On Modeling the Quantum and Anticipatory Systems by Passive Neural Nets

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Abstract

This paper presents some conjectures on modeling the quantum systems using lossless-orthogonal neural nets. Structure of these nets consists of n compatibly connected (entangled) pairs of neurons-qubits.

Keywords: quantum and anticipatory systems

1. Introduction

It seems more and more, that tending from different directions the scientific world is approaching the following thesis: The Nature, the Universe exists as an information aether or more technically: the Structures exist as information flow fields. Such impressive thoughts have been presented by various authors also at this conference (see Mitchell, 1999). Moreover, it seems that the notion of information is a fundamental category in the description of quantum reality and it can be defined independently from the notion of probability.

Quantum information processing arising from quantum theory comprises two related parts: quantum computation and quantum information theory. To process quantum information one needs a type of quantum mechanics based system - quantum computers. Currently, nobody knows how to build a quantum computer, although quantum computer technology continues to develop and some suggestions have been made as possible designs for such computers. Even if no useful quantum computer is ever built, some models of quantum computers have been studied (Shor 1997, Bennet 1998).

Recently, we have shown how the properties of passive neural networks could be interpreted in order to model the quantum systems (Sienko 1999). The purpose of this paper is to give some further interpretations. As a result of such considerations we obtain an oscillatory "quantum" model of artificial neural nets that could be relevant for biological nets. We point out that such a quantum computer model could consist of n pairs of entangled lossless neurons–qubits. Thus, one obtains a lossless neural net with an orthogonal matrix of connections forming certain, maybe useful, type of unitary transformation used in quantum computation algorithms.

At first let us note some fundamentals of quantum mechanics used in quantum information processing:
a) Superposition of states
b) Unitarity of state transformations
c) Entanglement

Quantum entanglement discovered by Einstein, Podolsky, Rosen (EPR) is one of the most interesting phenomenon leading to non classical effects revealed by contemporary physics. The simplest example of such a entangled state is the singlet state of two – ½ spin particles labeled by 1 and 2:

$$|\Psi^-\rangle_{12} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right)$$

The properties of the states of this kind are responsible for phenomena like quantum cryptography, quantum dense coding, quantum teleportation and quantum computation. Such an entangled pair is a quantum system in an equal superposition of the states

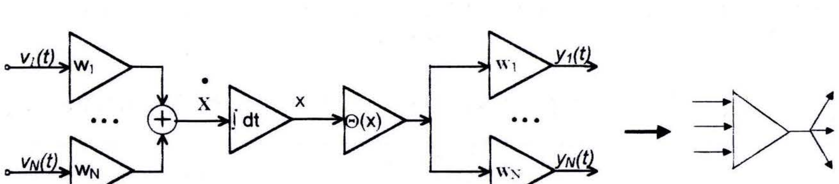
$|\uparrow\rangle_1|\downarrow\rangle_2$ and $|\downarrow\rangle_1|\uparrow\rangle_2$

The entangled state contains no information on the individual particles – it only indicates that the two particles will be in opposite states (Horodecki 1998).

2. Lossless Neural Net as a Model of a Quantum System

A concept of Passive Neural Network has been proposed few years ago (see for example (Luksza 1998)). The unique characteristics of these neural nets are:
- passivity of neurons
- compatibility of connections

It is worth noting that passive neurons can be lossless if there is no dissipation of pseudoenergy-information energy (Arbeitsenergie). For completeness of this presentation let us remind some basic features of the lossless neural nets. A first order model of the lossless neuron in the form of information – flow network is given in Fig.1.



where: Θ(x)-activation function (sigmoidal), w_i-synaptic weights

Fig.1. A first order model of the lossless neuron as information-flow network.

It can be seen that the passivity reads for technical formulation of the following relation:

$$E = \sum_{i=1}^{N} \int_{-\infty}^{t} v_i y_i dt \geq 0, \ \forall t \qquad v_i, y_i \in L_2 \tag{1}$$

where: E – information energy absorbed by the neuron.

To model a quantum system using lossles neural net one postulates the following conjectures:

Conjecture 1.
Lossless neuron ≡ physical model of one bit (classical) or
≡ elementary building block of vacuum.

A structure of lossless neural net can be obtained as compatible connections of N lossless neurons. Thus

$$E = \sum_{i=1}^{N} E_i \tag{2}$$

where: E_i – pseudoenergy absorbed by i-th neuron.

Hence compatibility of connections preserves the losslessness of the net.
An example of 2 neuron net is shown in Fig.2.

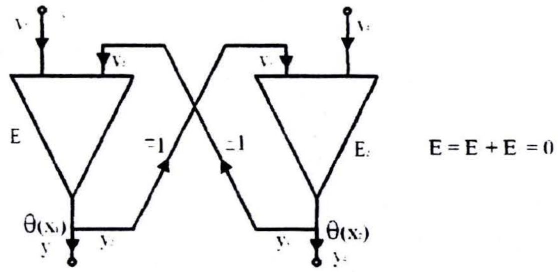


Fig. 2. Two lossless neurons compatibly connected –a model of one qubit or ½ spin particle.

It can be seen that the state – space description of the net from Fig.2 is following:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & \pm 1 \\ \mp 1 & 0 \end{bmatrix} \begin{bmatrix} \Theta(x_1) \\ \Theta(x_2) \end{bmatrix} \tag{3}$$

Generally, the lossless neural net composed of N neurons is described by the following state – space equation :

$$\dot{\mathbf{x}} = \mathbf{W}\Theta(\mathbf{x}) \tag{4}$$

where: **W** - matrix of weights (information flow connections) is skew – symmetric

Conjecture 2.

a Compatible connection can be seen as a model of entanglement i.e. Compatible connection ≡ Entanglement

b. Compatible connection of 2 lossless neurons i.e. one entangled pair of two neurons ≡ model of one qubit or ≡ ½ spin particle (Fig.2 and Eq.(3)).

Def.1. Lossless–orthogonal neural net (LONN)

Neural net composed of n pairs of entangled lossless neurons i.e. qubits will be called lossless – orthogonal neural net, if weight matrix **W** is orthogonal (and skew – symmetric as given by Eq. (4)).

Example 1.

Two entangled pairs of qubits forming a lossless – orthogonal neural net is shown in Fig.3. This can be seen as a model of EPR pair.

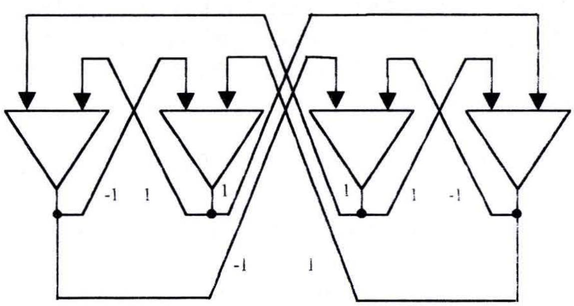


Fig. 3. Lossless – orthogonal neural net created by 2 entangled pairs.

The weight matrix of this neural net is given by:

$$\mathbf{W}_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix} ; \ \mathbf{W}_2 \cdot \mathbf{W}_2^{\mathrm{T}} = 2 \cdot \mathbf{1} \tag{5}$$

and can be easily orthogonalized.

The neural net from Fig.3. can be used as a building block to create N pair orthogonal net. For example the structures of 4 pair and 8 pair neural nets are given by the following weight matrices:

$$\mathbf{W}_4 = \begin{bmatrix} \mathbf{W}_2 & \mathbf{1} \\ -\mathbf{1} & -\mathbf{W}_2 \end{bmatrix}, \mathbf{W}_8 = \begin{bmatrix} \mathbf{W}_4 & \mathbf{1} \\ -\mathbf{1} & -\mathbf{W}_4 \end{bmatrix} \tag{6}$$

where: **1**- unity matrix.

It is worth noting, that changing the sign of connections in matrix $\mathbf{W}_2$ i.e.:

$$\mathbf{W}_2 = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \tag{7}$$

one obtains a model consisting of 2 entangled pairs of qubits revealing the features of a 0 spin particle. This type of structures have been used in synthesis of losless (not orthogonal) neural net based associative memories-static and oscillatory/chaotic (Citko 1997).

Quantum description of analysed here structures needs that their states are determined by vectors (rays) in appropriate Hilbert spaces. Thus, the states of lossless-orthogonal neural nets are vectors from Hilbert space spanned by spinors (eigenfunctions of information-flow matrix $\mathbf{W}$) with scalar product given by Eq. (1). On the other hand, the states of lossless neural nets having the structure based on 0 spin particles are spanned by Fock-like basis.

Some basic properties of the lossless–orthogonal neural networks can be driven from Eq.(4):

1. Structure of these nets creates the following nonlinear vector field:

$$\mathbf{W}\,\theta\,(\mathbf{x}) = \nu(\mathbf{x}) \tag{8}$$

where : $\mathbf{W}$ (dim.n x n) – orthogonal and skew – symmetric (n even).
$\mathbf{v}(\mathbf{x})$ is tangent vector field on $S^{n-1} \subset R^n$ satisfying:

$$\langle \mathbf{v}(\mathbf{x}), \mathbf{x} \rangle = 0 \tag{9}$$

$$|\mathbf{v}(\mathbf{x})|^2 = \langle \mathbf{v}(\mathbf{x}), \mathbf{v}(\mathbf{x}) \rangle > 0 \tag{10}$$

for all $\mathbf{x} \in S^{n-1}$(sphere)

It means that the only equlibrium point of the net is $\mathbf{x} = \mathbf{0}$ i.e.

$$\nu(\mathbf{x}) = \mathbf{W}\,\theta\,(\mathbf{x}) = \mathbf{0} \rightarrow \mathbf{x} = \mathbf{0} \tag{11}$$

2. LONN determines a type of orthogonal transformation, namely:

$$\mathbf{W}\,\theta\,(\mathbf{x}) + \mathbf{I}_{\mathbf{in}} = \mathbf{0} \tag{12}$$

where :$\mathbf{I}_{\mathbf{in}}$ - input vector (input data or information)

Hence, output of the net

$$\theta\,(\mathbf{x}) = -\,\mathbf{W}^{T}\,\mathbf{I}_{in} \tag{13}$$

can be seen as a Walsh–like spectrum of input information. It could be called as n entangled pair based spectrum.

3. The most interesting aspect of LONN theory seems to be given by the famous Adams theorem (Eckmann 1999):

> The maximum number of continues orthogonal tangent vector fields on S^{n-1} is $\rho(n)$-1, where $\rho(n)$ is Radon number of n. Hence, one has the following conclusion:
>
> Let $\mathbf{W}_1,\dots,\mathbf{W}_s$ be a set of orthogonal Hurwitz–Radon matrices and let $\alpha_1, \dots ,\alpha_s$ be real numbers with $\sum \alpha_j^2 = 1$.

Then:

$$\mathbf{W}(\alpha) = \sum \alpha_j\,\mathbf{W}_j \tag{14}$$

is orthogonal, where $s_{max} = \rho(n) - 1$

Technically, number s_{max} establishes a maximum of LONN capacity. On the other hand, LONN with weight matrix $\mathbf{W}(\alpha)$ can be seen as an implementation of homotopy groups.

Conjecture 3.

LONN with weight matrix $\mathbf{W}(\alpha) \equiv$ Model of quantum system.

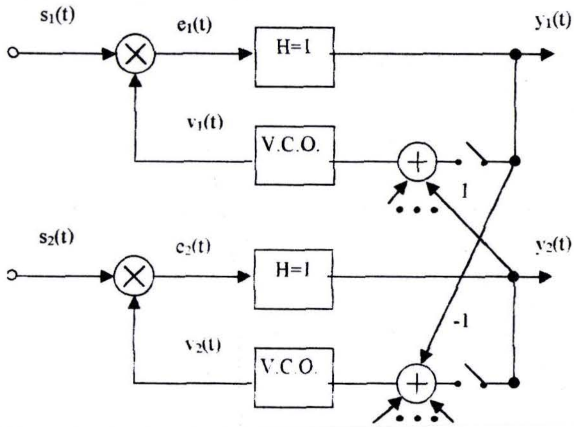
Since, such LONN are physically implementable in VLSI technology with very small (probably) time of decoherence, hence :

Conjecture 4.

A quantum computer being a real–time very large scale parallel processor can be implemented as a LONN.

3. Oscillatory "Quantum" Model of Neural Nets

It is known, that only unitary transformations of quantum systems are allowed. In the case of considered here lossless neural nets, such transformations are implemented by changing the information-flow matrix (i.e. connections) **W**. From theoretical point of view, many proper mathematical tools can be here exploited (for example symplectic groups). Technically, one should propose a structure with variable weights. We have found that one of the most interesting solutions can be achieved by using the phase-lock loop principle. Such an oscillatory, PLL based structure implementing one qubit is shown in Fig.4.



$s_i(t) = A_{Ci}\sin(\omega_{Ci}t + \phi_{i1})$
$v_i(t) = A_{Vi}\cos(\omega_{Ci}t + \phi_{i2})$, $i = 1,2$

Fig.4. PLL based structure of one qubit.

Indeed, the phase-signals are governed by the following equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = 2\pi \begin{bmatrix} 0 & \pm k_{v1}k_{m2}A_{C2}A_{v2} \\ \mp k_{v2}k_{m1}A_{C1}A_{v1} & 0 \end{bmatrix} \begin{bmatrix} \sin x_1 \\ \sin x_2 \end{bmatrix} + \begin{bmatrix} \dot{\phi}_{11} \\ \dot{\phi}_{21} \end{bmatrix} \qquad (15)$$

where:

$x_i = \phi_{i1} - \phi_{i2} \quad i = 1,2., \quad |x_i| \le \frac{\pi}{2}$

$\phi_{12} = \pm 2\pi k_{v1} \int_{-\infty}^{t} y_2 d\tau$

$\phi_{22} = \mp 2\pi k_{v2} \int_{-\infty}^{t} y_1 d\tau$

$e_i(t) = k_{mi}A_{Ci}A_{Vi}\sin x_i$
k_{mi}- sensitivity of multiplier
k_{vi}- sensitivity of oscillator (V.C.O.)

It can be seen, that the weights of connections are here controled by the amplitudes A_{Ci} of input carriers. The structure from Fig. 4. can be easily scaled to the connection of N qubits. The analysis of properties of such neural nets will be published elsewhere.

3. Some Notes on Anticipation

A little thinking shows that at Rosen's concept of anticipation, there is no the mystery of time-reversal (Glaserfeld 1998). Dubois tried to make this concept more genuine by the notion of incursion (Dubois 1998). Using the incursion is, however, not necessary to realize anticipatory systems i.e. systems implementing the time-reversal. Indeed, given dynamical system can be realized by using e.g. integrators or differentiators (non-causal), delays or predictors (non-causal) etc. We guess, that Piaget's characterization of anticipatory systems based on the following assumptions (Zaus 1999):

1. conservation of information
2. recursivity of processes

is fully correct.

To illustrate the above characterization let us formulate the following conjecture:Lossless-orthogonal neural net $\mathbf{F}[\,.\,]$ with weight matrix $\mathbf{W}$, lossles-orthogonal neural net $\mathbf{F}^{-1}[\,.\,]$ with matrix $\mathbf{W}^{-1}$ and equilibrium equation of $\mathbf{F}[\,.\,]$ constitute an anticipatory system. First of all let us note, that the equilibrium equation (12) can be seen as an atemporal model of $\mathbf{F}[\,.\,]$. Hence we have:

$$\mathbf{\Theta}^{t+\Delta t} = \mathbf{F}\left[\mathbf{I}_{in}^{t}\right] \text{ - an analysis by dynamical system} \tag{16}$$

$$\mathbf{\Theta}^{t} = -\mathbf{W}^{T}\mathbf{I}_{in}^{t} \text{ - a solution from model} \tag{17}$$

$$\mathbf{I}_{in}^{t+\Delta t} = \mathbf{F}^{-1}\left[\mathbf{\Theta}^{t}\right] \text{ - a synthesis (anticipation)} \tag{18}$$

where: $\mathbf{I}^{t}_{in}$ input at the moment t

Θ^{t} – response at the moment t

$\Delta t > 0$

Since the model of $\mathbf{F}[\,.\,]$ is nonlinear (saturation of Θ), so by virtue of this rather simple anticipatory system one can formulate, maybe useful, conclusion: Anticipation is possible in information preserving, adaptable system under "weak" inputs, for example in lossles-orthogonal neural nets.

4. Concluding Comments

This paper presents some conjectures on modeling the quantum system by using LONN. It is however worth noting that the lossless orthogonal neural nets are interesting objects of searching per se. Their unique features can be summarized as follows:

losslessness $\leftrightarrow$ losslessness of elementary building blocks i.e. neurons
skew symmetry $\leftrightarrow$compatibility of connections
orthogonality $\leftrightarrow$ topology of connections

Hence, LONN should be, first of all, seen as a Walsh–like orthogonal transformers. Having such nets implemented, one could realize so called quantum computation algorithms and quantum information processing (e.g. Quantum Associative Memory, Quantum Computational Learning Algorithm, Quantum Coding and Information Compression). Presented in this paper structures of LONN rely on concept of general dynamical system. It is possible however to formulate such structures by using the concept of lossless gates i.e. memoryless, lossless neurons.

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