A Metric Tensor of the New General Lorentz Transformation Model

Branko Novakovic*, Dario Novakovic**, and Alen Novakovic*

* FSB - University of Zagreb, Luciceva 5, P. O. B. 509, 10000 Zagreb, Croatia
** Ministry of Labor and Social Protection, Dezeliceva 7, 10000 Zagreb, Croatia
Tel.- Fax.: (+385) 1 61 68 354, E-mail: branko.novakovic@fsb.hr

Abstract.
A new General Lorentz Transformation model (GLT-model) derived by Novakovic (1999) for the particle motion in x-axis only, has been extended to the full form including y and z - axes. Starting with this transformation model, a general line element and a corresponding general metric tensor of GLT - model have been derived. The general line element and the metric tensor are functions of two free parameters α and α', which are the functions of the space-time coordinates. The identification of two free parameters of GLT-model has been done for a weak and a strong gravitational field. The weak gravitational field solution of the two free parameters of GLT-model corresponds to the well-known Schwartzschild's metrics of the line element, for a spherically symmetric non-rotating body. It is very important to point out that the line element of GLT-model given in a non-diagonal form has got a very important property: non-singularity in a very strong gravitational field. Finally, a simple coordinate transformation procedure has been derived that transforms a general line element into diagonal one, with metric components (-1, 1, 1, 1), equal to the metrics in Special Relativity. Since the all items in SR and GR can be described as the functions of two free parameters of GLT-model, the possibilities of an unification of Einstein's Special and General Theories of Relativity, as well as a new unification of electromagnetic and gravitational fields are opened.

Keywords: New General Lorentz Transformations, General Line Element, General Metric Tensor, Special Relativity, General Relativity.

1. Introduction

As it is well-known, there are many derivations of Lorentz Transformations (Einstein, 1909, 1916, 1955; Miller, 1981; Supek, 1992). In all of the known derivations methods of the Lorentz Transformation model (LT-model), it is assumed that the observation signal is the light, which has the same (constant) speed in both observer and moving systems. Thus, one of the remaining questions is: are there possibilities to generalize LT-model in the sense of employing of a set of different observation signals including the light signal? The basic ideas for solution of this problem are considered by Novakovic (1998). It has been expected that this generalization should contain both Lorentz-Einstein and Galilean Transformations, and could be helpful in a new unification of electromagnetic and gravitational fields. This was the basic motivation for the paper (Novakovic, 2000), where a new General Lorentz Transformation model (GLT-model) has been derived. In that paper, it has been confirmed that GLT-model contains both Lorentz-Einstein and Galilean Transformations, as its special realizations. If GLT-model is correct then there exist four new observation phenomena (a length and time neutrality, and a length dilation and a time contraction). Besides, the well known phenomena (a length contraction, and a time dilation) are also the constituents of GLT-model. A special consideration has been devoted to a correlation between GLT-model and a limitation on particle velocities. Furthermore, there was the indication that GLT-model could be used for coordinate transformation in a gravitational field. This indication has been confirmed by this paper.

The GLT-model in the reference (Novakovic, 2000) has been derived for the particle motion in x-axis only. In this paper it is extended to the full form of GLT-model including y and z axes. Starting with this new full form of the GLT-model a general line element has been derived. This line element is a function of two free parameters, α and α'. These parameters determine observation signal velocities in systems O and O', where the system O' is moving relative to the system O with an arbitrary velocity v, along an arbitrary radius vector. From the general line element we obtained a corresponding general metric tensor of GLT - model. Generally, the free parameters of the GLT-model are functions of the space-time coordinates. It means that these two free parameters are functions of the state of the energetic potential of the fields in which observation signals propagate. Thus, it is expected that these two free parameters can be identified in each potential field, including the combination of two or more potential fields.

In order to identify two free parameters of GLT-model in a gravitational field, the general line element is transformed into spherical polar coordinates, which are appropriate to the problem. The next step was the diagonalisation of the line element. Thus, we obtained the Schwartzschild's like form of the line element for a spherically symmetric non-rotating body. The identification of that line element with the Schwartzschild's spherically symmetric vacuum solution (Schwartzschild, 1916) gives the solution of the free parameters of the general line element. The line element of GLT-model has two solutions. The first one is in a weak gravitational field, and the second solution is in a strong gravitational field. In the case of the weak gravitational field (like in our Solar System) the metrics of the line element of GLT-model corresponds to the well-known Schwartzschild's metrics of the line element.

In a strong gravitational field the solution of the line element contains a quadratic term of the gravitational potential. Of course, this term can be neglected in a weak gravitational field, what leads to the solution exactly equal to the Schwartzschild's metrics of the line element. The solution of two free parameters of GLT-model is equal to the solution of the same parameters, obtained in the reference (Novakovic, 2000), by employing the well-known gravitational red-shift experiment and the energy equations, derived from a null component of a four-momentum vector of GLT-model. It is very important to point out that the line element of GLT-model given in a non-diagonal form has got a very important property: non-singularity in a strong gravitational field. It seems that the metrics of GLT-model is valid both in a weak and in a very strong gravitational field. A definition of the free parameters of GLT-model includes the possibilities that an observation signal can be a subluminal signal (like sonar signal), the light signal, as well as a superluminal signal (like tachyons signal). In that sense, it seems that GLT-model can include gravitational influences to a Dual Relativity (Dubois and Nibart, 2000), and to the Tachyons Region of motions (Nibart, 2000). The possibility of an application of GLT-model to superluminal signals (tachyons) has been discussed by Novakovic (2000) in the sections Particle Speed Limits in GLT-model, and Particle Speed Limits in a Gravitational Field. Furthermore, it seams that the GLT-model approach can be combined with Computational Derivation of Quantum Relativist Electromagnetic Systems with Forward-Backward Space-Time Shifts (Dubois, 2000), and with a Relativistic Model of a Particle - Antiparticle Pair (Nibart, 2000).

The all items in Special and General Relativity, like the Lorentz-Einstein transformations, the Maxwell equations, a metric tensor, the Christoffel symbols (1869), the Riemann Tensor (1876), the Ricci Tensor (1901), the Einstein field equations (1916) and so on, can be described as functions of the two free parameters of GLT-model. Consequently, it seems that the possibilities of an unification of Einstein's Special and General Theories of Relativity, as well as a new unification of electromagnetic and gravitational fields are opened. Furthermore, in this paper it is shown that there exists a simple coordinate transformation procedure that transforms a general line element into diagonal one, with metric components (-1, 1, 1, 1), equal to the metrics of the line element in SR. If this transformation is correct, then the transformation of Riemann's metrics into de Cartesian or Minkowski one is possible. The final goal of applications of GLT-model could be an analysis of a possibility of unification of Quantum Mechanics and General Theory of Relativity, trying to avoid the well-known problems of that unification.

This paper is organized as follows. The second section presents a new full form of GLT-model, related to x, y, and z-axes. In the third section a general line element and a metric tensor of the full form of GLT-model have been derived. The fourth section describes the general line element and the metric tensor of the full form of GLT-model in spherical polar coordinates. In the fifth section an identification of free parameters of the full form of GLT-model in a gravitational field has been discussed. The sixth section presents a derivation of a general diagonal form of a line element and a metric tensor of the GLT-model. Finally, the conclusions are emphasized by the seventh section .

2. The Full Form of the GLT - Model

The GLT- model derived by Novakovic (2000) for the particle motion in x-axis only, can be extended to the full form including y and z - axes. In order to derive the full form of the GLT-model let define the coordinates of the two parallel systems O and O' by x, y, z, t, and x', y', z', t', respectively, and the time $t = t' = 0$, when the origins of the two systems coincide. The system O' is moving relative to the system O with an arbitrary velocity v, along an arbitrary radius vector. Let an observation signal (which is a bearer of information) has the velocity α c in system O, and α 'c in system O ', where c is a constant reference signal velocity, and α and α ' are free parameters which are positive constants, or functions of the space-time coordinates, $\alpha > 0$, and $\alpha' > 0$.

Generally, parameters α and α' are functions of the space-time coordinates, and velocity v is not a constant. In the derivation of the differential form of GLT-model (eqs.2 and 3) we supposed that in infinite small intervals of dx^i, and dx'^i, i = 0, 1, 2, 3, parameters α and α', and the particle velocity v are constants. Finally, for the convenience, we employ parameter δ, where $\delta = 1$ if an observation signal is emitted from the origin of the system O, and $\delta = -1$ if an observation signal is emitted from the origin of the system O'.

Let the displacement four-vectors dX and dX' are defined in frames O and O′ respectively, by the expression:

$$dX \to 0\left(\sqrt{\alpha\alpha'}cdt, dx, dy, dz\right) = \{dx^i\}$$

$$dX' \to 0'\left(\sqrt{\alpha\alpha'}cdt', dx', dy', dz'\right) = \{dx'^i\} \qquad i = 0,1,2,3, \tag{1}$$

where dX has the components in the frame O, and dX' has the components in the frame O'. Applying this four-vector concept the GLT-model for the events in O system is described by the tensor equation :

$$dx'^i = \Lambda^i{}_\beta \, dx^\beta, \qquad i, \beta = 0,1,2,3, \tag{2}$$

where $\Lambda^i{}_\beta$ is the element in the i-th line and β-th column of $[\Lambda^i{}_\beta]$, which is the (4x4) transformation matrix of GLT-model:

$$\left[\Lambda^i{}_\beta\right] = \begin{bmatrix} H & D\left[\delta(\alpha-\alpha')_x c - v^1\right] & D\left[\delta(\alpha-\alpha')_y c - v^2\right] & D\left[\delta(\alpha-\alpha')_z c - v^3\right] \\ -D\,v^1 & H & 0 & 0 \\ -D\,v^2 & 0 & H & 0 \\ -D\,v^3 & 0 & 0 & H \end{bmatrix} \tag{3}$$

In the equation (3), $D = H/\sqrt{\alpha\alpha'}c$, and v^1, v^2, and v^3 are projections of the particle velocity v to the x, y, an z - axes, respectively, while $\delta(\alpha-\alpha')_x$, $\delta(\alpha-\alpha')_y$, and $\delta(\alpha-\alpha')_z$ are the corresponding projections of the term $\delta(\alpha-\alpha')$ to the x, y, an z - axes. Finally, parameter H is described by the equation:

$$H = 1 \Big/ \left[1 - \frac{(v)^2}{\alpha\alpha' c^2} + \frac{\delta(\alpha-\alpha')v}{\alpha\alpha' c}\right]^{1/2}. \tag{4}$$

The corresponding full form of GLT-model for the events in O′ system can be described by the tensor equation:

$$dx^{i} = \Lambda'^{i}_{\beta}\, dx'^{\beta}, \qquad i, \beta = 0, 1, 2, 3, \tag{5}$$

where (4x4) transformation matrix $[\Lambda'^{i}_{\beta}]$ is given by the following form:

$$\left[\Lambda''_{\beta}\right] = \begin{bmatrix} H & D\left[v^1 - \delta(\alpha - \alpha')_x c\right] & D\left[v^2 - \delta(\alpha - \alpha')_y c\right] & D\left[v^3 - \delta(\alpha - \alpha')_z c\right] \\ D v^1 & H & 0 & 0 \\ D v^2 & 0 & H & 0 \\ D v^3 & 0 & 0 & H \end{bmatrix}. \tag{6}$$

Remarks. The GLT-model (eqs. 2 to 6), is general in the sense that it satisfies Lorentz - Einstein Transformation model in Special Relativity (for $\alpha = \alpha' = 1$, and c is the speed of the light), and the Galilean Transformation model (for $\delta = 1$, $\alpha = 1$, and $\alpha' c = (c - v)$, and for $\delta = -1$, $\alpha' = 1$, and $\alpha c = (c - v)$). The GLT-model can also be used in General Relativity by identification of parameters α and α' in a gravitational field (see the next sections). The possible observation phenomena related to the GLT-model (a length, a time and a mass neutrality, contraction and dilation) have been presented in the reference (Novakovic, 2000).

3. A General Line Element and a Metric Tensor of the Full Form of GLT-Model

In order to derive a line element ds^2 of the full form of GLT-model one can start with the equations:

$$d\tau^2 = \frac{-ds^2}{\alpha\alpha' c^2} = \frac{1}{H^2} dt^2, \qquad ds^2 = -\frac{1}{H^2}\alpha\alpha' c^2 dt^2. \tag{7}$$

where $d\tau$ is a proper time of the moving particle. Applying H from (4) to the equation (7) we obtain the line element in the form:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \delta(\alpha - \alpha')vc\,dt^2 + (v)^2 dt^2. \tag{8}$$

Now, one can make the following substitutions:

$$\delta(\alpha - \alpha')v = \delta(\alpha - \alpha')_x v^1 + \delta(\alpha - \alpha')_y v^2 + \delta(\alpha - \alpha')_z v^3,$$
$$(v)^2 = (v^1)^2 + (v^2)^2 + (v^3)^2, \qquad v^i = \frac{dx^i}{dt}, \qquad i = 1,2,3. \tag{9}$$

This substitution transforms the equation (8) into the final form of a line element of GLT-model:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \delta(\alpha - \alpha')_x dx^1 cdt - \delta(\alpha - \alpha')_y dx^2 cdt - \\ - \delta(\alpha - \alpha')_z dx^3 cdt + (dx^1)^2 + (dx^2)^2 + (dx^3)^2. \tag{10}$$

Since $dx^1 = dx$, $dx^2 = dy$ and $dx^3 = dz$, we can describe a general line element of the full form of GLT-model by the equation:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \delta(\alpha - \alpha')_x cdtdx - \delta(\alpha - \alpha')_y cdtdy - \\ - \delta(\alpha - \alpha')_z cdtdz + dx^2 + dy^2 + dz^2. \tag{11}$$

It is very easy to see that for ($\alpha = \alpha' = 1$) the general line element (eq. 11) is transformed into the well-known line element in Special Relativity:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2. \tag{12}$$

We expected this result, because in Special Relativity there is no a gravitational field, and parameters α and α' are constants and equal to one.
Applying displacement four-vector dX from eq. 1, the line - element (eq. 11) can be transformed into the form:

$$ds^2 = -(dx^0)^2 + 2b_x dx^0 dx^1 + 2b_y dx^0 dx^2 + 2b_z dx^0 dx^3 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2, \tag{13}$$

where the elements b_x, b_y and b_z are given by the equations:

$$b_x = \frac{-\delta(\alpha - \alpha')_x}{2\sqrt{\alpha\alpha'}}, \quad b_y = \frac{-\delta(\alpha - \alpha')_y}{2\sqrt{\alpha\alpha'}}, \quad b_z = \frac{-\delta(\alpha - \alpha')_z}{2\sqrt{\alpha\alpha'}}. \tag{14}$$

As it is well-known, the corresponding Riemann's line element is given by the form:

$$ds^2 = g_{00}(dx^0)^2 + 2g_{01}dx^0 dx^1 + 2g_{02}dx^0 dx^2 + \\ +2g_{03}dx^0 dx^3 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 + g_{33}(dx^3)^2 \tag{15}$$

Comparing equations (13) and (15), one can derive a general covariant metric tensor of the full form of the GLT-model:

$$[g_{ij}] = \begin{bmatrix} -1 & b_x & b_y & b_z \\ b_x & 1 & 0 & 0 \\ b_y & 0 & 1 & 0 \\ b_z & 0 & 0 & 1 \end{bmatrix}, \tag{16}$$

which is symmetric and has ten non-zero elements, as we expected.

In the case $\alpha = \alpha' = 1$, the parameters $b_x = b_y = b_z = 0$, and the metric tensor (eq. 16) is transformed into the well-known metric tensor of Lorentz-Einstein Transformation model:

$$[\eta_{ij}] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (17)$$

The contra-variant general metric tensor g^{ij}, of GLT-model can be derived by inversion of the covariant one using eq. 16:

$$[g^{ij}] = \begin{bmatrix} g^{00} & g^{01} & g^{02} & g^{03} \\ g^{10} & g^{11} & g^{12} & g^{13} \\ g^{20} & g^{21} & g^{22} & g^{23} \\ g^{30} & g^{31} & g^{32} & g^{33} \end{bmatrix}, \quad (18)$$

where the elements g^{ij} are given as follows:

$$\begin{aligned} &g^{00} = \frac{-1}{1+b^2}, && g^{01} = g^{10} = \frac{b_x}{1+b^2}, && g^{02} = g^{20} = \frac{b_y}{1+b^2}, \\ &g^{03} = g^{30} = \frac{b_z}{1+b^2}, && g^{11} = \frac{1+b_y^2+b_z^2}{1+b^2}, && g^{12} = g^{21} = \frac{-b_x b_y}{1+b^2}, \\ &g^{13} = g^{31} = \frac{-b_x b_z}{1+b^2}, && g^{22} = \frac{1+b_x^2+b_z^2}{1+b^2}, && g^{23} = g^{32} = \frac{-b_y b_z}{1+b^2}, \\ &g^{33} = \frac{1+b_x^2+b_y^2}{1+b^2}, && b^2 = b_x^2 + b_y^2 + b_z^2 . \end{aligned} \quad (19)$$

The parameters b_x, b_y, and b_z are given by eq. 14. The determinants of the metric tensor (eqs. 16 and 18) can be calculated by the equations:

$$\det[g_{ij}] = -(1+b^2), \qquad \det[g^{ij}] = -1/(1+b^2) \quad . \quad (20)$$

The traces of the metric tensors $[g_{ij}]$ and $[g^{ij}]$ are identical:

$$T_R[g_{ij}] = T_R[g^{ij}] = 2 \quad , \quad (21)$$

what we expected that should be.

Remarks. The metric tensor of GLT-model (eqs. 16 and 18), is general in the sense that it satisfies the metric tensor in Special Relativity (for $\alpha = \alpha' = 1$, and c is the speed of the light), and can be used in General Relativity by identification of parameters α and α' in a gravitational field (see the next sections). This tensor can probably be employed in the others potential fields requiring the corresponding identification of parameters α and α'.

4. A General Line Element and a Metric Tensor of the Full Form of GLT-Model in Spherical Polar Coordinates

In order to identify two free parameters α and α′, of GLT-model in a gravitational field, the general line element (eq. 11) should be transformed into spherical polar coordinates, which are appropriate to the problem. For this purpose one can use the corresponding coordinate transformations:

$$x = r\sin\theta\cos\phi, \qquad y = r\sin\theta\sin\phi, \qquad z = r\cos\theta, \tag{22}$$

where r is a radius vector, θ is an angle between radius vector r and z-axis, and ϕ is an angle between projection of a radius vector r on (x-y) plane and x-axis. The corresponding derivations of x, y, and z coordinates give the following equations:

$$\begin{aligned} dx &= dr\sin\theta\cos\phi + r\left[\cos\theta\cos\phi d\theta - \sin\theta\sin\phi d\phi\right], \\ dy &= dr\sin\theta\sin\phi + r\left[\cos\theta\sin\phi d\theta + \sin\theta\cos\phi d\phi\right], \\ dz &= dr\cos\theta - r\sin\theta d\theta. \end{aligned} \tag{23}$$

The projections of the term $\delta(\alpha-\alpha')$, which is collinear with the radius vector r, on x, y, and z axes have the form:

$$\begin{aligned} \delta(\alpha-\alpha')_x = \delta(\alpha-\alpha')\sin\theta\cos\phi, \qquad \delta(\alpha-\alpha')_y = \delta(\alpha-\alpha')\sin\theta\sin\phi, \\ \delta(\alpha-\alpha')_z = \delta(\alpha-\alpha')\cos\theta. \end{aligned} \tag{24}$$

The substitutions of eqs. 23 and 24 into the general line element (eq. 11) give the corresponding general line element in spherical polar coordinates:

$$ds^2 = -\alpha\alpha' c^2 dt^2 - \delta(\alpha-\alpha')\, cdtdr + dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2. \tag{25}$$

This line element contains the usual two-dimensional spherical surface element as we expected. If the displacement four-vector of spherical polar coordinates is determined by the form:

$$dx \to 0(cdt, dr, d\theta, d\phi) = \left\{dx^i\right\}, \quad i = 0,1,2,3., \tag{26}$$

then the covariant metric tensor of the line element (eq. 25) can be described by the expression:

$$[g_{ij}] = \begin{bmatrix} -\alpha\alpha' & \frac{-\delta(\alpha-\alpha')}{2} & 0 & 0 \\ \frac{-\delta(\alpha-\alpha')}{2} & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}, \tag{27}$$

where there are six non-zero elements as it is expected. The determinant and the trace of this covariant metric tensor (eq. 27) can be calculated by the relations:

$$\det[g_{ij}] = -\frac{(\alpha+\alpha')^2}{4} r^4 \sin^2\theta, \qquad T_R[g_{ij}] = 1 - \alpha\alpha' + r^2(1+\sin^2\theta) \tag{28}$$

The contra-variant metric tensor, g^{ij}, of the line element (eq. 25) can be obtained by inversion of the covariant one, using eq. 27:

$$[g^{ij}] = \begin{bmatrix} g^{00} & g^{01} & 0 & 0 \\ g^{10} & g^{11} & 0 & 0 \\ 0 & 0 & g^{22} & 0 \\ 0 & 0 & 0 & g^{33} \end{bmatrix}, \tag{29}$$

where the non-zero elements of eq. 29 are given as follows:

$$\begin{aligned} g^{00} &= \frac{-4}{(\alpha+\alpha')^2}, \qquad g^{01} = g^{10} = \frac{-2\delta(\alpha-\alpha')}{(\alpha+\alpha')^2}, \\ g^{11} &= \frac{4\alpha\alpha'}{(\alpha+\alpha')^2}, \qquad g^{22} = \frac{1}{r^2}, \qquad g^{33} = \frac{1}{r^2\sin^2\theta}. \end{aligned} \tag{30}$$

The determinant and the trace of this contra-variant metric tensor (eq. 29) have the forms:

$$\det[g^{ij}] = -4/\left[(\alpha+\alpha')^2 r^4 \sin^2\theta\right] \qquad T_R[g^{ij}] = \frac{4(\alpha\alpha'-1)}{(\alpha+\alpha')^2} + \frac{1+\sin^2\theta}{r^2\sin^2\theta}. \tag{31}$$

In the case $\alpha = \alpha' = 1,\ \ r = 1,\ \ \theta = \pi/2$, the both metric tensor (eqs. 27 and 29) have the same determinants and the same traces:

$$\det[g_{ij}] = \det[g^{ij}] = -1, \qquad T_R[g_{ij}] = T_R[g^{ij}] = 2, \tag{32}$$

what is equal to the corresponding determinants and traces of the metric tensors in S.R. If the displacement four-vector of spherical polar coordinates is given by the expression:

$$dx \to 0\left(\sqrt{\alpha\alpha}\ cdt, dr, d\theta, d\phi\right) = \{dx^i\} \qquad i = 0,1,2,3. \tag{33}$$

then, the line element (eq. 25) can be transformed into the form:

$$ds^2 = -\left(dx^o\right)^2 + 2bdx^0dx^1 + \left(dx^1\right)^2 + r^2\left(dx^2\right)^2 + r^2 \sin^2\theta\left(dx^3\right)^2, \tag{34}$$

where the element b is given by the equation:

$$b = \frac{-\delta(\alpha - \alpha')}{2\sqrt{\alpha\alpha'}}. \tag{35}$$

Comparing eq. 34 with Riemann's line element (eq. 15) one can derive a general covariant metric tensor of the GLT-model in spherical polar coordinates, related to the line element (eq. 34):

$$\left[g_{ij}\right] = \begin{bmatrix} -1 & b & 0 & 0 \\ b & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}, \tag{36}$$

which is symmetric and has six non-zero elements as we expected.
As we can see from eqs. 35 and 36, the parameters $\alpha = \alpha' = 1$, transform the metric tensor (eq. 36) into diagonal form, which corresponds to the metric tensor in non-gravitational field with spherical polar coordinates:

$$\left[\eta_{ij}\right] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{bmatrix}. \tag{37}$$

This metric tensor has also the usual two-dimensional spherical surface element. The trace of this tensor can be calculate by the equation:

$$T_R\left[\eta_{ij}\right] = T_R\left[g_{ij}\right] = r^2\left(1 + \sin^2\theta\right), \tag{38}$$

which is the same as the trace of the metric tensor from eq. 36.
The contra-variant metric tensor g^{ij}, of the line element (eq. 34) of the GLT-model in spherical polar coordinates, can be derived by inversion of the covariant one using eq. 36:

$$\left[g^{ij}\right]=\begin{bmatrix} g^{00} & g^{01} & 0 & 0 \\ g^{10} & g^{11} & 0 & 0 \\ 0 & 0 & g^{22} & 0 \\ 0 & 0 & 0 & g^{33} \end{bmatrix}, \tag{39}$$

where the non-zero elements in eq. 39 are given as follows:

$$\begin{gathered} g^{00}=\frac{-1}{1+b^2},\quad g^{01}=g^{10}=\frac{b}{1+b^2},\quad g^{11}=\frac{1}{1+b^2}, \\ g^{22}=\frac{1}{r^2},\quad g^{33}=\frac{1}{r^2\sin^2\theta}, \end{gathered} \tag{40}$$

Here parameter b is given by eq. 35. The determinants of the metric tensor (eqs. 36 and 39) are given by the equations:

$$\det\left[g_{ij}\right]=-\left(1+b^2\right)r^4\sin^2\theta,\qquad \det\left[g^{ij}\right]=-\frac{1}{\left(1+b^2\right)r^4\sin^2\theta}, \tag{41}$$

For $\theta=\pi/2$ and $r=1$ the determinants of the tensors (eqs. 36 and 39) are equal to the determinants of the tensors (eqs. 16 and 18) given by eq. 20. The traces of the metric tensors (eqs. 36 and 39) can be calculated by the equations:

$$T_R\left[g_{ij}\right]=r^2\left(1+\sin^2\theta\right),\qquad T_R\left[g^{ij}\right]=\frac{1+\sin^2\theta}{r^2\sin^2\theta}. \tag{42}$$

For $\theta=\pi/2$ and $r=1$, the traces of the tensors (eqs. 36 and 39) are identical :

$$T_R\left[g_{ij}\right]=T_R\left[g^{ij}\right]=2, \tag{43}$$

what we expected that should be.

Remarks. The non-diagonal line element in eq. 25 has got a very important property: non-singularity in a strong gravitational field. It has been proved in the next section. In the case $\alpha=\alpha'=1$, $r=1$, $\theta=\pi/2$, the both metric tensor (eqs. 27 and 29), as well as the metric tensors (eqs. 36 and 39) are transformed into the corresponding tensors in Special Relativity.

5. Identification of Free Parameters of the Full Form of GLT-Model in a Gravitational Field

In this section it is presented the process of the identification of the free parameters α and α' in a gravitational field. It has been done by using the comparison of the line element (eq. 25) with the well-known spherically symmetric vacuum solution of the line element for non-rotating body, first found by Schwartzschild in 1916. In order to compare the line element of the GLT-model given by eq. 25 with the Schwartzschild form of the line element:

$$ds^2 = -e^{\nu}c^2dt^2 + e^{\lambda}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \tag{44}$$

the line element (eq. 25) should be diagonalized. As we can see from eq. 25 the non-diagonal term is:

$$-\delta(\alpha - \alpha')cdt\, dr, \tag{45}$$

which interacts between cdt and ds^2 and dr and ds^2 in the equation of the line element (eq. 25). In the process of diagonalization of the line element (eq. 25) the term (eq. 45) should be eliminated, and its influence to the ds^2 must be added to the corresponding diagonal elements in the diagonalized line element $\underline{ds}^2$ of eq. 25. In order to transform the line element of GLT-model (eq. 25) into the corresponding diagonal one, we can start with the equation:

$$\underline{ds}^2 = \frac{\partial(ds^2)}{\partial(c^2dt^2)}c^2dt^2 + \frac{\partial(ds^2/\alpha\alpha')}{\partial(dr^2)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2, \tag{46}$$

where ds^2 is from eq. 25, and $\underline{ds}^2$ is the diagonalized form of ds^2. In the process of diagonalization we suppose that parameters α and α' are constants in the infinite small space-time differential elements (dt, dr, $d\theta$, $d\phi$). The line element (eq. 25) can be diagonalised for a weak and a strong gravitational field. In the case of a week gravitational field (like in our Solar System), and for the case where the observation signal is the light, one can use the following substitution in the non-diagonal element (eq. 45):

$$dr = cdt, \quad or \quad cdt = dr, \tag{47}$$

where the first term is employed in $\partial(ds^2)/\partial(c^2dt^2)$ and the second one in $\partial(ds^2/\alpha\alpha')/\partial(dr^2)$ Thus, employing the equations (45) to (47) and using eq. 25 one can derive the unknown coefficients of the diagonal line element (eq. 46):

$$\frac{\partial(ds^2)}{\partial(c^2dt^2)} = -[\alpha\alpha' + \delta(\alpha - \alpha')], \quad \frac{\partial(ds^2/\alpha\alpha')}{\partial(dr^2)} = \left[\frac{1}{\alpha\alpha'} - \frac{\delta[\alpha - \alpha']}{\alpha\alpha'}\right] = [1 - \delta(\alpha - \alpha')]/\alpha\alpha'. \tag{48}$$

Finally, the diagonalized line element of the GLT-model in a weak gravitational field has the form:

$$\underline{ds}^2 = -[\alpha\alpha' + \delta(\alpha - \alpha')]c^2dt^2 + \frac{[1 - \delta(\alpha - \alpha')]}{\alpha\alpha'}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2. \tag{49}$$

Comparing this line element with the Schwartzschield's one, (eq. 44), we can identify the following parameters:

$$e^{\nu} = [\alpha\alpha' + \delta(\alpha - \alpha')], \qquad e^{\lambda} = [1 - \delta(\alpha - \alpha')] / \alpha\alpha'. \qquad (50)$$

Employing the well-known gravitational red-shift experiment and the energy equation, it has been shown in the reference (Novakovic, 2000) that the parameters α and α' in a gravitational field can be replaced by the quantities:

$$\delta = 1,\ \alpha = 1 + \frac{\phi}{c^2} = 1 - \frac{GM}{rc^2},\ \alpha' = 1, \quad or\ \delta = -1,\ \ \alpha = 1,\ \alpha' = 1 + \frac{\phi}{c^2} = 1 - \frac{GM}{rc^2}. \qquad (51)$$

where ϕ is a gravitational potential, G is a gravitational constant, M is a total gravitational mass and r is a gravitational radius from a center of gravity to a general space point of interest.

The parameters α and α' from eq. 51 should satisfy the well-known Schwartzshield's solution for coefficients e^{ν} and e^{λ}. Applying (51) to (50) we obtain (in both cases, $\delta=1$ and $\delta=-1$) the solution:

$$e^{\nu} = 1 - \frac{2GM}{rc^2}, \qquad e^{\lambda} = \frac{1 + \frac{GM}{rc^2}}{1 - \frac{GM}{rc^2}}. \qquad (52)$$

Now, for $r < \infty$, one can define a weak gravitational field by the property $(GM / r c^2)^2 \approx 0$, while a strong gravitational field has the property $(GM / r c^2)^2 >> 0$. It means that, for the same G, r and c, a mass of a source of a strong gravitational field is much greater then a mass of a source of a weak gravitational field. For an example, the gravitational field of our Sun can be taken as the weak gravitational field, while objects like black holes have got strong gravitational fields, both in a limited radius r. Of course, a gravitational field of a particular gravitational source is not present in the region $r \to \infty$. On the other hand, if a gravitational radius r of a mass M goes to zero ($r \to 0$), then the condition of a strong gravitational field is always satisfied. Meanwhile, we do not expect $r = 0$, because a mass M of a gravitational source must have a minimal volume different from zero, and consequently $r > 0$. Following the definition of the weak gravitational field, equation (52) can be transformed in the form valid in a weak gravitational field:

$$e^{\nu} = 1 - \frac{2GM}{rc^2}, \qquad e^{\lambda} = \frac{\left(1+\frac{GM}{rc^2}\right)\left(1-\frac{GM}{rc^2}\right)}{\left(1-\frac{GM}{rc^2}\right)\left(1-\frac{GM}{rc^2}\right)} = \frac{1-\left(\frac{GM}{rc^2}\right)^2}{1-\frac{2GM}{rc^2}+\left(\frac{GM}{rc^2}\right)^2},$$

$$\left(\frac{GM}{rc^2}\right)^2 \approx 0 \;\rightarrow\; e^{\lambda} \approx \frac{1}{1-\frac{2GM}{rc^2}}. \tag{53}$$

This solution of the line element of the GLT-model , eq. 25, in a weak gravitational field is equal in e^{ν} and approximately equal in e^{λ} to the spherically symmetric vacuum solution of non-rotating body, first found by Schwartzschild in 1916:

$$\underline{ds}^2 = -\left(1-\frac{2GM}{rc^2}\right)c^2dt^2 + \left(1-\frac{2GM}{rc^2}\right)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2 d\phi^2. \tag{54}$$

This solution has been derived from the well-known Einstein field equations in General Relativity, for the static vacuum field, where the energy-momentum tensor vanishes, i.e. $T_{\mu\beta} = 0$. Thus, the Cristoffel symbols and the Riemann and Ricci tensors of diagonalised line element of GLT-model (eq. 49) are equal to the corresponding ones of the Schwartzschild line element (eq. 54). Following the previous considerations one can conclude that the GLT-model is verified in General Relativity for a weak static vacuum gravitational field of a spherically symmetric non-rotating body.
In the case of a strong gravitational field , $(GM / r c^2)^2 >> 0$, and if we use the light as an observation signal, one can employ the following substitutions in the non-diagonal element (eq. 45):

$$dr = \alpha \; c\,dt, \quad or \quad cdt = dr / \alpha, \tag{55}$$

where the term $dr = \alpha\, cdt$ is employed in $\partial\left(ds^2\right)/\partial\left(c^2dt^2\right)$, while the term $cdt = dr/\alpha$ is used in $\partial\left(ds^2/\alpha\alpha'\right)/\partial\left(dr^2\right)$. Repeating the previous procedure one can derive the coefficients e^{ν} and e^{λ} of the line element (eq. 25) in diagonalized form, valid for a strong gravitational field and $\delta = 1$:

$$\delta = 1, \quad e^{\nu} = \left[\alpha\alpha' + \delta(\alpha-\alpha')\alpha\right] = \alpha^2, \qquad e^{\lambda} = \left[1 - \frac{\delta(\alpha-\alpha')}{\alpha}\right]/\alpha\alpha' = \frac{1}{\alpha^2}. \tag{56}$$

Applying the substitution of parameter α from eq. 51 into the equations (56), for the case $\delta = 1$, we obtain the solution:

$$e^{\nu} = \left(1 - \frac{GM}{rc^2}\right)^2 = 1 - \frac{2GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2, \quad e^{\lambda} = \frac{1}{\left(1 - \frac{GM}{rc^2}\right)^2} = \frac{1}{1 - \frac{2GM}{rc^2} + \left(\frac{GM}{rc^2}\right)^2}. \quad (57)$$

Thus, the diagonalized line element of the GLT-model in a strong gravitational field, for δ=1, has the form:

$$\underline{ds}^2 = -\alpha^2 c^2 dt^2 + \alpha^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (58)$$

where $\alpha = (1 - GM / r c^2)$. In the case of a weak gravitational field, $(GM / r c^2)^2 \approx 0$, the quadratic term $(GM/rc^2)^2$ can be neglected, and the strong field solution (eq. 57) is transformed into the weak field solution given by eq.53.

In the case $\delta = -1$, the equations (55) and (56) have the forms:

$$dr = \alpha' c dt, \quad or \quad c dt = dr / \alpha' , \quad (59)$$

$$\delta = -1, \quad e^{\nu} = [\alpha\alpha' + \delta(\alpha - \alpha')\alpha'] = \alpha'^2, \quad e^{\lambda} = \left[1 - \frac{\delta(\alpha - \alpha')}{\alpha'}\right] / \alpha\alpha' = \frac{1}{\alpha'^2}.$$

what leads to the solution of a strong gravitational field for δ = 1 ,eq.57.

Remarks. The line element of GLT-model, given in spherical polar coordinates and diagonalized in a weak gravitational field corresponds to the Schwartzschild spherically symmetric vacuum solution of the line element for non-rotating body. The quadratic term $(GM/rc^2)^2$ in the strong field solution (eq. 57) can be neglected in a weak gravitational field. In that case the strong field solution (eq. 57) is transformed into the weak field solution given by eq. 53. The Cristoffel symbols and the Riemann and Ricci tensors of diagonalised line element of GLT-model (eq. 49) in a weak gravitational field are equal to the coresponding ones of the Schwartzschild line element (eq. 54). These elements for a strong gravitational field will be presented in the next paper.

From eq. 54 one can see the well-known fact that the Schwartzschild's line element is singular when $r = 2GM/c^2$, because the coefficient of dr^2 becomes an infinite number. This corresponds to the equation $(2GM/rc^2) = 1$, or $(GM/rc^2) = 0.5$, or $(GM/rc^2)^2 = 0.25$. Thus, the singularity of the Schwartzschild's line element appears in a strong gravitational field. As it is well-known, this singularity makes a big problem in an analysis of strong gravitational fields of objects like black holes. The same singularity problem, at the same singularity radius, has the line element of GLT-model diagonalised in a weak gravitational field (eq. 53). The line element of GLT-model diagonalised in a strong gravitational field (eq. 57 and 58) has the singularity when $r = GM/c^2$, that corresponds to the equation $(GM/rc^2) = 1$, or $(GM/rc^2)^2 = 1$.

It is very important to point out that the line element of GLT-model given in **a non-diagonal form (eq. 25)** has got a very important property: **non-singularity in a very strong gravitational field.** This can be proved by substituting the solution of free parameters α and α' in a gravitational field (eq. 51) into the non-diagonal line element of GLT-model (eq. 25). As the result we obtain the following non-diagonal form of the line element:

$$ds^2 = -\left(1 - \frac{GM}{rc^2}\right) c^2 dt^2 + \left(\frac{GM}{rc^2}\right) c\, dt\, dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2 d\phi^2. \quad (60)$$

It is easy to see that the line element (60) has no singularity, except for r = 0, what we expect that can not be happen, because a mass M of a gravitational source must have a minimal volume different from zero, and consequently $r > 0$. If the gravitational radius satisfies the equation $r = GM / c^2$, then eq. 60 is transformed into the new relation:

$$ds^2 = c\, dt\, dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2 d\phi^2, \quad (61)$$

which also has got non-singularity property. Thus, the non-diagonal form of the line element of GLT-model (eq. 60) can be used for analysis of very strong gravitational fields of objects like black holes, because in eq. 60 the singularity problem does not exist.

Following the previous considerations one can conclude that the GLT-model is verified in G.R. for a static vacuum gravitational field of a spherically symmetric non-rotating body.

6. Derivation of a General Diagonal Form of a Line Element and a Metric Tensor of the GLT-Model

In order to derive a general line element in a diagonal form, with metric components (-1, 1, 1, 1) as we have in Special Relativity, one can apply the following coordinate transformation procedure to the equation (13):

$$\begin{gathered} dz^0 = a dx^0 = \sqrt{1+b^2}\, dx^0, \qquad dz^1 = b_x dx^0 + dx^1, \qquad dz^2 = b_y dx^0 + dx^2, \\ dz^3 = b_z dx^0 + dx^3, \qquad b^2 = b_x{}^2 + b_y{}^2 + b_z{}^2, \end{gathered} \quad (62)$$

where b_x, b_y, and b_z are given by eq. 14. The contra-variant coordinates dx^i, i = 0, 1, 2, 3, are generate by the equations:

$$dx^0 = \sqrt{\alpha\alpha'}\, cdt, \qquad dx^1 = dx, \qquad dx^2 = dy, \qquad dx^3 = dz. \quad (63)$$

The quadratic terms of the components in eq. 62 can be presented by the equations:

$$\begin{gathered} \left(dz^0\right)^2 = \left(1+b^2\right)\left(dx^0\right)^2, \qquad \left(dz^1\right)^2 = b_x{}^2\left(dx^0\right)^2 + 2b_x dx^0 dx^1 + \left(dx^1\right)^2, \\ \left(dz^2\right)^2 = b_y{}^2\left(dx^0\right)^2 + 2b_y dx^0 dx^2 + \left(dx^2\right)^2, \ \left(dz^3\right)^2 = b_z{}^2\left(dx^0\right)^2 + 2b_z dx^0 dx^3 + \left(dx^3\right)^2. \end{gathered} \quad (64)$$

Thus, a general diagonal line element of GLT-model can be derived by employing the transformations (eqs. 62 and 64):

$$ds^2 = -\left(dz^0\right)^2 + \left(dz^1\right)^2 + \left(dz^2\right)^2 + \left(dz^3\right)^2. \tag{65}$$

This can be proved by substituting eq. 64 into eq. 65. As the result we obtain the line element in the following form:

$$ds^2 = -\left(dx^0\right)^2 + 2b_x dx^0 dx^1 + 2b_y dx^0 dx^2 + 2b_z dx^0 dx^3 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2. \tag{66}$$

This line element is exactly equal to eq. 13, what confirms the coordinate transformations given by eq. 62. The new coordinates dz^i can be calculated from the coordinates dx^i, i = 0, 1, 2, 3, by using the tensor equations:

$$dz^i = A^i{}_\beta\, dx^\beta, \quad i, \beta = 0,1,2,3, \tag{67}$$

where $A^i{}_\beta$ is the element in the i-th line and β-th column of the transformation matrix A:

$$A = \begin{bmatrix} a & 0 & 0 & 0 \\ b_x & 1 & 0 & 0 \\ b_y & 0 & 1 & 0 \\ b_z & 0 & 0 & 1 \end{bmatrix}, \quad \det A = a = \sqrt{1+b^2}, \quad b^2 = b_x{}^2 + b_y{}^2 + b_z{}^2. \tag{68}$$

If one want to calculate the coordinates dx^i from dz^i, then it can be employed the inverse procedure:

$$dx^i = B^i{}_\beta\, dz^\beta, \quad i, \beta = 0,1,2,3, \tag{69}$$

where B is the inverse matrix of the matrix A from eq. 68:

$$B = A^{-1} = \begin{bmatrix} 1/a & 0 & 0 & 0 \\ -b_x/a & 1 & 0 & 0 \\ -b_y/a & 0 & 1 & 0 \\ -b_z/a & 0 & 0 & 1 \end{bmatrix}, \quad \det B = \frac{1}{a}, \quad a = \sqrt{1+b^2}, \quad b^2 = b_x{}^2 + b_y{}^2 + b_z{}^2. \tag{70}$$

Remarks. The coordinate transformations (eq. 62), transforms the Riemann's metrics (eq. 66) into the de Cartesian or Minkowski metrics (eq. 65). It includes the possibility that in the first step some problems can be solved in the coordinate system dz^i, i = 0, 1, 2, 3, because of the simple metric tensor. The second step should be the calculation of the coordinates dx^i, i = 0, 1, 2, 3 by employing the equations (69) and (70).

7. Conclusion

The proposed GLT-model is general in the sense that it satisfies both Lorentz - Einstein and Galilean coordinate transformations, and can also be used for coordinate transformation in General Relativity. If GLT-model is correct, then one can observe in a gravitational field a length, a time and a mass neutrality, contraction and dilation (Novakovic, 2000). The metric tensor of GLT-model is general in the sense that it contains the metric tensors of Special and General Relativity. This tensor can probably be employed in the others potential fields requiring the corresponding identification of free parameters α and α'. The line element of GLT-model, given in spherical polar coordinates and diagonalized in a weak gravitational field corresponds to the Schwartzschild spherically symmetric vacuum solution of the line element for non-rotating body.

The quadratic term $(GM/rc^2)^2$ in a strong field solution of the line element can be neglected in a weak gravitational field. In that case the strong field solution is transformed into the weak field solution. It is very important to point out that the line element of GLT-model, given by a non-diagonal form, has got a very important property: non-singularity in a very strong gravitational field. Finally, it is shown that there exists a simple coordinate transformation procedure that transforms a general line element into diagonal one with metric components (-1, 1, 1, 1). This is equal to the metrics of the line element in Special Relativity. If this transformation is correct, then the transformation of Riemann's metrics into de Cartesian or Minkowski one is possible.

Following the previous considerations one can conclude that the GLT-model is verified in Special Relativity (Novakovic, 2000) and in General Relativity for a static vacuum gravitational field of a spherically symmetric non-rotating body. The next goal is the investigation of the possibility of an unification of Einstein's Special and General Theories of Relativity, as well as the possibility of an unification of electromagnetic and gravitational fields, applying GLT-model. The previous investigation results give the hope that GLT-model can help in solution of the mentioned problems.

Acknowledgments

This work was supported by grants from the National Scientific Foundation of Republic of Croatia. The authors wishes to thank the anonymous reviewers for a variety of helpful comments and suggestions.

References

Einstein Albert (1909). The Collected Papers of Albert Einstein. Vol. 2: The Swiss years: writings, 1900-1909. Princeton, NJ, (1989).

Einstein Albert (1916). Die Grundlagen der allgemeinen Relativitätstheorie. Ann. Physik. Vol. 49, pp. 769-822.

Einstein Albert (1955). The Meaning of Relativity. 5th ed. Princeton University Press, Princeton, N.J.

Miller, A. I. (1981). Albert Einstein's Special Theory of Relativity : Emergence (1905) and early interpretation (1905-1911). Reading, Mass.

Supek Ivan (1992). Theoretical physics and structure of matter. Zagreb: Skolska knjiga, ch. 7, pp. 419-439.

Novakovic Branko, Novakovic Dario and Novakovic Alen (1998). New Observation Phenomena in Special Theory of Relativity. In *http://www.geocities.com/SiliconValley/Lab/4429/bnovak1.html*.

Novakovic Branko, Novakovic Dario and Novakovic Alen (2000). A New General Lorentz Transformation model. Computing Anticipatory Systems: CASYS'99-Third International Conference. Edited by Daniel M. Dubois. Published by The American Institute of Physics, AIP-CP517, pp. 437-450.

Dubois Daniel (2000). Computational Derivation of Quantum Relativist Electromagnetic Systems with Forward-Backward Space-Time Shifts. CASYS'99-Third International Conference. Edited by Daniel M. Dubois. Published by The American Institute of Physics, AIP-CP517, pp. 417-429.

Dubois Daniel, and Nibart Gilles (2000). Toward a Computational Derivation of a Dual Relativity with Forward-Backward Space-Time Shifts. CASYS International Journal of Computing Anticipatory Systems. Edited by Daniel M. Dubois. Published by CHAOS, Liège, Belgium, Volume 5, pp. 25-37.

Nibart Gilles (2000). Do Tachyions Violate the Causality Principle? Computing Anticipatory Systems: CASYS'99-Third International Conference. Edited by Daniel M. Dubois. Published by The American Institute of Physics, AIP-CP517, pp. 383-390.

Nibart Gilles (2000). A Relativistic Model of a Particle - Antiparticle Pair may Break up the E.P.R. Paradox. CASYS International Journal of Computing Anticipatory Systems. Edited by Daniel M. Dubois. Published by CHAOS, Liège, Belgium, Volume 5, pp. 95-115.