Fuzzy Techniques in Image Processing: Three Case Studies

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Abstract Fuzzy techniques can be applied in several domains of image processing [10]. In this paper we will show how fuzzy techniques are used in mathematical morphology, in establishing measures for image quality evaluation, and in constructing filters for image noise reduction.
Keywords: image processing, morphology, filters, noise reduction, similarity measures.

1 Mathematical Morphology

Within the domain of image processing, mathematical morphology is a theory based on geometrical concepts and set operators, used to analyze the shape or shape related properties of images. The theory was originally developed for binary images (images with only black and white), and has been extended to gray-scale images (images allowing shades of gray instead of only black and white).

In this section, we will discuss the different ways in which a fuzzy morphology can be constructed; fuzzy morphology is an extension of binary morphology to gray-scale morphology, using techniques from fuzzy set theory.

1.1 Binary Morphology

Binary images are mathematically represented as subsets of $\mathbb{R}^n$. The basic morphological operators are dilation and erosion. A morphological operation transforms an image A by means of a structuring element B into a new image. The structuring element can be chosen by the user; shape and size depend upon the application [20].

Definition 1 *Let $A, B \in \mathcal{P}(\mathbb{R}^n)$. The binary dilation $D(A,B)$ and the binary erosion $E(A,B)$ are given by:*

$$D(A,B) = \{y \in \mathbb{R}^n | T_y(B) \cap A \neq \emptyset\} \quad and \quad E(A,B) = \{y \in \mathbb{R}^n | T_y(B) \subseteq A\},$$

with $T_y(B) = \{x \in \mathbb{R}^n | x - y \in B\}$, i.e. the translation of B over the vector y.

These definitions have a very natural interpretation: a point y belongs to the dilation $D(A, B)$ if the translate $T_y(B)$ of the structuring element B hits A; it belongs to the erosion $E(A, B)$ if the translate $T_y(B)$ completely fits into A.

1.2 Fuzzy Morphology

Gray-scale images are mathematically represented as $\mathbb{R}^n$-$[0, 1]$ mappings, and can be identified with fuzzy sets in $\mathbb{R}^n$. The class of fuzzy sets in $\mathbb{R}^n$ is denoted by $\mathcal{F}(\mathbb{R}^n)$.

Several approaches to fuzzy morphology are possible. In the following, we will discuss approaches based on the fuzzification of logical operators, and on the fuzzification of set inclusion. Regarding the latter, the fuzzified set inclusions of Zadeh, Sinha & Dougherty, Kitainik and Bandler & Kohout will be discussed.

1.2.1 Approaches based on the fuzzification of logical operators

The set notions $\cap$ (intersection) and $\subseteq$ (inclusion), and consequently the underlying logical operations of conjunction and implication, play an important role in the definition of dilation and erosion. A fuzzification of the binary logical operators will lead to a fuzzification of the binary dilation and erosion [6].

Definition 2 *A binary operator $\mathcal{C}$ on [0,1] is a conjunctor if it is a mapping with increasing partial mappings that coincides with the Boolean conjunction on $\{0, 1\}^2$, i.e. $\mathcal{C}(0, 0) = \mathcal{C}(0, 1) = \mathcal{C}(1, 0) = 0$ and $\mathcal{C}(1, 1) = 1$. A t-norm is a commutative and associative conjunctor that also satisfies $(\forall x \in [0, 1])(\mathcal{C}(1, x) = \mathcal{C}(x, 1) = x)$.*

A binary operator $\mathcal{I}$ on [0,1] is an implicator if it is a mapping with decreasing first and increasing second partial mappings that coincides with the Boolean implication on $\{0, 1\}^2$, i.e. $\mathcal{I}(0, 0) = \mathcal{I}(0, 1) = \mathcal{I}(1, 1) = 1$ and $\mathcal{I}(1, 0) = 0$.

The minimum T_M, the algebraic product T_P and the Łukasiewicz t-norm T_W are very popular t-norms: $T_M(x, y) = \min(x, y)$, $T_P(x, y) = x \cdot y$ and $T_W(x, y) = \max(0, x + y - 1)$; the Łukasiewicz implicator I_L, the Kleene-Dienes implicator I_{KD} and the Reichenbach implicator I_R are well-known implicators: $I_L(x, y) = \min(1, 1 - x + y)$, $I_{KD}(x, y) = \max(1 - x, y)$ and $I_R(x, y) = 1 - x + x \cdot y$.

The definitions of the binary dilation and the binary erosion can then be fuzzified as follows.

Definition 3 *Let $A, B \in \mathcal{F}(\mathbb{R}^n)$, let $\mathcal{C}$ be a conjunctor and let $\mathcal{I}$ be an implicator. The fuzzy dilation $D_{\mathcal{C}}(A, B)$ and the fuzzy erosion $E_{\mathcal{I}}(A, B)$ are given by:*

$$\begin{aligned} D_{\mathcal{C}}(A, B)(y) &= \sup_{x \in T_y(d_B) \cap d_A} \mathcal{C}(B(x - y), A(x)), \quad \forall y \in D(d_A, d_B) \\ E_{\mathcal{I}}(A, B)(y) &= \inf_{x \in T_y(d_B)} \mathcal{I}(B(x - y), A(x)), \quad \forall y \in E(d_A, d_B), \end{aligned}$$

with $d_A = \{x \in \mathbb{R}^n | A(x) > 0\}$ and $d_B = \{x \in \mathbb{R}^n | B(x) > 0\}$.

The fuzzy dilation and erosion, as well as the practical consequences of the choice of the logical operators, are illustrated in Fig. 1.

Fig. 1: Fuzzy dilation and erosion. Top: original image; second row: dilation D_{T_M}, erosion $E_{I_{KD}}$ and edge image $D_{T_M} - E_{I_{KD}}$; bottom row: dilation D_{T_W}, erosion E_{I_W} and edge image $D_{T_W} - E_{I_W}$. Regarding the edge images: note that in the first case both large and fine structures are detected, while in the second case only large structures remain. This is due to the specific choice of logical operators.

If A and B are crisp subsets of $\mathbb{R}^n$, then one can easily show that the fuzzy dilation and erosion coincide with their binary counterparts. In [13, 14] also the relationship between fuzzy morphology and classical gray-scale morphologies is investigated.

Related approaches.

(1) In [3] a similar approach to define the fuzzy dilation and erosion is followed. There are however two differences: (i) a t-norm instead of a more general conjunctor is used; (ii) the t-norm and implicator are connected in a specific way. Consequently, the resulting framework is less general than the one presented above.

(2) In [5] one uses a fuzzification of the Minkowski addition $\oplus$ to achieve a fuzzification of the binary dilation. This approach is based on the crisp relation $A \oplus B = D(A, -B)$, and turns out to fit in the logical framework.

1.2.2 Approaches Based on the Fuzzification of Set Inclusion

Given a fuzzified set inclusion Inc, we can use it to extend the binary erosion to an operation on fuzzy sets in $\mathbb{R}^n$ by putting $E_{Inc}(A, B)(y) = Inc(T_y(B), A)$.

Fuzzy set inclusion of Zadeh.

For $A, B \in \mathcal{F}(\mathbb{R}^n)$, the fuzzy set inclusion $\subseteq_z$ of Zadeh is defined as [28]: $A \subseteq_z B \Leftrightarrow (\forall x \in \mathbb{R}^n)(A(x) \leq B(x))$. The drawback of this definition is that it doesn't allow any degree of subsetness: a fuzzy set is a subset of another fuzzy set, or it is not. This also implies that the corresponding "fuzzy" erosion E_z will be crisp:

$$y \in E_z(A, B) \Leftrightarrow \inf_{x \in T_y(d_B)} A(x) - B(x - y) \geq 0.$$

The Zadeh-erosion can be linked to the logical framework. If we define:

$$I_z(x, y) = \begin{cases} 1 & \text{if } x \leq y \\ 0 & \text{else} \end{cases}, \quad \forall (x, y) \in [0,1]^2,$$

then we have that $E_z(A, B) = E_{I_z}(A, B)$.

Fuzzy Set Inclusion of Sinha and Dougherty.

In [21] a general indicator Inc_{SD} for fuzzified set inclusion, based on nine intuitive acceptable axioms, is introduced:

$$Inc_\lambda(A, B) = \inf_{x \in \mathbf{R}^n} \min(1, \lambda(A(x)) + \lambda(1 - B(x))),$$

where λ is a $[0,1] - [0,1]$ mapping satisfying a set of properties. Some examples of possible λ-mappings are given by $\lambda_n(x) = 1 - x^n$ $(n \geq 1)$, and $\lambda_n(x) = \frac{1-x}{1+nx}$ $(n \in]-1, 0])$. Using the indicator Inc_λ, the fuzzy erosion E_λ is obtained as follows [22]:

$$E_\lambda(A, B)(y) = \inf_{x \in T_y(d_B)} \min\left(1, \lambda(B(x - y)) + \lambda(1 - A(x))\right), \quad \forall y \in E(d_A, d_B).$$

This approach can be linked to the logical framework. Indeed, if we define

$$\mathcal{I}_\lambda(x, y) = \min(1, \lambda(x) + \lambda(1 - y)), \quad \forall (x, y) \in [0, 1]^2,$$

then it follows that $E_\lambda(A, B) = E_{\mathcal{I}_\lambda}(A, B)$.

Fuzzy set inclusion of Kitainik.

Another axiom-based approach towards fuzzified set inclusion is given in [11]. The general form of the indicator Inc_φ for the fuzzified set inclusion is:

$$Inc_\varphi(A, B) = \inf_{x \in \mathbf{R}^n} \varphi\left(\max(A(x), 1 - B(x)), \min(A(x), 1 - B(x))\right),$$

with φ belonging to a certain class of $T - [0, 1]$ functions, $T = \{(x, y) | (x, y) \in [0, 1]^2 \text{ and } x \geq y\}$.

The corresponding fuzzy erosion E_φ is given by, for $y \in E(d_A, d_B)$:

$$E_\varphi(A, B)(y) = \inf_{x \in T_y(d_B)} \varphi\left(\max(B(x - y), 1 - A(x)), \min(B(x - y), 1 - A(x))\right).$$

Also in this case the φ-erosion coincides with a specific choice of an implicator $\mathcal{I}$ in the logical framework. If we define $\mathcal{I}_\varphi$ as:

$$\mathcal{I}_\varphi(x, y) = \varphi\left(\max(x, 1 - y), \min(x, 1 - y)\right), \quad \forall (x, y) \in [0, 1]^2,$$

then $E_\varphi(A, B) = E_{\mathcal{I}_\varphi}(A, B)$. The relationship with the logical framework is much stronger than the above formula suggests. This is explored in great detail in [14].

Fuzzy set inclusion of Bandler and Kohout.

The inclusion-based approach in [1] is a logical one, and is based on the fuzzification of the binary expression $A \subseteq B \Leftrightarrow (\forall x \in \mathbf{R}^n)(x \in A \Rightarrow x \in B)$, using the notion of an implicator $\mathcal{I}$:

$$Inc_{\mathcal{I}}(A, B) = \inf_{x \in \mathbf{R}^n} \mathcal{I}(A(x), B(x)).$$

It immediately follows that the corresponding fuzzy erosion equals $E_{\mathcal{I}}(A, B)$.

1.3 Conclusion

We have discussed several "fuzzy ways" to extend binary morphology to morphology for gray-scale images. All these different approaches fit into one general logical framework; we refer to [13, 14] for a complete study of their interdependencies. This observation suggests that further research should concentrate on the logical framework.

2 Similarity Measures for Images

An important problem in image processing is the comparison of images: if different algorithms are applied to an image, we need an objective measure to compare the different output images. It is well-known that classical measures, such as the MSE (mean square error), do not always give convincing results. Since gray-scale images can be identified with fuzzy sets, it is interesting to investigate whether similarity measures, i.e. measures that are developed to express the degree of similarity between fuzzy sets, can also be applied in image processing.

In this section, we will discuss some properties w.r.t. which similarity measures can be evaluated in the context of image processing. We will give examples of satisfactory measures, and illustrate their behaviour with some examples.

Notational remark: in this section $A, B \in \mathcal{F}(X)$, with $X = \{(x,y)\ 0 \leq x \leq M, 0 \leq y \leq N\}$ a *discrete* set of image points.

2.1 Definition

In the literature a lot of measures that express the similarity between two fuzzy sets can be found. In most cases, a similarity measure is formally defined as a fuzzy binary relation in $\mathcal{F}(X)$, i.e. a $\mathcal{F}(X) \times \mathcal{F}(X) \rightarrow [0,1]$ mapping that is reflexive, symmetric and min-transitive. However, not every measure in the literature satisfies this definition. Therefore, we give a larger interpretation to the notion of a similarity measure: a similarity measure is any measure to compare two fuzzy sets.

2.2 Relevant Properties for Image Processing

For our investigation [24, 15], we have considered the following properties:

Reflexivity: for two identical images one may expect that the similarity measure has output 1.

Symmetry: the output of the similarity measure is expected to be independent of the order in which the two input images are considered.

Reaction to noise (e.g. salt & pepper noise or gaussian noise): a good similarity measure should not be affected too much due to noise (a noisy image is coming from, and is consequently similar to, an original image), and should be decreasing with respect to an increasing noise-percentage.

Reaction to enlightening and darkening: if one enlightens or darkens an image with a constant value, the similarity measure should return a high value (indeed, one considers almost identical images). One also expects a decreasing behaviour with respect to an increasing enlightening- or darkeningpercentage.

Reaction to binary images: similarity measures should also yield good results when applied to binary images. In particular, one may expect that the similarity measure produces a value between 0 and 1, and not only the crisp values 0 or 1.

This list should not be considered as complete: depending on the application or the type of images that have to be compared, some properties will be less relevant and sometimes completely different properties will have to be investigated.

2.3 Similarity Measures Applicable in Image Processing

From a total of 30 different measures [24], only the following six satisfied the above mentioned properties. The first similarity measure is based on the fuzzy Minkowski distance, and is given by [4, 27]:

$$S_1(A,B) = 1 - \left(\frac{1}{MN} \sum_{(x,y)\in X} |A(x,y) - B(x,y)|^r \right)^{\frac{1}{r}} \quad \text{with } r \in \mathbb{N}_0.$$

The measures S_2 [16, 26] and S_3 [7] are based on the sigma count; this is an extension of the notion of cardinality. The sigma count of a fuzzy set A (with finite support) in a universe U is given by:

$$|A| = \sum_{u\in U} A(u).$$

$$S_2(A,B) = \frac{|A \cap B|}{|A \cup B|} = \frac{\sum_{(x,y)\in X} \min(A(x,y), B(x,y))}{\sum_{(x,y)\in X} \max(A(x,y), B(x,y))}$$

$$S_3(A,B) = \frac{|co\, A \cap co\, B|}{|co\, A \cup co\, B|} = \frac{\sum_{(x,y)\in X} \min(1 - A(x,y), 1 - B(x,y))}{\sum_{(x,y)\in X} \max(1 - A(x,y), 1 - B(x,y))}.$$

Note that the intersection is modelled by the minimum, the union by the maximum and the complement by the $1-$ operator.

For the similarity measures S_4 [4, 16, 25, 26], S_5 [25] and S_6 [2] it is less obvious to give an intuitive interpretation:

$$S_4(A,B) = 1 - \frac{\sum_{(x,y)\in X} |A(x,y) - B(x,y)|}{\sum_{(x,y)\in X} (A(x,y) + B(x,y))}$$

$$S_5(A,B) = \frac{1}{MN} \sum_{(x,y)\in X} \left[\frac{\min(A(x,y), B(x,y))}{\max(A(x,y), B(x,y))} \right]$$

$$S_6(A,B) = 1 - \frac{1}{MN \cdot 2\ln 2} \sum_{(x,y)\in X} \left[(A(x,y) - B(x,y)) \cdot \ln\left(\frac{1 + A(x,y)}{1 + B(x,y)} \right) + (B(x,y) - A(x,y)) \cdot \ln\left(\frac{2 - A(x,y)}{2 - B(x,y)} \right) \right].$$

2.4 Some Examples

In Fig. 2 we have added salt & pepper noise to the cameraman (1% and 5%), and we have enlightened the image twice (+0.1 and +0.2). These images are compared to the original image, and the numerical results are displayed in Table 1. One can verify that the returned values are relatively high, and that the similarity values slightly decrease w.r.t. an increasing noise or enlightening level. For comparison, also the classical MSE measure is displayed; for $A, B \in \mathcal{F}(X)$ the MSE is defined as: $MSE(A,B) = \frac{1}{MN}\sum_{(x,y)\in X} |A(x,y) - B(x,y)|^2$.

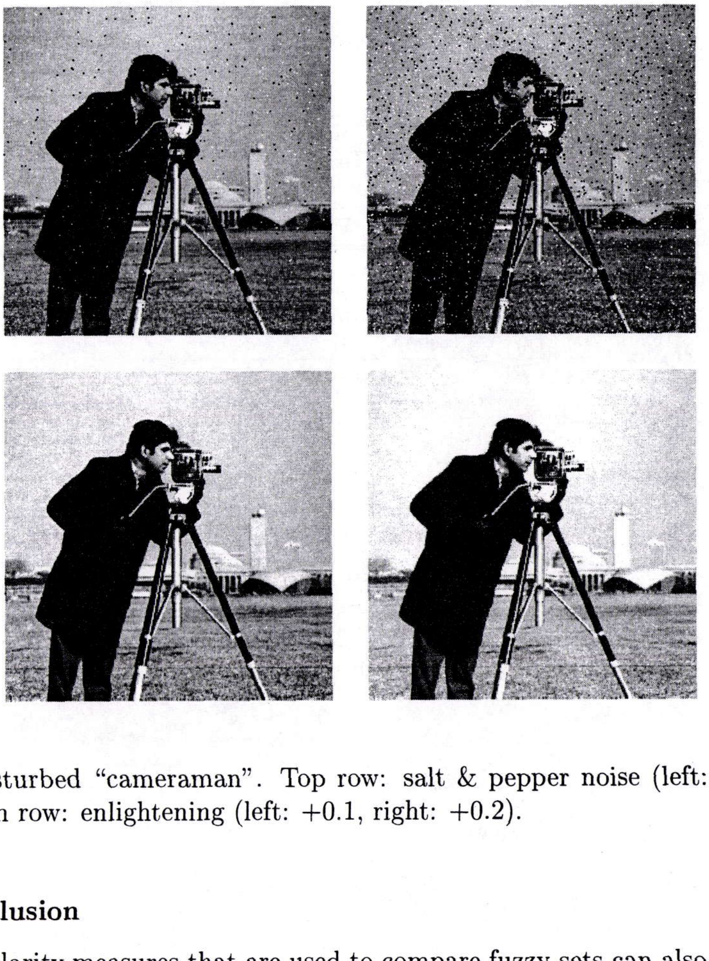

Fig. 2: Disturbed "cameraman". Top row: salt & pepper noise (left: 1%, right: 5%); bottom row: enlightening (left: +0.1, right: +0.2).

2.5 Conclusion

Several similarity measures that are used to compare fuzzy sets can also be successfully applied in image processing. Based on a list of relevant properties, we obtained 6 measures that show a very good behaviour.

Table 1: Values of the similarity measures when the "cameraman" image is disturbed by noise (column 2 and 3) or by enlightening (column 4 and 5).

measure	1% noise	5% noise	+0.1 enlightening	+0.2 enlightening
$S_1(A, B)$	0.94508	0.87654	0.90207	0.80450
$S_2(A, B)$	0.98980	0.94849	0.82627	0.70453
$S_3(A, B)$	0.99110	0.95478	0.81683	0.63463
$S_4(A, B)$	0.99488	0.97356	0.90487	0.82666
$S_5(A, B)$	0.99280	0.96236	0.74136	0.62594
$S_6(A, B)$	0.99699	0.98480	0.99051	0.96211
$MSE(A, B)$	193.02	981.73	613.90	2446.40

3 A Fuzzy Filter for Image Noise Reduction

In this section we wil focus on fuzzy techniques for image filtering. Already several fuzzy filters for noise reduction have been developed, e.g. the well-known FIRE-filter from Russo [17, 18, 19], the weighted fuzzy mean filter from Lee [12], and the iterative fuzzy control based filter from Farbiz and Menhaj [8]. However, most techniques are not specifically designed for gaussian(-like) noise or do not give convincing results when applied to this type of noise.

In this section, we will present a new technique for filtering gaussian noise by a fuzzy filter [23]. Two important features are presented: first, the filter estimates a 'fuzzy gradient' in order to be less sensitive to local variations due to noise; second, the membership functions are adapted accordingly to the noise level to perform 'fuzzy smoothing'.

3.1 Construction of the Filter

The general idea behind the filter is to average a pixel using other pixels from its neighbourhood, but simultaneously to take care of important image structures such as edges. The main concern of the proposed filter is to distinguish between local variations due to noise and due to image structure.

In order to accomplish this, for each pixel we derive a value that expresses the degree in which the gradient in a certain direction is small. Such a value is derived for each direction corresponding to the neighbouring pixels of the processed pixel, by means of a fuzzy rule (subsection 3.1.1).

The further construction of the filter is then based on the observation that a small fuzzy gradient most likely is caused by noise, while a large fuzzy gradient most likely is caused by an edge in the image. Consequently, for each direction we will apply two fuzzy rules that take this observation into account, and that determine the contribution of the neighbouring pixel values. The result of these rules is defuzzified and a correction term is obtained for the processed pixel (subsection 3.1.2).

3.1.1 Fuzzy Gradient Estimation

Consider the neighbourhood of a pixel (x, y) as displayed in Fig. 3. The gradient $\nabla_D(x, y)$ is defined as the difference between the central pixel (x, y) and its neighbour in the direction D; $D \in dir = \{NW, W, SW, S, SE, E, NE, N\}$. For example:

$$\begin{aligned} \nabla_N(x,y) &= I(x,y-1) - I(x,y), \\ \nabla_{SE}(x,y) &= I(x+1,y+1) - I(x,y). \end{aligned}$$

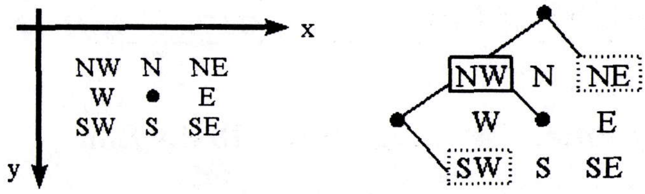

Fig. 3: Left: the neighbourhood of a central pixel (x, y); right: pixels involved to calculate the gradient values in the NW-direction.

The gradient information should be consistent perpendicular to the direction in which the gradient is calculated. Therefore, for each direction we consider 3 pixels perpendicular to that direction. For example, in the NW-direction we calculate the values $\nabla_{NW}(x, y)$, $\nabla_{NW}(x - 1, y + 1)$ and $\nabla_{NW}(x + 1, y - 1)$; see Fig. 3. In Table 2 we give an overview of the pixels that are involved in the calculations for every direction.

Table 2: Pixels involved to calculate the fuzzy gradients: each direction (column 1) corresponds to a fixed position (column 2), and the sets in column 3 specify which pixels are considered w.r.t. that fixed position in the calculations.

direction	position	set
NW	$(x-1, y-1)$	{(-1,1),(0,0),(1,-1)}
W	$(x-1, y)$	{(0,1),(0,0),(0,-1)}
SW	$(x-1, y+1)$	{(1,1),(0,0),(-1,-1)}
S	$(x, y+1)$	{(1,0),(0,0),(-1,0)}
SE	$(x+1, y+1)$	{(1,-1),(0,0),(-1,1)}
E	$(x+1, y)$	{(0,-1),(0,0),(0,1)}
NE	$(x+1, y-1)$	{(-1,-1),(0,0),(1,1)}
N	$(x, y-1)$	{(-1,0),(0,0),(1,0)}

To derive the value $\nabla_D^F(x, y)$ that expresses the degree in which the gradient in direction D is small, we make use of the fuzzy set `small`. The membership function

m_K for the property `small` is the following; see Fig. 4:

$$m_K(u) = \begin{cases} 1 - \frac{|u|}{K}, & 0 \leq u \leq K \\ 0, & u > K \end{cases} \tag{1}$$

where K is an adaptive parameter [1].

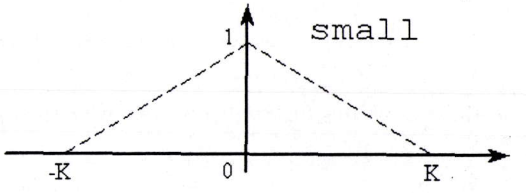

Fig. 4: The membership function `small`.

For example, the value of the fuzzy gradient $\nabla^F_{NW}(x,y)$ is calculated by the following rule:

$$\begin{array}{ll} \text{if} & (\nabla_{NW}(x,y) \text{ is } \texttt{small} \text{ and } \nabla_{NW}\,(x-1,y+1) \text{ is } \texttt{small}) \text{ or} \\ & (\nabla_{NW}(x,y) \text{ is } \texttt{small} \text{ and } \nabla_{NW}\,(x+1,y-1) \text{ is } \texttt{small}) \text{ or} \\ & (\nabla_{NW}(x-1,y+1) \text{ is } \texttt{small} \text{ and } \nabla_{NW}\,(x+1,y-1) \text{ is } \texttt{small}) \\ & \text{then } \nabla^F_{NW}\,(x,y) \text{ is } \texttt{small}. \end{array} \tag{2}$$

Eight rules are applied, computing the fuzzy gradients $\nabla^F_D(x,y)$, $D \in dir$. These rules are implemented using the minimum to represent the AND operator, and the maximum for the OR-operator.

3.1.2 Fuzzy Smoothing

To compute the correction term Δc for the processed pixel we use a pair of fuzzy rules for each direction. Consider for example the direction NW. Using the values $\nabla^F_{NW}(x,y)$ and $\nabla_{NW}(x,y)$ we fire the following rules, and compute their truthvalue λ^+_{NW} and λ^-_{NW}:

$$\begin{array}{l} \lambda^+_{NW} : \text{if } \nabla^F_{NW}(x,y) \text{ is } \texttt{small} \text{ and } \nabla_{NW}(x,y) \text{ is } \texttt{positive} \text{ then } \Delta c \text{ is } \texttt{positive}, \\ \lambda^-_{NW} : \text{if } \nabla^F_{NW}(x,y) \text{ is } \texttt{small} \text{ and } \nabla_{NW}(x,y) \text{ is } \texttt{negative} \text{ then } \Delta c \text{ is } \texttt{negative}. \end{array}$$

For the properties `positive` and `negative` we also use linear membership functions, as illustrated in Fig. 5.

[1]Instead of making use of larger windows to obtain better results for heavier noise, the filter is applied iteratively. The shape of the membership function `small` is adapted each iteration according to an estimate of the (remaining) amount of noise. We refer to [23] for technical details.

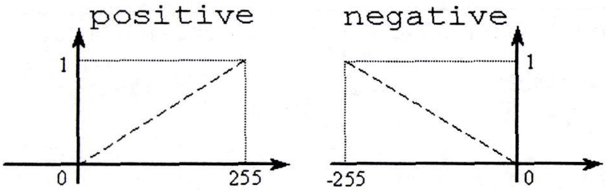

Fig. 5: The membership functions `positive` and `negative`.

If we have calculated these values for all directions, we can compute the correction term Δc as follows:

$$\Delta c = 255 \cdot \sum_{D \in dir} (\lambda_D^+ - \lambda_D^-). \tag{3}$$

This correction term is added to the pixel value of the considered pixel (x, y).

3.2 Some Examples

To illustrate the performance of the presented filter, we have applied it to two noisy "cameraman" images; see Fig. 6. For comparison, we have also applied the classical Wiener filter, and the Weighted Fuzzy Mean (WFM) filter [12].

The numerical results are displayed in Table 3. Besides the MSE-values, we have also calculated the values of the similarity measure S_1 $(r = 1)$.

Table 3: Numerical results corresponding to the images in Fig. 6.

	$\sigma = 5.7$		$\sigma = 18.0$	
	MSE	S_1	MSE	S_1
Noisy image	32.10	0.98217	300.22	0.94546
New filter	21.97	0.98611	95.53	0.97292
Wiener filter	43.34	0.98311	93.21	0.97173
WFM filter	232.56	0.95792	434.98	0.92750

The results not only confirm the good performance of our filter, but also illustrate that similarity measures (in particular the used measure S_1) are a better tool for image comparison:

(1) In the case of low noise ($\sigma = 5.7$) our new filter performs best w.r.t. both the MSE and S_1 measure. Regarding the WFM filter: performs worst (MSE much higher, similarity measure lower). Regarding the Wiener filter: performs bad w.r.t. MSE, but shows an improvement w.r.t. S_1. The latter is in accordance with the visual result, and illustrates that the similarity measure S_1 better reflects the visual observations than the MSE measure.

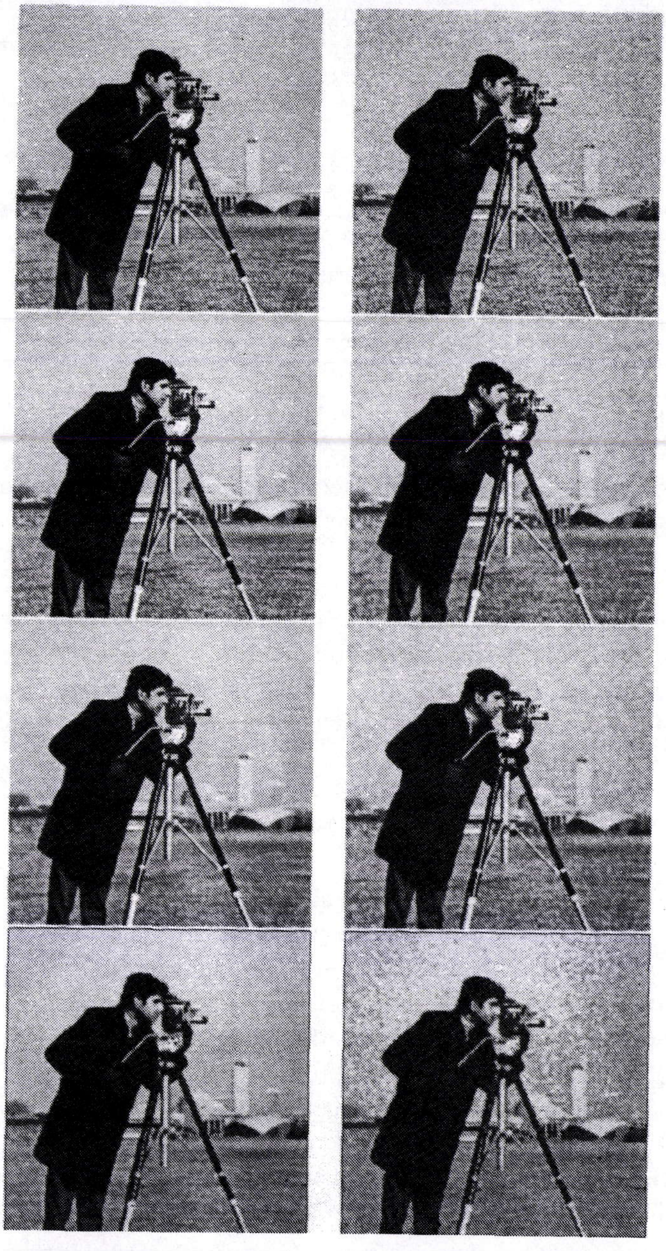

Fig. 6: Disturbed and filtered "cameraman". Top row: gaussian noise (left: $\sigma = 5.7$, right: $\sigma = 18.0$); second row: results of the proposed filter (left: $\alpha = 1.0$ and 1 iteration; right: $\alpha = 2.0$ and 2 iterations); third row: results of the classical Wiener filter; bottom row: results of the WFM filter.

(2) In the case of heavier noise ($\sigma = 18.0$) both our new filter and the Wiener filter perform best (the new filter is best w.r.t. the similarity measure S_1, Wiener is best w.r.t. MSE). Regarding the WFM filter: performs worst.

3.3 Conclusion

We presented a new fuzzy filter for additive noise reduction. Its main feature is that it distinguishes between local variations due to noise and due to image structures, using a fuzzy gradient estimation. Fuzzy rules are fired to consider every direction around the processed pixel. Although its relative simplicity, the fuzzy filter is able to compete with state-of-the-art filter techniques for noise reduction

4 General Conclusion

In this paper we have illustrated that fuzzy techniques can be used for both the extension of existing theories in image processing and the development of new techniques. In this way, fuzzy techniques are applied to extend binary morphology to morphology for gray-scale images. Futhermore, notions of fuzzy set theory were used to develop a measure of comparison for images. And finally, we proposed a new filter using fuzzy rules for the reduction of additive gaussian noise.

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