resultant forced path. The predetermined effect is a spatiotemporal "clock-work". The clock-work just mirrors a quasi-periodical

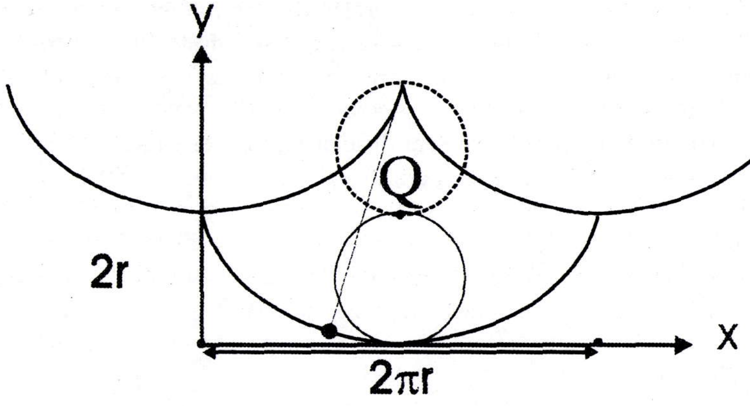

Fig. 6: In human locomation the legs act like a cycloidal pendulum to generate a quasi-periodic crank-driven locomotion.

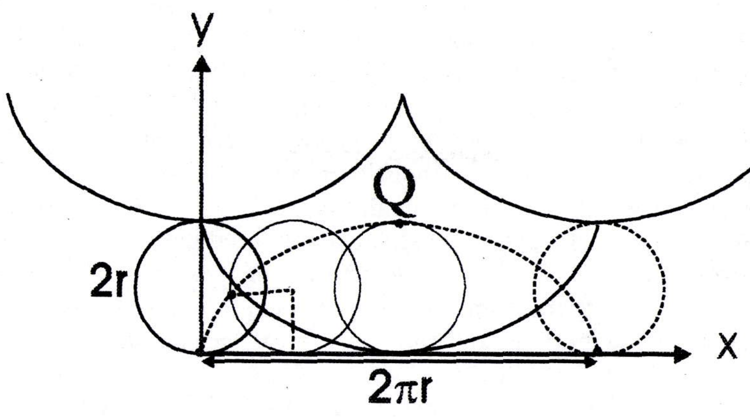

Fig. 7: A cycloidal Huygens pendulum also may serve the constraints to generate crank-driven quasi-periodicy in locomotion.

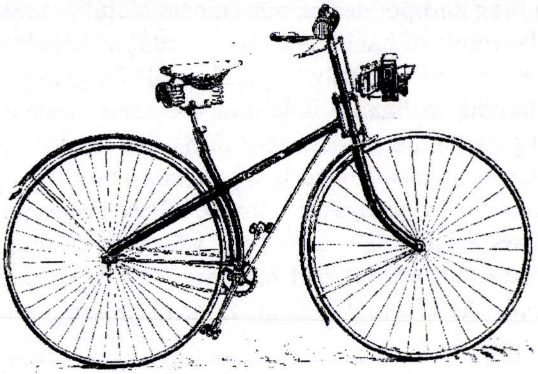

Fig. 8: Bike with still adjustable crank-radius and unelaborated frame, 1889.

physical event.

Bikes adapt the resultant path by the quasi-periodicity of the pedals (Fig.6). This geometry binds clockwork-like harmony and spatiotemporal motion. The Huygens pendulum like stroke, by its geometrical constraints, fits $t = t'' = t^n$. Accordingly, there should be a factor guiding r from within the kinematic chain to fit for the environment by measure. This spatial factor would complete the kinematic spatiotemporal pair.

5 The puzzling standard in locomotion

It is known from bicycle industry, that all grown-up people around the world in all sorts of environments, women and men, tall and small, young and old, with long or short legs, on bikes with large and small wheels, with large and small frames, with different handlebars and gender related construction, use for their bikes pedals with the same standard crank of 170 - 175 mm (Fig.8). The diameter of the related driving rotary leg motion consequently is 34-35 cm. Optionally there are longer cranks available. But people using these, later come back to the standard cranks, which cover worldwide 99 % of the industrial production. In the early days of biking, the crank length had been adjustable, but as the result of experience, the standard length was found optimal and became with 170 mm (alternatively 175 mm) the de facto standard more than a hundred years ago. To this measure also fall unicycles and tandem cranks.

Any of the given anthropometric measures, related to gender, age etc., have a normal distribution, but not the cranks. The standard itself mirrors the optimal measure for sensorimotor activity. And, apparently, this should be a key to understand the driving kinematic chain. But normal biking is a hand and leg-work. In this respect it is to be remembered, that the legs are just the front end of the driving forces, while guiding and pondering movements stabilize and harmonize motion. These mainly are the result of handlebar movements, which also are symmetric and have an axis of rotation and a spatiotemporal result for $r(t)$.Across the handlebar, the distance from handle to handle falls into the same measure of a circle of the crank radius, falling to the same diameter of 34-35 cm. If we focus this motion while biking, we realize the measure to synchronize the motion of the arms and legs on their forced equal circular path. Accordingly, all the industrial standards for bike pedals and handlebars as well as car steering wheels and push pedals accordingly fall to diameters between 330 mm and 400 mm. These common measures in the driving paths indicate the kinematic chain to meet external environmental forcing (Table 1).

Table 1: Crank diameters for bicycles, cars and motor boats.

bike pedal: ————	330 mm (165 mm crank)
car steering wheel: ———	340 mm (VW min.)
bike pedal: ————	340 mm (170 mm crank)
bike pedal: ————	350 mm (175 mm crank)
bike pedal: ————	360 mm (180 mm crank)
car steering wheel: ———	370 mm (Smile)
car steering wheel: ———	380 mm (Porsche 911)
car steering wheel: ———	390 mm (Citroen)
bike handlebar: ———	400 mm (max.)
boat steering wheel: ——	406 mm (max.)

There is a general binding factor a to result in a specific measure of any locomotion related devices. This measure joins the motor activity of arms and legs, accordingly Reuleaux (Reuleaux, 1875 [19]) focuses crank-driven as well as revolving ladder driven stational mechanics. A further locomotion mechanics related step (stairs included) leads towards the discussed synchronized motor activity of arms and legs. Additional information, to understand the binding and guiding of vertebrate mechanics, is found in the common frame of reference organizing the environmental contact.

6 A frame for sensori-motor activity

The motor activity of riders (Fig. 9) shows a clear framework of reference. All the rotating mechanical wheels and the related driving motor activity runs on circles, and all these axes of rotation are bound by a unique frame which is partially reproduced by the layout of the frame and the forks of the bike. This frame was not available in 1889 (Fig. 8). The actual design follows the scheme of the closed chain in the view of grapho-statics (Cullmann, 1866 [3]) and refers to a specific location

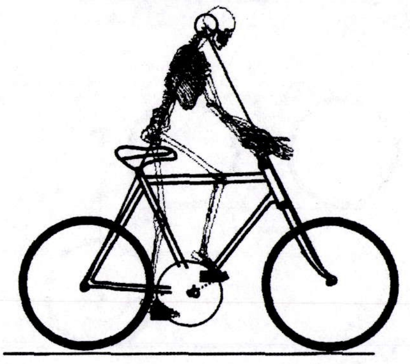

Fig. 9: Handlebars and cranks fall to a common diameter to fit the static frame of reference in vertebrate kinematics.

of the vertebrate mechanics. This is not realized in all literature on vertebrate mechanics (McGowan, 1999 [15]), there is good related experimental research available (Lengsfeld, 1975 [13], Knese, 1936 [9], Knese, 1948 [10]. The same scheme is applicable to horse-riding (where the motion of the horse-legs is at the place of the two bicycle wheels) as well as to the historical construction of bicycles from Drais on. The pattern of development is bound to this atlas-anchored frame of reference. The "atlas" joints the crane on the vertebrate column. Visibly the vertebrate mechanics and its attachments are anchored there (Fig. 9)

The cycloidal drive in bipedal locomotion (Fig.9) maybe be reduced from the bicycle to the physics of the unicycle, which has the advantage of reducing the parts to have a better view on the related quasi-periodical phase. There is no second wheel and there are no handlebars. Though the atlas related pattern is the same (Fig. 10)

7 Cycloids and fractals

Regular rotary motion generates quasi-periodical phases. The effort on the body side, taken from the movements of the legs and arms and the body has a rather irregular pattern (Crowinshield, 1978 [2], Hagemann, 1995 [7], Murray, 1967 [16], Perry, 1992 [18]), but the kinematic-chain driven rotation aims to be regular. Accordingly, we have to suggest the kinematic chain of walking and biking humans to aim regular phase patterns. Here we are already at a point, to outlook at the phase pictures of locomotion. The stabilizing factors for the locomotion of the vertebrate system has been detected in the motor activity of the legs. It is a cycloidal-pendulum stroke which is bound to a specific stabilizing factor $2r$.This stabilizes the locomotion of the kinematic chain down towards the contact at $t'(r')$ with the surface at

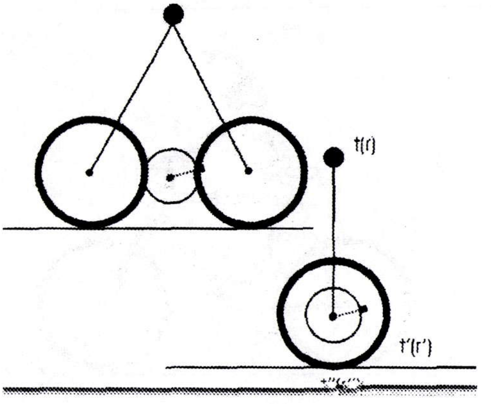

Fig. 10: A bicycle may be reduced to the unicycle.

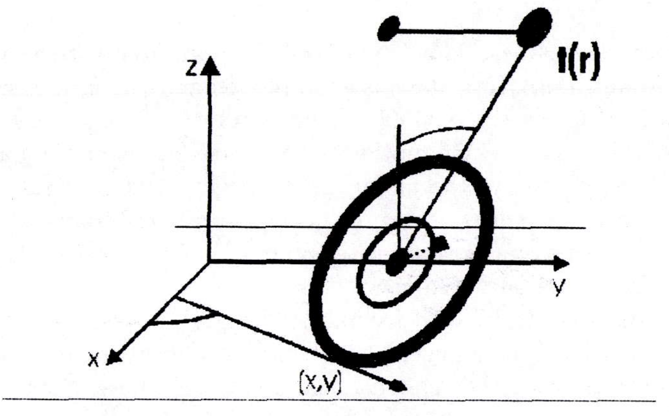

Fig. 11: The unicycle driven by cranks.

$t''(r'')$ - while the system is controlled from above the "atlas" at $t(r)$ (Fig. 11) This result from inside the system and related findings (Hespanha, 1999 [8]) are conform with the suggestion, to see the same system in the view from the observer side and also in respect to gravity (Kram, 1997 [12]). In this view the rider at $t(r)$ becomes the pendulum bob , that the acting locomotive system is using the geometrical constraints of specific vertebrate mechanics be stabilized by a quasi-periodicy. This may be described as a nonlinear feedback stabilization (Buzug, 1994 [1], Zenkov, 1999 [21], Zenkow, 1998 [22]).

How may these ideas may be seen together? The moving system rules across its attachments and contacts the stable spatiotemporal condition $t(r) = t''(r'')$. This stable state is different at the initial stroke. There is no information on the effective borders in respect of stability. Any initial stroke has no force feedback. Only the second stroke has access to this information. The same is true for a predetermined

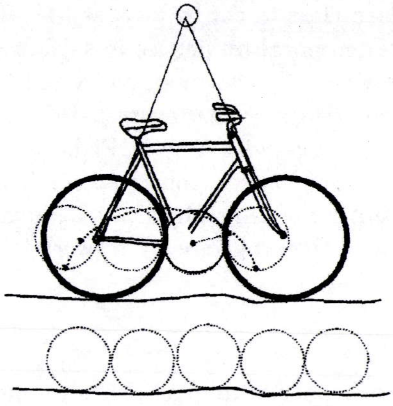

Fig. 12: Iteration by full turns of the crank as local self-similarity.

path to force the kinematic chain. The system has to set its geometrical constraints to generate a sort of "rail". Descartes has illustrated this rotation and rail binding by the thread-determination to draw the ellipse (Fig. 5). And because a drawing is a predetermined path on the surface of a paper to which the mind only has access across the pencil, such a drawing is not only predetermined by the forcing elliptic constraints, but also by the spatiotemporal conditions of the graphite contact on the surface of the paper. Furthermore these initial conditions have to refer to the complete iteration of the ellipse.

Any first stroke is a primary action under self-determined and spatiotemporally predefined iteration. This apparently meets the facts. Parkinsonian patients may be "defreezed" by primary external contact forces at a dislocated $t''(r'')$ to simulate the commensurable self-generation of an initial stroke towards the environment. But this method of defreezing only has an effect on the initialized repetitive locomotion and its deliberate end (Dietz, 1990 [4], Dunne, 1987 [5]). In other words: a normal person is self-defining its initial state to build the "rail" spatiotemporal path. The "defreezing" effect is the related inversion. This inversion has the same view "inverted kinematics" takes. Reaction is focused, not the goal and path aiming sensori-motor activity.

If we take it for granted, that the stability of a system may be described by the Lyapunov exponent, it must be realized, that the nonholonomic kinematics approach to path planning supposes an initial state, while also the equations have the form of time-independent equations. The observations above according to time (cycloidal pendulum) and spatial fit (equal radius of push pedal, handlebar and steering wheel) show fundamental properties in the geometry of the vertebrate and related mechanics to maintain an initial state in prestabilized harmony with external forcing according to Kepler's laws by definition (Fig. 1). The geometrical binding of time and space of vertebrate and related mechanics and their self-defined initial conditions at one hand and the Lyapunov exponent in the other view suggests the vertebrate mechanics for each class to act by local self-similarity (Fig. 12). The full turns followed by the spatiotemporal properties in surface contact generates a self-silimarity according to the actor. This locomotion by full turns has the quality of a mapping process with the consequence, that every path is a local self-similar pattern of its generator (Peitgen, 1992, appendix A by Y. Fisher [17]). Each individual owns by its vertebrate-implemented geometrical constraints independently the same clock-work to generate independently the same physical path qualities. This is the reason to suggest a gravity controlled fractal geometry to work in sensori-motor activity.

8 Conclusion

Sensori-motor "Eccentric anticipation" across vertebrate mechanics allows to generate a patiotemporal behaviour to meet external forcing and contact. The spatiotemporal qualities of the cycloidal pendulum (isochronism, tautochronism) actually are predictive. The cycloid related full turn apparently sets out a circle-based local self-similarity. In respect to eccentric anticipation this becomes an active fractal mapping process.

References

[1] Buzug T. (1994) Analyse chaotischer Systeme, BI Wissenschaftsverlag: Mannheim.

[2] Crowinshield RD, Brand RA, Johnson RC. (1978) Effects of walking velocity and age on hip kinematics and kinetics. Clinical Orthopedy, 132, pp. 140-144.

[3] Culmann Karl. (1866) Graphische Statik. Zürich: Mayer & Zeller.

[4] Dietz Mark A. Goetz Christopher G. Stebbins Glenn T. (1990) Evaluation of a modified inverted walking stick as a treatment for parkinsonian freezing episodes. Movement Disorders 5(3), pp. 243–247.

[5] Dunne John W. Hankey Graeme J. Edis Robert H. (1987) Parkinsonism: Upturned walking stick as an aid to locomotion. Archive of Physical and Medical Rehabilitation, 68, pp. 380–381.

[6] Full RJ. (1997) Invertebrate locomotor systems. In W.H. Dantzler, ed. Handbook of Physiology. Sec. 13, pp. 853–930. Comparative Physiology. Oxford University Press, New York.

[7] Hagemann Patricia A. (1995) Gait Characteristics of Healthy Elderly: A Literature Review. Physical Therapy, 18, pp. 210 (http://www.geriatricspt.org/pubs/ioa/V18n2/V18n2p14.html).

[8] Hespanha JP. Liberzon D. Morse AS. (1999) Logic-based switching control of a nonholometric system with parametric modeling uncertainty. Systems and Control Letters, 38, pp. 167-177.

[9] Knese KH. (1936) Das Kopfgelenk der aquatilen Säugetiere, Morphologisches Jahrbuch, 78, pp. 313-376.

[10] Knese KH. (1948) Kopfgelenk, Kopfhaltung und Kopfbewegung des Menschen, Zeitschrift für Anatomie, 114, pp. 67-107.

[11] Kornhuber HH. Deecke L. Grözinger B. (1976) Voluntary finger movements in man: celebral potientials and theory, Biological Cybernetics, 23, pp. 99-119.

[12] Kram Rodger. Domingo Antoinette. Ferris Daniel P. (1997) Effect of reduced gravity on the preferred walk-run transition speed, The

[13] Lengsfeld Klaus-Peter. (1975) Über den formenden Einfluss des Cervidengeweihs auf Hinterhaupt und erste Halswirbel. Mathematical-biological thesis. Christian-Albrechts-University, Kiel.

[14] Libet Benjamin. Wright EW. Feinstein B. Pearl DK. (1979) Subjective referral of the timing for a conscious sensory experience, Brain, 102, pp. 193-224.

[15] McGowan Christopher. (1999) A Practical Guide to Vertebrate Mechanics. Cambridge University Press: Cambridge.

[16] Murray MP. Sepic SB. Barnard EJ. (1967) Patterns of Sagittal Rotation of the Upper Limbs in Walking, Physical Therapy, 47, pp. 272-284.

[17] Peitgen HO. Juergens H. Saupe D. (1994) Chaos and Fractals. Springer: Heidelberg.

[18] Perry J. (1992) Gait Analysis: Normal and Pathological Function. Slack Incorporated: Thorofare, NJ.

[19] Reuleux F. (1875) Theoretische Kinematik. Vieweg: Braunschweig (English edition by Kennedy, 1876).

[20] Worg R. (1993) Deterministisches Chaos: Wege in die nichtlineare Dynamik. BI Wissenschaftsverlag: Mannheim.

[21] Zenkov DV. Bloch AM. Marsden JE. (1998) The Energy-Momentum Method for Stability of Nonholonomic Systems. Dynamics and Stabilty of Systems 13, pp. 123-165.

[22] Zenkov DV. Bloch AM. Marsden JE. (1999) Lyapunov-Malkin Theorem and Stabilization of the Unicyle with . (http:/www.math.lsa.umich.edu/ zenkov/papers/unicycle.ps.gz)