The Theory of Infinite Momentum Frames (continued)

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« The real voyage of discovery consists not in seeking new landscapes, but in having new eyes. »
Marcel Proust

Abstract
In a previous communication [1], G. NIBART (2001) has proposed a general definition of infinite momentum frames (IMF) from a mathematical point of view which allows to consider IMF with any number of dimensions.
In the present communication, we try to build a new concept of space time with IMF.
For this purpose, we study some assumptions about infinite momentum frames having n dimensions (IMF-n): a) usual referential frames can be deduced from IMF-n (with $n \geq 4$), b) IMF basis vectors can be associated to spinors. We also illustrate some particular IMF-n instances with the following examples: definition of spinors in an IMF-2, expression in an IMF-2 of the longitudinal Doppler effect including the case of tachyons, and application of IMF-6 to the « 6 dimensional universe » [2] which has been defined by G. NIBART (2000).
Keywords: Infinite Momentum Frame, IMF, Spinor, Tachyon, Doppler

1 Introduction

1.1 History of Infinite Momentum Frames

Infinite momentum frames (IMF) have been first introduced in the theory of partons [3] by J. KOGUT and L. SUSSKIND (1973) as ordinary referential frames (ORF) moving with almost the light velocity. The system of two light cone coordinates (IMF-2) has been later re-developed by R. DUTHEIL (1984, 1990) on the basis of complex rotations group in a pseudo Euclidean space [4,5]. R. DUTHEIL and G. NIBART (1986) have proposed a generalization [6] to a four dimensional infinite momentum frame (IMF-4).

In a previous communication [1], G. NIBART (2001) has criticized the definitions of IMF proposed by these authors and he has proposed a new definition of infinite momentum frames, having any number of dimensions (IMF-n) and based on vectors having a null geodesic, i.e. vectors which are isotropic in the sense of mathematics.

1.2 Why to use Infinite Momentum Frames ?

Since they have been introduced [3] by J. KOGUT and L. SUSSKIND (1973), infinite momentum frames have been used again in the theory of partons [7] by Matthias BURKARDT (2000).

Infinite momentum frames have been also used in the calculation of semileptonic decays of heavy Lambda baryons [8] by Barbara KONIG, Jurgen G. KORNER, Michael KRAMER and Peter KROLL (1997), in the quark-parton model of the proton spin structure [9] by Petr ZAVADA (1997), in the quark distributions of the nucleon [10] by D.I. DIAKONOV, V. Yu PETROV, P.V. POBYLITSA, M.V. POLYAKOV and C. WEISS (1997), in the type-IIB string theory [11] by Pei Ming HO and Yong Shi WU (1998), in the sine-Gordon model [12] by Silvio PALLUA and Predrag PRESTER (1999), in string and field theory [13] by Charles B. THORN (1999), in the type-IIA string perturbation theory [14] by G. GRIGNANI, P. ORLAND, L.D. PANIAK and G.W. SEMENOFF (2000), in the scalar quantum field theory[i] [15] by Joel S. ROZOWSKY and Charles B. THORN (2000), and for the quantization on a spacelike surface close to a light front in the perturbative scalar field theory [16] by A. HARINDRANATH, L. MARTINOVIC and J.P. VARY (2000).

1.3 Trying to Build a new Concept of Space Time

The usual concept of space time is not obvious to me, and in the present communication, I am trying to build a new concept of space time.

1.3.1 Is Space Time Just a Primary Container ?

Space time is most often considered as an invariable container of matter and energy, but the general theory of Relativity has shown that space has a curvature which depends on all the masses which it contains. In the Big Bang theory, space time seems to "appear" with the energy of the universe. It suggests that both matter energy and space time are complementary[ii].

In cosmology, the universe expansion of both matter energy and space time is usually related to the $t = 0$ time of the universe, i.e. to the relativist singularity. Although it may be considered as an objection to the standard model of cosmology, a possible alternative might be that the usual space time concept is not pertinent for cosmology. So it suggests that a new concept of space time should be proposed in relation to the light cone.

1.3.2 Heuristic Considerations About Spinors and Space Time

In a previous communication [2] a « 6 dimensional universe » U_6 has been defined by G. NIBART (2000) to build a relativistic model of particle antiparticle pair with relativist E.P.R. correlations. He has shown that the behavior of a pair of boson

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i : it is a non relativist interpretation of quantum field theory.

ii : in the sense of N. Bohr's Complementary Principle.

antiboson can be represented, with their non local correlations, with the following KLEIN, GORDON and FOCK equation in U_6 (eq. 115 in ref. 2):

$$G^{\mu}\left(1^{\mu}+R^{\mu^2}\right)\frac{\partial}{\partial X^{\mu^2}}\psi=-2\chi^2\psi \quad (\mu,\nu=1,2,3,4,5,6) \tag{1}$$

where

$$G^{\mu}=G^{\mu\mu} \quad (\mu,\nu=1,2,3,4,5,6) \tag{2}$$

is the diagonal of the metrics tensor of U_6 which is equal to the signature $(+++---)$ in the framework of the special theory of Relativity, where X^{μ} are the coordinates in U_6, where R^{μ} is the directions bivector (eq. 65-68 in ref. 2) of the pair of bosons in U_6, and χ the usual KLEIN, GORDON and FOCK constant.

He has also shown that the behavior of a pair of fermion antifermion can also be represented, with their non local correlations, with the following DIRAC equation in U_6 (eq. 146 in ref. 2):

$$\left(-\gamma^0 R^{\mu}G^{\mu\mu}+i\Gamma^{\mu}\right)J^{\mu\mu}\frac{\partial}{\partial X^{\mu}}\psi+2\chi\psi=0 \quad (\mu=1,2,3,4,5,6) \tag{3}$$

where γ^0, Γ^{μ} are the Dirac matrix in U_6 (eq. 142, 143 in ref. 2) and where $J^{\mu\nu}$ is a "square root" of the metrics tensor $G^{\mu\nu}$ as shown below (eq. 145 in ref. 2):

$$G^{\mu\nu}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&-1&0&0\\0&0&0&0&-1&0\\0&0&0&0&0&-1\end{bmatrix} \qquad J^{\mu\nu}=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&i&0&0\\0&0&0&0&i&0\\0&0&0&0&0&i\end{bmatrix} \tag{4}$$

The possibility of a relativistic representation of the non local correlations within a pair of fermion antifermion shows that the localization in space time is not a primary requirement. B.J. HILEY (2000) has recalled [17] that "there is no necessity to start an explanation of quantum processes in space time". Moreover the introduction of a "square root" of the metrics tensor, and the aspect of $J^{\mu\nu}$ with imaginary numbers has suggested to me that the primary concept, before the locality, should be built with spinors.

However, in relativist quantum mechanics there is a spinor representation of 4-vectors, and thus a spinor may represent a position in space time.

1.3.3 Study of Some Assumptions About IMF

We think that it is too early to give a pertinent new definition of space time. Thus before proposing a new postulate of physics, we are studying some assumptions concerning light hypercone coordinates having n dimensions, i.e. infinite momentum frames (IMF-n).

In the present communication, we study the following assumptions:

- Infinite momentum frames having 4 dimensions (IMF-4) can be defined from usual referential frames or tachyonic referential frames (see section 2.2),

- usual pseudo Euclidean referential frames can be deduced from an IMF-n having $n \geq$ 4 dimensions (see section 2.3) and thus
- an IMF-n having $n \geq 4$ dimensions may be considered as a generator of usual pseudo Euclidean referential frames (see section 2.3).
- The light hypercone may be built from spinors, because all basis vectors of an IMF-n can be associated to spinors (see section 2.4).

To illustrate these assumptions, some applications of particular instances of IMF-n have been presented:

- definition of spinors in an IMF-2 (see section 3.2.2),
- application of IMF-2 to the longitudinal Doppler effect (see section 3.3), including the case of tachyons (see section 3.3.2),
- application of IMF-6 to the « 6 dimensional universe » (see section 4) which has been defined in a previous communication [2] by G. NIBART (2000),
- discussion of some problems of the space time duality in IMF-6 (see section 4.3).

2 The Light Cone as a new Foundation of Physical Space Time

2.1 A General Theory of Infinite Momentum Frames

2.1.1 General Definition of an Infinite Momentum Frame

In a previous communication [1] we have proposed the following definition of an infinite momentum frame:

« An infinite momentum frame is a referential frame generated by any number of isotropic basis vectors ».

Important remark: We use the word "isotropic" in the sense of mathematics. Isotropic vectors are vectors which have a null geodesic but non zero components.

The most general equation of a light cone with n dimensions, is

$$ds^2 = 0 \tag{5}$$

and we may imagine a hypercone having more than 4 dimensions. The left hand side of the equation 5 is a quadratic form, so its solutions are vectors with complex components or vectors with real components if the metrics signature contains different signs (i.e. + and –). The pseudo Euclidean space (one minus sign) is a particular case.

Because all the basis vectors ε_μ of an IMF are isotropic, i.e. have a null geodesic, the metrics tensor $\eta_{\mu\nu}$ has a null diagonal:

$$\varepsilon_\mu \cdot \varepsilon_\mu = \eta_{\mu\mu} = 0 \qquad (\mu = 0,1,\ldots n) \tag{6}$$

so all IMF coordinate axis are isotropic, and we may say that an IMF is an « isotropic referential frame ». Consequently a relativist interval in an IMF can be expressed as

$$ds^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu \qquad (\mu \neq \nu \quad \mu,\nu = 0,1,\ldots n) \tag{7}$$

where ξ^μ are IMF coordinates, and it is equivalent to

$$ds^2 = (\eta_{\mu\nu} + \eta_{\nu\mu}) d\xi^\mu d\xi^\nu \qquad (\mu < \nu \quad \mu,\nu = 0,1,\ldots n) \tag{8}$$

Here, in equation 8, we can see the two particular types of IMF defined below.

2.1.2 Degenerated IMF

If the IMF metrics tensor $\eta_{\mu\nu}$ is antisymetric, we have

$$\eta_{\mu\nu} = -\eta_{\nu\mu} \qquad (\mu, \nu = 0,1,...n) \tag{9}$$

we deduce from equation 8 that the relativist interval is always null:

$$ds^2 = 0 \tag{10}$$

In such a case all vectors have a null geodesic, and we say that the IMF is degenerated.

2.1.3 Perfectly Isotropic IMF

If the IMF metrics tensor $\eta_{\mu\nu}$ is symetric, i.e.

$$\eta_{\mu\nu} = \eta_{\nu\mu} \qquad (\mu, \nu = 0,1,...n) \tag{11}$$

the equation 8 simplifies into:

$$ds^2 = 2\,\eta_{\mu\nu} d\xi^{\mu} d\xi^{\nu} \qquad (\mu < \nu \quad \mu, \nu = 0,1,...n) \tag{12}$$

In such a case we may say that the IMF is a « perfectly isotropic referential frame ».

2.2 An IMF Built From a Pseudo Euclidean Referential Frame

Let us consider a pseudo Euclidean referential frame with n dimensions. For example, we may consider either an ordinary referential frame (ORF-4) with the real metrics $g_{\mu\nu}$ having the signature $(+---)$ or a tachyonic referential frame (TRF-4) with the real metrics $\widetilde{g}_{\mu\nu}$ [19] having the signature $(-+++)$. Anyway the equation 7 of the light cone can be expressed in the given referential frame as

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = 0 \qquad (\mu, \nu = 0,1,...n) \tag{13}$$

and the equation 13 allows to define vectors of the light cone, which are the basis vector of an IMF. Moreover an IMF can be deduced from a pseudo Euclidean referential frame with a linear transformation of the coordinates, such as

$$x^{\mu} = a^{\mu}_{\nu} \xi^{\nu} \qquad (\mu, \nu = 0,1,...n) \tag{14}$$

where a^{μ}_{ν} are constants. Thus the IMF metrics can be classically deduced from the pseudo Euclidean metrics with the equation

$$\eta_{\mu\nu} = g_{\lambda\rho} a^{\lambda}_{\mu} a^{\rho}_{\nu} \qquad (\lambda, \mu, \nu, \rho = 0,1,...n) \tag{15}$$

In the framework of the special theory of Relativity, using the not null values of $g_{\mu\nu}$ for an ORF-4 or a TRF-4, we would have the particular equations:

$$\eta_{\mu\mu} = g_{\lambda\lambda} a^{\lambda}_{\mu} a^{\lambda}_{\mu} = 0 \qquad (\lambda, \mu = 0,1,2,3) \tag{16}$$

$$\eta_{\mu\nu} = g_{\lambda\lambda} a^{\lambda}_{\mu} a^{\lambda}_{\nu} \qquad (\mu \neq \nu \quad \lambda, \mu, \nu = 0,1,2,3) \tag{17}$$

which can be developped as

$$\left(a^0_{\mu}\right)^2 - \left(a^1_{\mu}\right)^2 - \left(a^2_{\mu}\right)^2 - \left(a^3_{\mu}\right)^2 = 0 \qquad (\mu = 0,1,2,3) \tag{18}$$

$$\eta_{\mu\nu} = \pm\left(a_{\mu}^{0}a_{\nu}^{0} - a_{\mu}^{1}a_{\nu}^{1} - a_{\mu}^{2}a_{\nu}^{2} - a_{\mu}^{3}a_{\nu}^{3}\right) \qquad (\mu \neq \nu \quad \mu, \nu = 0,1,2,3) \tag{19}$$

where the sign $\pm$ is + for an ORF-4 and − for a TRF-4. Some examples of IMF deduced from an ORF-4 or a TRF-4 have already been given, with 2 dimensions [4,5] and with 4 dimensions [6].

The given pseudo Euclidean referential frame and the deduced IMF have the same number of dimensions, and so they may be considered as equivalent for the description of physics.

2.3 An IMF as a Generator of Usual Referential Frames

Firstly let us consider a light hypercone which generates an IMF-n of n dimensions (we may imagine that it has more than 4 dimensions) and let us name ξ^{μ} its coordinates.

Secondly let us introduce a natural observer: he has a clock in his pocket which implies a time arrow, he also has a concept of a 3 dimensional space which implies a local Euclidean geometry, and the relativist requirement implies a 4 dimensional pseudo Euclidean metrics, i.e. an ordinary referential frame (ORF-4).

From such a point of view, the 4 dimensional pseudo Euclidean referential frame (ORF-4) emerges from the introduction of a natural observer within an IMF-n having 4 or more dimensions. A pseudo Euclidean referential frame can be deduced from the given IMF-n with the linear transformation of the coordinates which is expressed as

$$x^{\mu} = a_{\nu}^{\mu}\xi^{\nu} \qquad (\mu = 0,1,2,3 \quad \nu = 0,1\ldots n \quad n > 3) \tag{20}$$

where the matrix of the tensor a_{ν}^{μ} is not a square matrix but a rectangle. The IMF metrics can be related to the pseudo Euclidean metrics with the following equation

$$\eta_{\mu\nu} = g_{\lambda\rho}a_{\mu}^{\lambda}a_{\nu}^{\rho} \qquad (\lambda, \rho = 0,1,2,3 \quad \mu, \nu = 0,1\ldots n \quad n > 3) \tag{21}$$

where the matrix of the tensors $\eta_{\mu\nu}$, $g_{\mu\nu}$ are always square matrix.

The problem of the emergence of a 4 dimensional pseudo Euclidean referential frame (ORF-4) from a given IMF-n having more than 4 dimensions may obviously have several solutions: it depends on the coefficients of the IMF-n metrics tensor $\eta_{\mu\nu}$. Thus the IMF-n and the ORF-4 are not equivalent for the description of physics, because of the extraneous dimensions of the IMF-n.

The rectangle tensor a_{ν}^{μ} may be interpreted as a « localization operator ». In the case of a correlated particles pair, two « localization operators » may be considered, but the unique description of the pair as one quantum might be done in an IMF-n.

2.4 A Spinor Representation of IMF Basis Vectors

Several types of spinors may be defined in relation to infinite momentum frames. Mathematically, we can always define spinors associated to isotropic[iii] vectors. We recall it with the example of how a 2 components spinor can be associated to a 3 dimensional isotropic vector (section 3.2.1).

[iii] The word "isotropic" is still used in the sense of mathematics.

Because basis vectors of infinite momentum frame are isotropic, they can be associated to spinors. Consequently the basis vectors of any infinite momentum frame can be represented with a set of spinors. We give an example of a definition of spinors in an IMF-2 (section 3.2.2).

2.5 Conclusion About Referential Frames Generated From an IMF

We would suggest to search for an IMF-n where we could express the most general equations of the laws of physics, including non local properties. If such an IMF-n does exist, it would be understood as the fundamental IMF-n. It should be represented with a set of spinors.

3 Applications of IMF-2

3.1 Recall of Definitions of IMF-2 Coordinates

R. DUTHEIL (1984, 1990) has demonstrated [4,5] the following relations between a two dimensional subluminal referential frame (ORF-2) with the two light cone coordinates[iv] τ, ζ of a two dimensional infinite momentum frame (IMF-2):

$$\begin{aligned} \tau &= \tfrac{1}{\sqrt{2}}(ct + x) \\ \zeta &= \tfrac{1}{\sqrt{2}}(ct - x) \end{aligned} \tag{22}$$

He has also demonstrated the following relations between a two dimensional superluminal referential frame (TRF-2) with two other light cone coordinates[v] $\tilde{\tau}$, $\tilde{\zeta}$ defined in the same IMF-2:

$$\begin{aligned} c\tilde{t} &= \tfrac{1}{\sqrt{2}}\left(\tilde{\tau} + \tilde{\zeta}\right) \\ \tilde{x} &= \tfrac{1}{\sqrt{2}}\left(\tilde{\zeta} - \tilde{\tau}\right) \end{aligned} \tag{23}$$

and he has shown that the four light cone coordinates τ, ζ, $\tilde{\tau}$, $\tilde{\zeta}$ are related by the following equations:

$$\begin{aligned} \tilde{\tau} &= \zeta \\ \tilde{\zeta} &= -\tau \end{aligned} \tag{24}$$

3.2 Applications of IMF-2 to Spinors

3.2.1 Recall of Mathematics: Spinor Associated to an Isotropic Vector

A spinor (ψ,ϕ) can be associated to an isotropic vector $\vec{\mathbf{A}}$ [18] as explained below. Let us consider a 3 dimensional isotropic vector $\vec{\mathbf{A}}$ having complex components a_1, a_2, a_3. We have

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[iv] He called them « inherent light cone coordinates ».

[v] He called them « tachyonic light cone coordinates ».

$$\|\vec{\mathbf{A}}\| = a_1^2 + a_2^2 + a_3^2 = 0 \tag{25}$$

Let us write the equation 25 as

$$a_3^2 = -(a_1 + ia_2)(a_1 - ia_2) \tag{26}$$

and let us define two complex numbers ψ, ϕ such as

$$\begin{aligned} a_1 + ia_2 &= -2\phi^2 \\ a_1 - ia_2 &= 2\psi^2 \end{aligned} \tag{27}$$

Reciprocally the vector $\vec{\mathbf{A}}$ can be deduced from a spinor (ψ, ϕ) with the following equations

$$a_1 = \psi^2 - \phi^2 \tag{28}$$

$$a_2 = i(\psi^2 + \phi^2) \tag{29}$$

$$a_3 = \pm 2\psi\phi \tag{30}$$

where the sign of a_3 can be arbitrarily chosen[vi].

3.2.2 Definition of Spinors Associated to Isotropic Vectors in an IMF-2

A 2 components spinor can be associated to every basis vector of an IMF-2. Here is an example. In the framework of the special theory of Relativity, the metrics is usually expressed as

$$s^2 = c^2t^2 - x^2 - y^2 - z^2 \tag{31}$$

Any isotropic 4-vector $\vec{\mathbf{V}}$ satisfies the light cone equation

$$s^2 = 0 \tag{32}$$

and the vector components are such as

$$c^2t^2 - x^2 - y^2 - z^2 = 0 \tag{33}$$

Let us consider the x axis as a privileged direction and write the equation 33 as

$$c^2t^2 - x^2 = y^2 + z^2 \tag{34}$$

Comparing with the equation 26 we have

$$a_1^2 = -y^2 \tag{35}$$

$$a_2^2 = -z^2 \tag{36}$$

$$a_3^2 = c^2t^2 - x^2 \tag{37}$$

so we may define the complex numbers a_1, a_2 as

$$a_1 = iy \tag{38}$$

$$a_2 = iz \tag{39}$$

and thus we may associate the isotropic vector $\vec{\mathbf{V}}$ with the spinor (ψ, ϕ) such as

$$iy - z = -2\phi^2 \tag{40}$$

$$iy + z = 2\psi^2 \tag{41}$$

[vi] E. CARTAN used to choose the minus sign.

$$c^2t^2 - x^2 = 4\psi^2\phi^2 \tag{42}$$

If we introduce the vector components in an IMF-2 built with the x axis

$$\begin{aligned}\tau &= \tfrac{1}{\sqrt{2}}(ct + x)\\ \zeta &= \tfrac{1}{\sqrt{2}}(ct - x)\end{aligned} \tag{43}$$

the equation 42 may be written

$$\tau\zeta = 2\psi^2\phi^2 \tag{44}$$

Finally, the squared product of the components of a spinor (ψ,ϕ) associated to an isotropic 4-vector $\vec{\mathbf{V}}$ corresponds to the product of the IMF-2 components τ, ζ of the vector. More precisely when $\tau\zeta > 0$ we have

$$\sqrt{\tau\zeta} = \sqrt{2}\,\psi\,\phi \tag{45}$$

and when $\tau\zeta < 0$ we have

$$\sqrt{-\tau\zeta} = \sqrt{2}\,i\,\psi\,\phi \tag{46}$$

3.2.3 Conclusion About Spinors in an IMF-2

With this simple example of the definition of a spinor in an IMF-2, we see that spinors may be used to represent isotropic vectors which are the generator of an infinite momentum frame.

3.3 Application of IMF-2 to the Longitudinal Doppler Effect

3.3.1 Subluminal Longitudinal Doppler Effect in an IMF-2

Let us consider a particle at rest in an ordinary referential frame K, and an other subluminal referential frame K' moving along the x axis with a subluminal velocity $v=c\beta$. The particle emits a photon in the direction of the x axis with a frequency ν in K and ν' in K'.

If the photon is moving away from the observer in K', the longitudinal Doppler effect is usually represented with the following equation

$$\nu' = \nu\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \tag{47}$$

which is similar to the IMF transformation equation of the τ coordinate (see eq. 67 in ref. 1) i.e.

$$\tau' = \tau\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \tag{48}$$

so when the photon is moving away, the longitudinal subluminal Doppler effect in an IMF-2 may be expressed with the τ coordinates, as

$$\frac{\nu'}{\nu} = \frac{\tau'}{\tau} \tag{49}$$

If the photon is moving towards the observer in K', the longitudinal Doppler effect is usually represented with the following equation

$$\nu' = \nu \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \tag{50}$$

which is similar to the IMF transformation equation of the ζ coordinate (see eq. 68 in ref. 1) i.e.

$$\zeta' = \zeta \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \tag{51}$$

So when the photon is moving nearer, the subluminal longitudinal Doppler effect in an IMF may be expressed with the ζ coordinates, as

$$\frac{\nu'}{\nu} = \frac{\zeta'}{\zeta} \tag{52}$$

3.3.2 Superluminal Longitudinal Doppler Effect in an IMF-2

Let us consider a superluminal particle related to its proper tachyonic referential frame $\widetilde{K}$, and an other superluminal referential frame $\widetilde{K}'$ moving along the $\widetilde{x}$ axis with a velocity $\widetilde{v} = c\widetilde{\beta}$ such as $|\widetilde{\beta}| > 1$. The particle emits a photon in the direction of the $\widetilde{x}$ axis with a frequency $\widetilde{\nu}$ in $\widetilde{K}$ and $\widetilde{\nu}'$ in $\widetilde{K}'$.

R. DUTHEIL has established [19] an equation of what he called « the tachyonic Doppler effect », but it concerns the superluminal longitudinal Doppler effect, only when the photon is moving away. His equation expressed in a tachyonic referential frame (TRF)

$$\widetilde{\nu}' = \widetilde{\nu} \frac{\sqrt{\widetilde{\beta}-1}}{\sqrt{\widetilde{\beta}+1}} \tag{53}$$

is similar to the IMF transformation equation of the $\widetilde{\tau}$ coordinate (see the system of equations 82 in ref. 1) i.e.

$$\widetilde{\tau}' = \widetilde{\tau} \frac{\sqrt{\widetilde{\beta}-1}}{\sqrt{\widetilde{\beta}+1}} \tag{54}$$

but his demonstration was restricted to $\widetilde{\beta} > 1$. When $\widetilde{\beta} < -1$ he would have found

$$\widetilde{\nu}' = \widetilde{\nu} \frac{\sqrt{-\widetilde{\beta}+1}}{\sqrt{-\widetilde{\beta}-1}} \tag{55}$$

which is similar to the IMF transformation equation of the $\widetilde{\tau}$ coordinate when $\widetilde{\beta} < -1$ (see the system of equations 84 in ref. 1) i.e.

$$\widetilde{\tau}' = -\widetilde{\tau} \frac{\sqrt{-\widetilde{\beta}+1}}{\sqrt{-\widetilde{\beta}-1}} \tag{56}$$

So when the photon is moving away, the longitudinal superluminal Doppler effect in an IMF may be expressed as

$$\frac{\tilde{\nu}'}{\tilde{\nu}} = \left|\frac{\tilde{\tau}'}{\tilde{\tau}}\right| \tag{57}$$

In the case when the photon is moving nearer to the hypothetical superluminal observer, he would have found the equation

$$\tilde{\nu}' = \tilde{\nu}\frac{\sqrt{\tilde{\beta}+1}}{\sqrt{\tilde{\beta}-1}} \tag{58}$$

which is similar to the following IMF transformation equation of the $\tilde{\zeta}$ coordinate when $\tilde{\beta} > 1$ (see the system of equations 82 in ref. 1)

$$\tilde{\zeta}' = \tilde{\zeta}\frac{\sqrt{\tilde{\beta}+1}}{\sqrt{\tilde{\beta}-1}} \tag{59}$$

or he would have found the equation

$$\tilde{\nu}' = \tilde{\nu}\frac{\sqrt{-\tilde{\beta}-1}}{\sqrt{-\tilde{\beta}+1}} \tag{60}$$

which is similar to the following IMF transformation equation of the $\tilde{\zeta}$ coordinate when $\tilde{\beta} < -1$ (see the system of equations 84 in ref. 1)

$$\tilde{\zeta}' = -\tilde{\zeta}\frac{\sqrt{-\tilde{\beta}-1}}{\sqrt{-\tilde{\beta}+1}} \tag{61}$$

So when the photon is moving nearer, the longitudinal superluminal Doppler effect in an IMF may be expressed as

$$\frac{\tilde{\nu}'}{\tilde{\nu}} = \left|\frac{\tilde{\zeta}'}{\tilde{\zeta}}\right| \tag{62}$$

Finally the IMF-2 transformations are directly related to the longitudinal Doppler effect, in both the spacelike region and the timelike region.

3.3.3 Graphs of the Superluminal and Subluminal, Longitudinal Doppler Effect

In the same communication [19] R. DUTHEIL has proposed to draw a common graph of both the superluminal longitudinal Doppler effect and the subluminal longitudinal Doppler effect, but it concerns the only case when the photon is moving away from the observer.

We have re-drawn it in the figure 1 below, with the *Mathcad 6.0 SE* software, using the following function of the β velocity in the interval [-5,+ 5]:

$$N_1(\beta) = \sqrt{\frac{|\beta-1|}{|\beta+1|}} \tag{63}$$

where N_1 is a frequency ratio corresponding to the following τ coordinate ratio of the IMF transformation

$$N_1 = \frac{\tilde{\nu}'}{\tilde{\nu}} = \left|\frac{\tilde{\tau}'}{\tilde{\tau}}\right| \quad \text{or} \quad N_1 = \frac{\nu'}{\nu} = \frac{\tau'}{\tau} \tag{64}$$

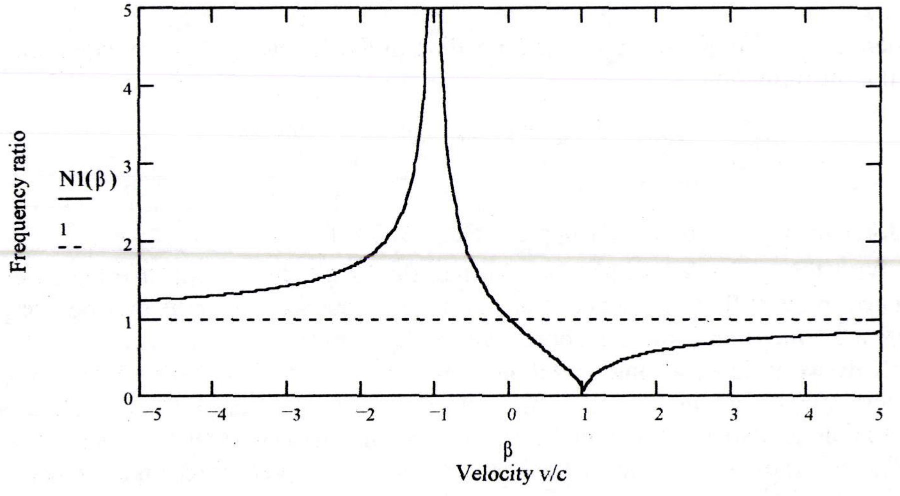

Figure 1: Longitudinal Doppler effect when the photon is moving away from the observer

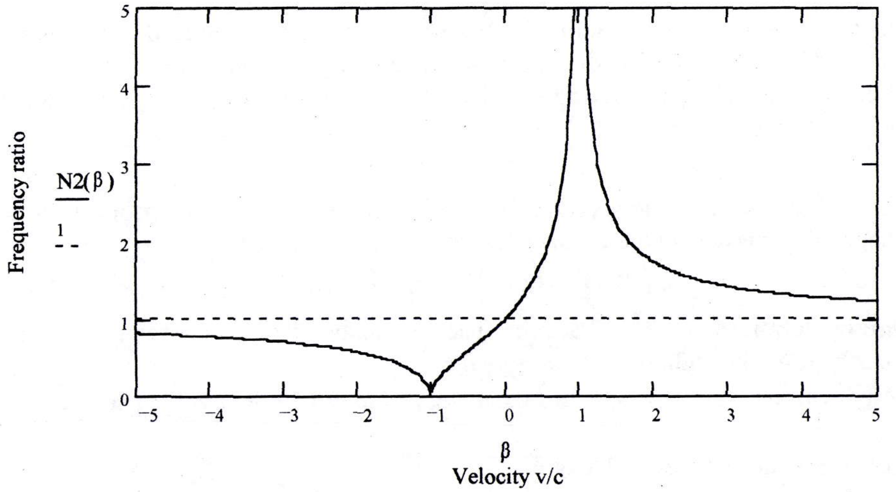

Figure 2: Longitudinal Doppler effect when the photon is moving nearer to the observer

We have also drawn a graph of the longitudinal Doppler effect when the photon is moving towards the observer, in the figure 2 above, with the *Mathcad 6.0 SE* software, using the following function of the β velocity in the same interval [-5,+ 5]:

$$N_2(\beta) = \sqrt{\frac{|\beta+1|}{|\beta-1|}} \tag{65}$$

where N_2 is a frequency ratio corresponding to the following ζ coordinate ratio of the IMF transformation

$$N_2 = \frac{\tilde{\nu}'}{\tilde{\nu}} = \left|\frac{\tilde{\zeta}'}{\tilde{\zeta}}\right| \quad \text{or} \quad N_2 = \frac{\nu'}{\nu} = \frac{\zeta'}{\zeta} \tag{66}$$

3.3.4 Conclusion About the Doppler Effect in an IMF-2

With this example, we have shown that the longitudinal Doppler effect is directly related to the IMF-2 coordinates transformation. Thus star red shifts may be interpreted with a 2 dimensional infinite momentum frame (IMF-2).

Now we propose an interpretation of the graphs of the Doppler effect, concerning superluminal velocities: If tachyons exists, they would produce a Doppler effect which is similar to that of subluminal particles. So the frequency ratio N would not be a sufficient knowledge to distinguish a subluminal Doppler effect and a superluminal Doppler effect.

4 Application of IMF-6

4.1 Recall About the « 6 Dimensional Universe »

In a previous communication, G. NIBART (2000) has introduced a 6 dimensional manifold U_6 named the « six dimensional Universe » [2], to build a relativist model of a particle antiparticle pair having relativist E.P.R. correlations. It has the following metrics with the signature $(+++---)$

$$ds^2 = G_{\mu\nu} dX^{\mu} dX^{\nu} \quad (\mu,\nu = 1,2,3,4,5,6) \tag{67}$$

In the framework of the special theory of Relativity, the metrics tensor $G^{\mu\nu}$ simplifies to diagonal terms and so the U_6 interval can be expressed as

$$ds^2 = \left(dX^1\right)^2 + \left(dX^2\right)^2 + \left(dX^3\right)^2 - \left(dX^4\right)^2 - \left(dX^5\right)^2 - \left(dX^6\right)^2 \tag{68}$$

where by definition X^1, X^2, X^3 are the spacelike (superluminal) coordinates and X^4, X^5, X^6 are the timelike (subluminal) coordinates.

Any isotropic vector $\vec{\mathbf{V}}$ satisfies the general equation of the light hypercone

$$ds^2 = 0 \tag{69}$$

which can be developped in U_6 as

$$\left(dX^1\right)^2 + \left(dX^2\right)^2 + \left(dX^3\right)^2 - \left(dX^4\right)^2 - \left(dX^5\right)^2 - \left(dX^6\right)^2 = 0 \tag{70}$$

4.2 Example of an IMF-6 Deduced From the « 6 Dimensional Universe »

An isotropic vector can be defined with a linear combination of a timelike vector and a spacelike vector of a given pseudo Euclidean space, as in the definition of IMF-2 coordinates (see section 3.1). The following space time association in the IMF-2 equation 22

$$x \leftrightarrow ct \tag{71}$$

in which the x axis has been privileged, suggests the naive definition of an IMF-6 written below

$$\begin{array}{ll} \tau^1 = \frac{1}{\sqrt{2}}\left(X^1 + X^4\right) & \zeta^1 = \frac{1}{\sqrt{2}}\left(X^1 - X^4\right) \\ \tau^2 = \frac{1}{\sqrt{2}}\left(X^2 + X^5\right) & \zeta^2 = \frac{1}{\sqrt{2}}\left(X^2 - X^5\right) \\ \tau^3 = \frac{1}{\sqrt{2}}\left(X^3 + X^6\right) & \zeta^3 = \frac{1}{\sqrt{2}}\left(X^3 - X^6\right) \end{array} \tag{72}$$

The system of equations 72 defines a system of 6 light hypercone coordinates: it is one possible IMF-6. However the system of equations 72 has priviledged the following space time association in U_6

$$\begin{array}{l} X^1 \leftrightarrow X^4 \\ X^2 \leftrightarrow X^5 \\ X^3 \leftrightarrow X^6 \end{array} \tag{73}$$

4.3 About Problems of the Space Time Duality

The space time duality in extending the usual Lorentz group to superluminal transformations is well known [20]: a transformation of a subluminal referential frame (ORF-4) into a superluminal referential frame (TRF-4), both having 4 dimensions, would produce a space time permutation, which is not possible because of the mismatch between the number of space and time dimensions.

Because infinite momentum frames allow to describe both subluminal and superluminal velocities, their generalization to more than two dimensions cannot avoid the problem of space time duality. An IMF-4 corresponding to both an ORF-4 and a TRF-4, all having 4 dimensions, does not avoid the mismatch between the 3 dimensional space and the one dimensional time.

A 6 dimensional infinite momentum frame (IMF-6) with a metrics having the signature $(+++---)$ has 3 "time" dimensions and 3 "space" dimensions, so there is no mismatch between space and time dimensions. The space time permutation required by the superluminal extension of the Lorentz transformations is quite possible within such an IMF-6.

The second problem of space time duality with several "time" dimensions is that we cannot avoid to privilege a particular space time permutation: in the naive example of IMF-6 (see section 4.2) the privileged space time association (eq. 73) defines one particular space time permutation in U_6.

There are several possible space time permutations in U_6. For example the spacelike axis X^1 may be simply associated with the timelike axis X^5 or with X^6 as shown below

$$X^1 \leftrightarrow X^6$$
$$X^2 \leftrightarrow X^4 \quad (74)$$
$$X^3 \leftrightarrow X^5$$

Moreover space time permutations in U_6 may also include rotations of the spacelike axis X^1, X^2, X^3 within the spacelike region and/or rotations of the timelike axis X^4, X^5, X^6 within the timelike region.

4.4 Introduction of Pauli Matrix in the U_6 Light Hypercone Equation

Every possible space time permutation in U_6, including rotations within the spacelike region and/or rotations within the timelike region, may be associated to a set of isotropic vectors of U_6, which are solutions of the light hypercone equation 70.

As it is usual in quantum mechanics, the sum of 3 real squares may be factorized with the use of hypercomplex numbers represented by Pauli matrix. So we may write in the spacelike region:

$$\left(dX^1\right)^2 + \left(dX^2\right)^2 + \left(dX^3\right)^2 = \left(\sigma_1 dX^1 + \sigma_2 dX^2 + \sigma_3 dX^3\right)^2 \quad (75)$$

where (σ_1, σ_2, σ_3) represents a permutation of the Pauli matrix, and we may write in the timelike region:

$$\left(dX^4\right)^2 + \left(dX^5\right)^2 + \left(dX^6\right)^2 = \left(\sigma_4 dX^4 + \sigma_5 dX^5 + \sigma_6 dX^6\right)^2 \quad (76)$$

where (σ_4, σ_5, σ_6) represents a second permutation of the Pauli matrix. Thus the U_6 light hypercone equation 70 is equivalent to the factorized equation below

$$\begin{aligned}&\left(\sigma_1 dX^1 + \sigma_2 dX^2 + \sigma_3 dX^3 + \sigma_4 dX^4 + \sigma_5 dX^5 + \sigma_6 dX^6\right)\\ &\times\left(\sigma_1 dX^1 + \sigma_2 dX^2 + \sigma_3 dX^3 - \sigma_4 dX^4 - \sigma_5 dX^5 - \sigma_6 dX^6\right) = 0\end{aligned} \quad (77)$$

4.5 Conclusion About the U_6 Light Hypercone Equation

An IMF-6 may be generated from the U_6 light hypercone equation. The introduction of Pauli matrix in the U_6 light hypercone equation also suggests to use spinor representations of the IMF-6.

5 Conclusion

We have shown that the longitudinal Doppler effect may be related to two transformed light cone coordinates (IMF-2). From this point of view, star redshifts should be interpreted with infinite momentum frames.

We have shown that spinors can be associated to isotropic vectors which are the basis vectors of an infinite momentum frame. We have shown that usual referential frames can be deduced from an infinite momentum frame (IMF-n), having $n \geq 4$ dimensions.

Can we represent with a set of spinors related to a fundamental IMF-n, the emergence of an observed time arrow and three observed oriented space directions ? This question should be studied in a next work.

Acknowledgements

The author thanks Professor George L. Farre for his pertinent remarks.

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