A Circular Qualitative Algebra

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Our society has replaced secret wisdom with information.
Secret wisdom results from journey into unknown like Odysseus',
and like his, it never ends.
"Care of the soul", Thomas Moore [1]

Abstract
Representation and manipulation of the world by models. Using world models for reasoning and action purposes. Two types of reasoning forms: symbolic and subsymbolic. Classification of subsymbolic reasoning forms by attributes of communicability enumerability and robustness. Roles of system observers realised with different subsymbolic technologies. Introductio of circular qualitative algebra (CQA) with its attributes of precision, robustness, learning ability, usability and interpretability. Application of CQA to: pattern recognition, modeling of power plant combustion process and traffic accident modeling.
Key words: model circularity, subsymbolic model, qualitative modeling, qualitative algebra

1 Introduction

This paper deals with modeling algorithm for implementation of the reasoning process in natural sciences. Such models are composed of the statement of the given situation, actual action and intention of what is to be achieved [2].
The modeling algorithm is based on data from world cases. The expected result of modeling algorithm is the model that enables action in the real world. Any meaningful action in the world requires anticipation of the expected results. There is no decision without anticipation. Results of previous actions in the world are actually exemplified in the world cases. Thus we are dealing here with modeling algorithm that is also an anticipative tool.
A general scenery of data acquisition from world cases: observation of the real world, expert role, intelligent machines and many facets of users critics and adaptive critics and anticipative decision is given in Fig.1. The world is by means of such intelligent machines represented and sometimes manipulated by humans. There is the necessity of building different particular models of each of the activities given in Fig.1. Such models

are built either by people, designers or by selforganized software programs – simple or artificial intelligence tools. However made such machines enable final reasoning and control about the particularities of the world, on the bases of which anticipative decisions about actions are taken and corrections to the actions are tuned. The majority of such actions occur in the real world or at least in the small part thereof – so it is our duty to make such actions as feasible and as transparent as possible. With adequate tools for reasoning and anticipative decision making we might be supporting both information and wisdom – a never ending task.

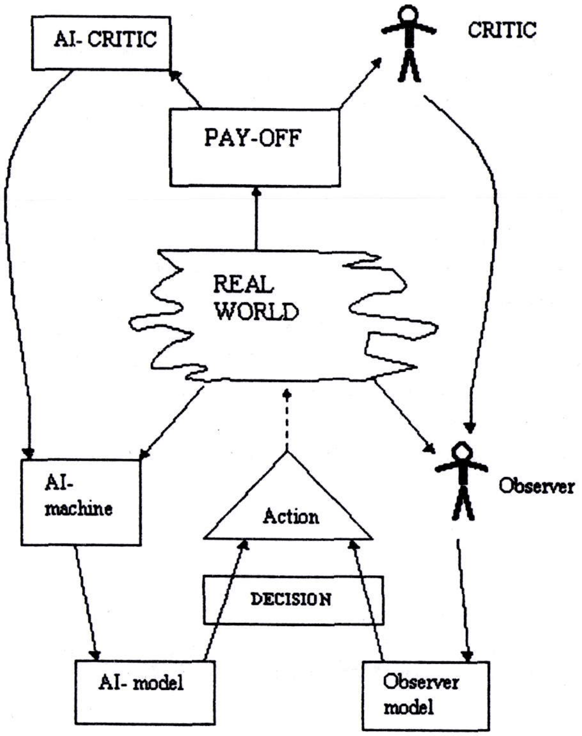

Fig.1: Observer – model – action – pay-off cycles in real world

2 Symbolic and subsymbolic reasoning forms

Reasoning is a circular anticipatory action of forming conclusions, judgement or inferences. Circularity is hidden in all reasoning forms as a result of basic information processing [3]. Circularity includes anticipative decision as illustrated in Fig. 1.

There are two basic reasoning forms: symbolic and subsymbolic. Symbolic reasoning forms are for example: logical rules, control rules, analytical mathematics, rule-based forms. Subsymbolic reasoning forms are generated in neural networks, fuzzy set logic, and genetic algorithms. Subsymbolic reasoning forms are explicit or implicit according to the nature of their results.

Explicit reasoning forms communicate the resulting reasoning model more or less clear to the user. Implicit forms retain the secret of the resulting model to some degree. Reasoning models can be described with a triplet of its internal states:

communicability, C, enumeration, E, and robustness, R, so that the relation

$$M_{ti}(C,E,R) < M_{tj}(C,E,R), \qquad t_i < t_e < t_j \qquad (2.1)$$

means: a reasoning model M has gained in its quality when between time instants t_2 and t_l if there happens a knowledge change event at t_e, ($t_i < t_e < t_j$) that enhances its communication ability, or decreases its enumeration space or increases its robustness.

An artificial neural network (ANN) communicates its outputs after its inputs have been applied according to its trained structure. Its communication can be and is wrapped into the implicit language of its own autopoiesis.

Such an ANN enumerates its communication in the frame of its output set discreteness as well. Thus when it possesses one output that is trained to be changed between -1 and 1 in relevance steps of 0.01 the total output enumeration set equals to 199. Its input and internal structural enumeration can be immense at the same time.

The robustness of such a network can be defined as the ability to work properly under input error, structure disturbances and model changes. ANN is robust to input error and structure disturbances and less robust to model changes, such as sigmoid function change, etc.

Put into much simpler content any such artificial means can be constructed from the observer and the actuator part, Fig. 2.

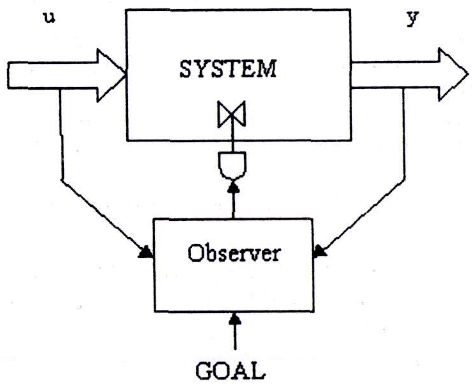

Fig.2: A simplified reasoning/action model: goal – observer – actuator

The basic formula for the linear observer is given when considering system input u(t), state x(t) and output y(t) signals [4] i.e.

$$x(k) = P1\ y(k) + P2\ u(k-1) \qquad (2.2),$$

where P1 and P2 are polynomials of the n-1 respectively n-2 order. Thus the state of the system is calculated from n observations. The number of calculation steps can be even higher in the presence of noise.
Actuators or better to say final process devices, possess nonlinearity that disables a neat linearity of the system observability calculation and points to the inevitability of application of heuristic or qualitative methods [4, 5].
Thus, let us reconsider the means of technical reasoning as observers and actuators in the reasoning and controlling of the simplified outer world.
Principal scheme of an PID controller coupled to the system is given in Fig. 3a, similar scheme with ANN in Fig. 3b, with fuzzy set controller FSL in Fig. 3c and with the circular qualitative algebra, CQA, in Fig. 3d.

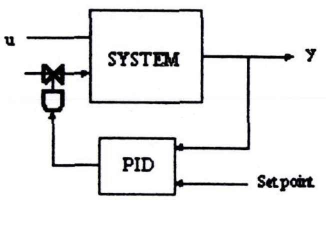

Fig. 3a: Pid controller

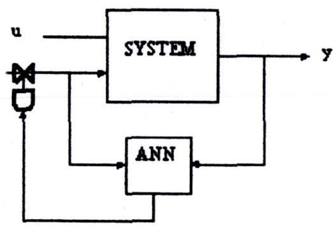

Fig. 3b: AN network

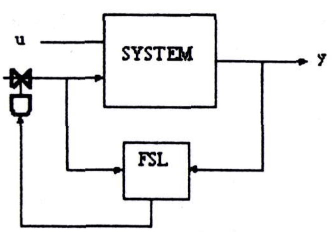

Fig. 3c: FSL machine

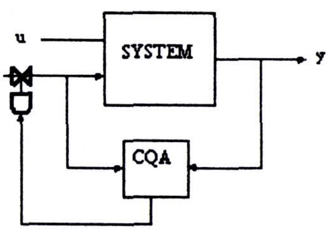

Fig. 3d: CQA tool

Fig. 3: Observer – actuator functions of four basic control mechanisms

As visible from the principal scheme the basic mechanisms of reasoning consist in

- observing the state of the system by comparing the input and /or output state with the expected or presetted one
- intervening into the control input(s) in order to obtain the change of the system state to the anticipated and more favourable one.

PID mechanism thereby compares the observed state with the presetted one, ANN uses the prelearned output of the network in order to compare it somehow with the measured output, fuzzy-set mechanism determines from the measured inputs and output the expert rule that must be applied for the actual system state, and CQA calculates the actual model according to obtained system input and output data and determines according to expert opinion about the system states and required anticipated interventions at the input.

System observers are thus machine parts with double functions:

- to determine the state of the system, or put into the simplest form to determine whether the state has been changed and in what amount
- to make the automatic and anticipative decision on what to do upon the determination of the actual system state.

The actuator is a machine part that executes the command issued from the observer or the decision maker.

A simple comparison of observer functions of four basic mechanisms depicted in Fig. 3 is given in Table 1.

Table 1: Basic functions of different observer means

Means	State determination	Decision type
PID	I/O signal difference	Automatic (expert defined)
ANN	Input signal pattern	Trained
FS	I/O signal grid	Expert based rules
CQA	I/O based model	Expert programmed

3 Structural parameters of subsymbolic forms

Structural parameters of subsymbolic forms are: precision, robustness, learning ability, usability and interpretability. Parameters of the circular qualitative algebra will be given after presentation of its basic algorithm.

3.1 Circular qualitative algebra - basic data processing

Any system can be described in terms of its input and output variables. A simple system with two input and one output variable given in measurement and ranked form is presented in Table 2.

Table 2: Measured and ranked variables from a three-event observation

Event	**1**	**2**	**3**	**Event**	**1**	**2**	**3**
Input 1	20	10	5	$\Re$(In1)*	1	2	3
Input 2	5	10	15	$\Re$(In2)	3	2	1
Output 1	4	2	2	$\Re$(Out1)	1	2.5	2.5

* $\Re$ - rank of the value in brackets

Let us suppose that all system variables are measurable in n successive moments. When these measured values are taken and converted into a qualitative form, or put into a ranking procedure of any kind, three n-point graphs can be obtained. If interested in system behavior, one would like to make any type of system model by means of some data processing. The simplest way to start would be to correlate qualitatively any combination of simple input and output n-point graphs. Intuitively one can make the following combinations of correlations: In 1 correlated to Out 1, and In 2 correlated to Out 1, Table 3.

Table 3: Qualitative evaluation of different input/output variable combinations

Event	Rank diff. at 1	Rank diff. at 2	Rank diff. at 3	Sum of squared rank diff. 1 – 3
$\Re$(In1)-$\Re$(Out1)	0	-0.5	0.5	0.5
$\Re$(In2)-$\Re$(Out1)	2	-0.5	-1.5	6.5

When the In1 *quantitative* data are added to the In 2 *quantitative* data and converted to a new n-point graph a better *qualitative correlation* with Out 1 data are obtained, Table 4. Thus turning again to the quantitative aspect of system information by making appropriate algebraic procedure and going back to the qualitative evaluation a *circular way* of system modeling is introduced [6].

Table 4: The case of complete qualitative correlation of the sum of input variables In1 and In2 and the output variable Out1

Event	**1**	**2**	**3**
In = In1 + In2	25	20	20
$\Re$(In)	1	2.5	2.5
$\Re$(Out)	1	2.5	2.5
$\Re$(In)- $\Re$(Out)	0	0	0

3.2 More complex operations in CQA

The subject of this part is to introduce some common features of algebraic operations on the quantitative aspects of n-graphs and analyse the results of these operations in qualitative aspect. Such n-graphs are strictly defined on dynamic across-the-time series of measurements or by experts estimatable processes although a lot of freedom in relative proportions of time measure is acceptable. The time intervals between steps depend on the nature of the process and are outside the scope of the paper. The ranking of time independent data series is considered elsewhere [7, 8].

Definition 3.1

An *n-point graph* is a single-valued discrete function defined in n points and obtained from the ranking procedure of a process measured values or from ranking procedure of a Gödel numeration of a more complex system behavior, where n>1.

However processed the sum of all n-graph values is equal to

$$S = 1+2+ ... + n = \frac{n(n+1)}{2}, \qquad (3.1)$$

and its mean value is $\frac{n+1}{2}$

Definition 3.2

The n-graphs $\Re\{g_i\}$, $\Re\{g_j\}$and $\Re\{g_k\}$will be designated as G_i^n, G_j^n and G_k^n.

Theorem 3.1

The n-point graph G_i^n is *transparent* to the scaling procedure, i.e., any linear operation on its quantitative part with the constants a and b, does not change the graph or

$$\Re\{a+ bg_i^n\} = \Re\{g_i^n\}, \text{ for } a \neq 0, b>0 \qquad (3.2).$$

The proof is trivial since multiplying all values with a constant does not change the order of ranked values, nor does adding a positive or negative constant. The meaning of relation (3.2) is the invariance of qualitative algebra under positive metric transformation.

Definition 3.3

The normalized quantitative values g_i^n of the n-graph G_i^n are those measured or estimated values of g_i^n put into unit scale i.e. $[g_i^n] \in \{0,1\}$.

Definition 3.4

Two n-graphs are complementary to each other if and only if their rank values for each period have complementary values i.e. the sum of ranks equals n+1 in each step of the n-graphs.

Theorem 3.2

The addition operation introduced as calculation noise w^n on normalized values of two complementary n-point graphs yields any n-point graph:

$$\Re\{g_i^n + g_j^n + w^n\}=\Re\{g_k^n\}, G_k^n \neq G_i^n \neq G_j^n \text{ for } G_i^n = (G_j^n)^{-1} \qquad (3.3),$$

where the actual shape of the G_k^n depends on the quantitative noise of the components g_i^n and g_j^n and $(\ )^{-1}$ denotes the qualitative complementarity feature.

Proof of theorem 3.2:

Introductory parts of the proof describes the nature of the quantitative – qualitative transformation:

When graphs g_i^n and g_j^n differ at only one point their addition in all points will give the same graph except at the point of difference. Let us suppose that at this point the values of the g_i^n a and of the g_j^n is b, and that at all other points both graphs possess the same values equal to a, such as the example on Table 5. The amount of noise at this point is ten times lower than the difference a-b.

Table 5: The example of addition of two very similar n-graphs $g_i^n+g_j^n=g_k^n$

period	1	2	3	4	5	6	7								
w^n	0	0	0	0.1	0	0	0	**period**	1	2	3	4	5	6	7
$g_i^n+w^n$	7	7	7	6.1	7	7	7	G_i^n	3.5	3.5	3.5	7	3.5	3.5	3.5
g_j^n	4	4	4	5	4	4	4	G_j^n	4.5	4.5	4.5	1	4.5	4.5	4.5
g_k^n	11	11	11	11.1	11	11	11	G_k^n	4.5	4.5	4.5	1	4.5	4.5	4.5

The slightly greater difference in the graphs than those in Table 5 are given in Table 6.

Table 6: Example of noiseless addition of two slightly less similar n-graphs: $g_i^n+g_j^n=g_k^n$

period	1	2	3	4	5	6	7	period	1	2	3	4	5	6	7
g_i^n	7	7	6	7	7	7	7	G_i^n	3.5	3.5	7	3.5	3.5	3.5	3.5
g_j^n	4	4	4	5	4	4	4	G_j^n	4.5	4.5	4.5	1	4.5	4.5	4.5
g_k^n	11	11	10	12	11	11	11	G_k^n	4.5	4.5	7	1	4.5	4.5	4.5

Proceeding with less and less similar graphs the question arises about the addition of the most disimilar n-graphs. These are the graphs that possess the feature of complementarity of both the n-graph components of the addition process. Such two complementary noiseless n-graphs are given in Table 7.

Table 7: Addition of two complementary noiseless n-graphs. Their quantitative parts are purposely chosen as being similar to the qualitative part

period	1	2	3	4	5	6	7	period	1	2	3	4	5	6	7
g_i^n	7	2	3	1	4	6	5	G_i^n	1	6	5	7	4	2	3
g_j^n	1	6	5	7	4	2	3	G_j^n	7	2	3	1	4	6	5
g_k^n	8	8	8	8	8	8	8	G_k^n	4	4	4	4	4	4	4

By the following operation on two complementary n-graphs

$$g_k^n = g_i^n + g_j^n + w_n \quad (3.4),$$

where w_n is a noise component, such as computation noise, any new shape of the resulting n-graph can be obtained, q.e.d.

3.3 Estimation of structural parameters of CQA

Precision is given approximately proportional to the length of the n-graph.
Robustness is approximately proportional to the inverse of the n-graph length.
Learning ability is instant and absolute – using expanded complementary graphs during system modeling the observer instantly selects the best model. Learning can be even enhanced by system evolutionary addition of best possible complex variables into a selection process.

Usability can be dubious because of the possibility of having a whole cohort of system models – thus the expert must decide about the most usable model.
Interpretability although explicit is also problematic when the qualitative form is complex. The possibility of using simpler qualitative analytic relations has been proved as highly effective [9].

4 Examples

4.1 Recognition of a simple shape

Two triangular forms are decomposed into respective x- and y-projections. The third triangle is obtained by shift and rotation from two initial trangular forms. The data of the new triangle projections are used as target function for the CQA algorithm. The result is given in Fig. 4.

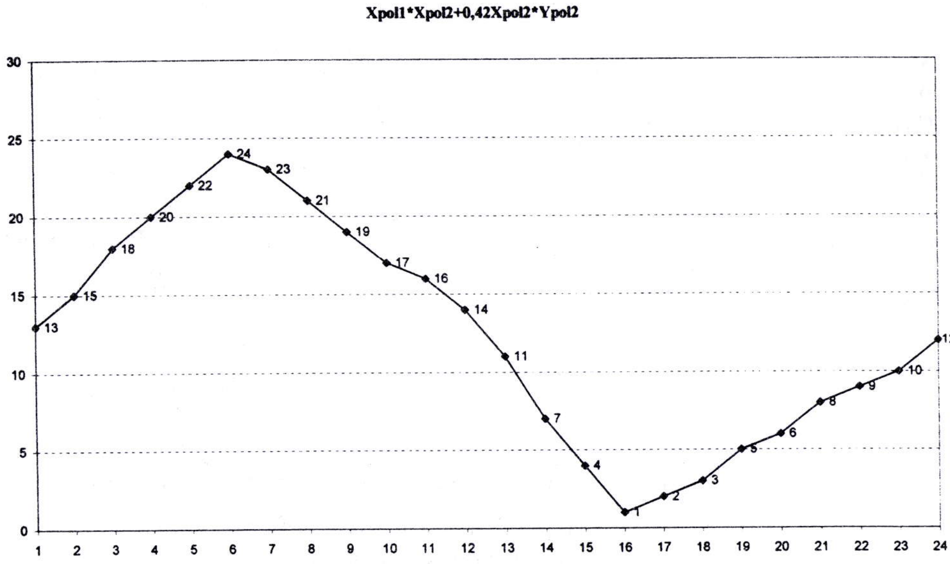

Fig. 4: Model of a shifted and rotated triangle x-projection from two other triangle and x and y projections

4.2 Level of determinancy of turbine input parameters from boiler data

Basic boiler data of the fully controlled power plant are used as input variables [10]. The goal functions are turbine steam parameters: temperature, pressure and flow. Determinancy is equal to squared qualitative correlation coefficient. Resulting determinancy coefficients δ for 24-hours observation on 21st February 1999. are:

- steam temperature, $\delta = 0.3364$
- steam pressure, $\delta = 0.7744$

- steam flow, $\delta = 0.9044$.

Similar results have been obtained for the whole period of 50 observation days.

4.3 Model of road traffic accidents

Model input variables are taken from the 36 months observation of the main and basic influential traffic parameters [11].

The resulting model of the injured people is, Fig. 5

$$M_{r=.925} = \text{(Gasoline consumption.)*(Mean temperature)+} \\ \text{0.570/ (No.of accidents*No. of motor vehicles)} \qquad (3.5).$$

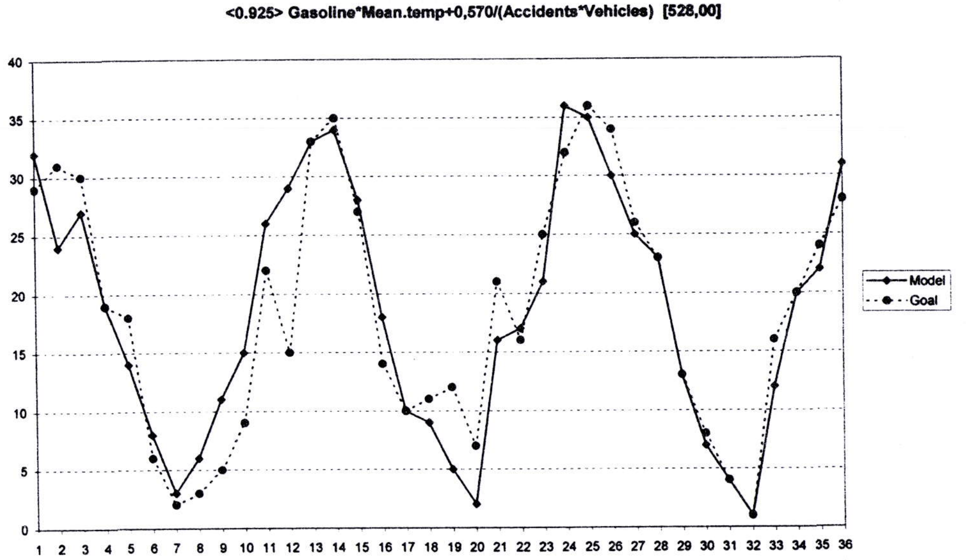

Fig. 5: Model of injured people in road traffic accidents from basic traffic variables

5 Discussion

The circular qualitative algebra introduces a subsymbolic reasoning tool that possesses the ability of explicitness not principally met in other subsymbolic reasoning tools such as artificial neural networks or fuzzy set logic.

Comparison of basic parameters of such tools gives:

- precision of CQA is lower than ANN for smaller data series
- robustness is of the order of ANN decreasing with greater data series
- learning is very fast, reproducible and selfenhancing
- usability of CQA depends highly upon the expert

- interpretability although explicit can be encapsulated, but set-similar interpretations can be introduced [12].

Regarding the modeling possibilities it can be pointed out that the method is specially well applicable for nonlinear stochastic processes.

Another problem of general optimality of the solution is solved with the proof of the Theorem 3.2. Such optimality is principally not the feature of ANN and FSL.

6 Conclusion

The circular qualitative algebra is introduced as a reasoning model that explicitly uses the circularity feature of qualitative and quantitative information aspects of the collected data. The method produces anticipative model(s) of the goal functions that are selected from the most similar model variables in a way that is algebraic simple, precise enough and practicaly instant. As presented in the work such models possess the feature of general optimum.

Usability of such models for reasoning and their interpretation are intuitively clear although somehow encapsulated because of the qualitative nature of the model form. Model conversion to quantitative values is straightforward for well correlated models.

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