The Theory of Infinite Momentum Frames

Gilles NIBART
Laboratoire de Physique Théorique Fondamentale de Paris
31 rue de l'Évêque, F-06140 Coursegoules (France)
fax / phone 33 (0)4 93 59 14 46

Abstract
Infinite momentum frames (IMF) have been first introduced by J. Kogut and L. Susskind (1973) in the theory of partons. The concept of infinite momentum frames (IMF) have been developed by R. DUTHEIL (1984) on the basis of complex rotations group in a pseudo Euclidean space.

In the present communication, we re-examine in section 2, the different definitions of IMF proposed by these authors: we criticize the not allowed renormalization of « *divergent coordinates* » done by J. Kogut and L. Susskind, we abstract the development by R. DUTHEIL of a two dimensional infinite momentum frame (IMF-2) from considerations on the subluminal and the superluminal Lorentz groups, we criticize the generalization to a four dimensional infinite momentum frame (IMF-4) proposed by R. DUTHEIL and G. NIBART.

In section 3, we study the relativist transformations of two dimensional infinite momentum frames (IMF-2), which correspond to a subluminal Lorentz transformation or a superluminal Lorentz transformation.

In section 4, we propose a new mathematical concept of IMF based on isotropic vectors and having any number of dimensions.

In section 5, we re-examine the relativist quantum theory in IMF-2 developed by R. DUTHEIL, we propose a generalization of the Klein, Gordon and Fock equations in IMF-4, and we discuss the generalization by R. DUTHEIL of the Dirac equations to 4 dimensions.

Keywords: Parton, Tachyon, Infinite Momentum Frame, IMF

1 Introduction

1.1 First uses of infinite momentum frames

J. KOGUT and L. SUSSKIND have introduced infinite momentum frames (IMF) to describe « the instantaneous distribution of partons present at any time within the hadron » [1]. They have defined an IMF as an ordinary referential frame (ORF) moving with almost the light velocity. In such an infinite momentum frame, time is so much dilated that hadrons appear as a static collection of partons.

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R. DUTHEIL has developped the concept of infinite momentum frames (IMF) on the basis of complex rotations group in pseudo Euclidean space (see section 2.2 of this paper) and he has applied infinite momentum frames to the theory of tachyons [2].

Infinite momentum frames have been used by R. DUTHEIL and G. NIBART [3] to show with timelike and spacelike Dirac equations in IMF, that a subluminal anti-fermion can be interpreted as a superluminal fermion having an electric charge of opposite sign. R. DUTHEIL has also shown with these Dirac equations in IMF, that a photon may be considered as composed of a subluminal fermion and a superluminal fermion [4,5].

So these authors have used IMF for specific purposes, but they do not have studied the theory of infinite momentum frames for itself.

1.2 Why infinite momentum frames ?

A Cartesian referential frame is a mathematical representation of three orthogonal measuring rods, and today ordinary referential frames are nothing else. A natural observer who holds a measuring rod in his hand and who has a clock in his pocket, would use tools which are not adapted to space time continuum experiments and so he may not hope to experiment space time duality beyond the light barrier.

The standard meter has changed: it is no more a rod but an electromagnetic wave length. To measure durations we have exchanged the old egg timer for an atomic clock: the standard second is given by an electromagnetic wave period. Moreover most relativist experiments use electromagnetic properties.

Because the standard space time units, the space time continuum and the space time duality, all are related to electromagnetic properties, we should rather use infinite momentum frames which are the proper referential frames of photons.

2 History of Infinite Momentum Frames

An infinite momentum frame corresponds to a relativist singularity, where momentum and energy become infinite, except for photons. It is not a referential having an infinite velocity, because a tachyon with an infinite velocity has a null energy and has a finite momentum which is the « fundamental momentum » [6].

2.1 First definition of an infinite momentum frame

J. KOGUT and L. SUSSKIND have defined an infinite momentum frame as a referential frame moving with nearly the velocity of light in vacuum, relatively to a natural observer, but they have only considered subluminal referential frames.

Let us consider a mass particle at rest in an ordinary referential frame K, and an other subluminal referential frame K' moving along the x axis with a velocity v which is almost equal to the light velocity. The energy of the particle in K is

$$E = m_0 c^2 \tag{1}$$

and in K' it is

$$E' = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \tag{2}$$

so when the relative velocity of K' increases and tends to the light velocity in vacuum

$$|v| \to c \tag{3}$$

the energy related to K' tends to infinity.

$$E' \to \infty \tag{4}$$

<u>Remark</u>: The divergence of energy is usually interpreted as a kinetic energy property, but this example shows that the relativist singularity depends on the choice of a referential frame. Actually a mass particle never has an infinite energy because there is no natural observer having the light velocity, and a mass particle cannot have an infinite energy because paradoxically it would contain the energy of the whole universe.

Let us write the Lorentz transformation K→K' as the system of equations

$$\begin{aligned} x' &= x\cos\theta + i\,ct\sin\theta \\ ct' &= i\,x\sin\theta + ct\cos\theta \end{aligned} \tag{5}$$

and

$$\begin{aligned} y' &= y \\ z' &= z \end{aligned} \tag{6}$$

with a complex rotation defined by the both equations

$$\begin{aligned} \cos\theta &= \frac{1}{\sqrt{1-\beta^2}} \\ \sin\theta &= \frac{i\beta}{\sqrt{1-\beta^2}} \end{aligned} \tag{7}$$

where

$$\beta = \frac{v}{c} \tag{8}$$

The complex rotation have an imaginary angle θ

$$\theta = \mathrm{Arctg}(i\,\beta) \tag{9}$$

<u>Remark</u>: In equations 7 the cosinus is always positive

$$\cos\theta > 0 \tag{10}$$

but the sign of the sinus depends on the sign of β, i.e. it depends on the direction of the referential K'. The relativist singularity corresponds to velocities almost equal to the light velocity. When $|v| \to c$ i.e. $|\beta| \to 1$ we have

$$\cos\theta \to +\infty \tag{11}$$

$$i\sin\theta \to \pm\infty \tag{12}$$

and the angle $i\theta$ tends to infinity

$$i\theta \to \pm\infty \tag{13}$$

with the same sign as the sinus in equation 12. When $\beta > 0$ we have

$$i\sin\theta \to -\infty \tag{14}$$

so the angle $i\theta$ tends to infinity negatively

$$i\theta \to -\infty \tag{15}$$

When $\beta < 0$ we have

$$i\sin\theta \to +\infty \tag{16}$$

and the angle $i\theta$ tends to infinity positively

$$i\theta \to +\infty \tag{17}$$

Anyway the relativist singularity results into infinite coordinates

$$\begin{aligned} x' &\to \pm\infty \\ ct' &\to \pm\infty \end{aligned} \tag{18}$$

so we might say that x' and ct' are « *divergent coordinates* ».

To propose an approximation of equations 5 which represents the Lorentz transformation K→K' near the singularity, J. KOGUT and L. SUSSKIND have incorrectly considered « it is convenient to define rescaled quantities with these divergences removed » [1], and they have defined a « dilated time » τ

$$\tau = \sqrt{2}\, ct' e^{+i\theta} \tag{19}$$

with a scaling factor $e^{i\theta}$ which is divergent as shown below

$$i\theta \to -\infty \quad \Rightarrow \quad e^{+i\theta} \to 0 \quad e^{-i\theta} \to +\infty \tag{20}$$

$$i\theta \to +\infty \quad \Rightarrow \quad e^{-i\theta} \to 0 \quad e^{+i\theta} \to +\infty \tag{21}$$

where the sign of $i\theta$ depends on the direction of K'. It is not correct at all !

To do their renormalization, these authors have divided the time coordinate by a divergent term. Their mathematical development is dubious and we cannot accept it. Well, it is not so easy to remove the relativist singularity !

However they have been led to express their « dilated time » as

$$\tau = \tfrac{1}{\sqrt{2}}\left(x + ct\right) \tag{22}$$

in the ordinary referential frame K, and so they have introduced a light cone coordinate. Since this work, IMF have been understood as being a system of light cone coordinates.

2.2 Second definition of an infinite momentum frame

R. DUTHEIL has introduced an infinite momentum frame (IMF) on the basis of complex rotations groups in a pseudo-Euclidean space, from considerations about the transformation of an ordinary referential frame (ORF) into a tachyonic referential frame (TRF).

Complex rotations groups in pseudo-Euclidean spaces have been introduced very early in the theory of Relativity, and A. EINSTEIN has already used them in a lecture at Princeton University [7], but considing only the subluminal Lorentz group. On the basis of complex rotations groups of four dimensions, R. DUTHEIL has introduced the superluminal Lorentz group [2,4,5] and also the concept of infinite momentum frames [2,4,5].

As the SO(3,1;C) and SO(1,3;C) groups of the 4×4 complex matrix are isomorphic together and with SO(4;C) there are two complete orthochronous Lorentz groups, which have the same Lie Algebra, and which are isomorphic with one another

$$\mathcal{L}_+^T \quad \widetilde{\mathcal{L}}_+^T \tag{23}$$

$\mathcal{L}_+^T$ is the usual Lorentz group (orthochronous, subluminal) which preserves the real metrics $g_{\mu\nu}$ having the signature $(+---)$ with the usual real coordinates, called subluminal coordinates

$$x^\mu \quad (\mu = 0.1.2.3) \tag{24}$$

$\widetilde{\mathcal{L}}_+^T$ is the superluminal (and orthochronous) Lorentz group which preserves the real metrics $\widetilde{g}_{\mu\nu}$ having the signature $(-+++)$ with the inherent real coordinates, called superluminal coordinates

$$\widetilde{x}^\mu \quad (\mu = 0.1.2.3) \tag{25}$$

They correspond respectively to ordinary referential frames (ORF) and tachyonic referential frames (TRF). The ORF and TRF metrics have opposite signatures, and in the framework of the special theory of Relativity the metrics tensors can be represented with the following constant matrix:

$$\left[\widetilde{g}_{\mu\nu}\right] = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \left[g_{\mu\nu}\right] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{26}$$

where the tensor symbol between brackets indicates the associated matrix. There is only one matrix operator S which transforms a metrics to the other and reciprocally [2], as shown in the matrix equations

$$\begin{aligned} \left[\widetilde{g}_{\mu\nu}\right] &= [S]\left[g_{\mu\nu}\right] \\ \left[g_{\mu\nu}\right] &= [S]\left[\widetilde{g}_{\mu\nu}\right] \end{aligned} \tag{27}$$

and the matrix operator $[S]$ is the opposite of the unity matrix

$$[S] = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{28}$$

From this remark R. DUTHEIL has shown that there is an operator T which enable to pass from $\mathcal{L}_+^T$ to $\widetilde{\mathcal{L}}_+^T$ and reciprocally, and he has defined the « transceic » operator T with the following matrix equation

$$[S] = [T][T] \tag{29}$$

The equation 29 has several solutions for T because the 4-matrix $[S]$ has several square roots, and two examples of solutions are given in equations 31,32 below.

Nevertheless R. DUTHEIL has proposed [2] only one solution: the operator T defined by

$$[T]=\begin{pmatrix} 0 & \pm i & 0 & 0 \\ \pm i & 0 & 0 & 0 \\ 0 & 0 & \pm i & 0 \\ 0 & 0 & 0 & \pm i \end{pmatrix} \tag{30}$$

where the imaginary number i represents the rotation operator of angle $\pi/2$. He decided to choose the case with all plus signs as in T_1:

$$[T_1]=\begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \tag{31}$$

Later he asserted [4] that there is only one operator T_1 which is defined by the equation 29. It is not true because the matrix $[S]$ has several square roots. Here is an other possible operator T_2:

$$[T_2]=\begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \end{pmatrix} \tag{32}$$

Next R. DUTHEIL has explained [2,4] that the operator T does not belong to SO(4;C), because the matrix $[T_1]$ defined by equation 31 has a negative determinant

$$\det[T_1]=-1 \tag{33}$$

but his demonstration is not correct because the other solution T_2 has a positive determinant

$$\det[T_2]=+1 \tag{34}$$

Reducing his framework to only two dimensions, he has considered the operator O defined by

$$[O]=\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} \tag{35}$$

where the imaginary number i represents the rotation operator of angle $\pi/2$.

I think that R. DUTHEIL has chosen the T operator with definition 31, because it contains the operator O where it applies to one time coordinate x_0 and one space coordinate x_1. The operator O belongs to the complex rotation group SO(2;C), and there are two dimensional Lorentz sub-groups ℓ_+^T $\tilde{\ell}_+^T$ which preserves their associated metrics having respectively the signatures $(+-)$ and $(-+)$. Finally considering the transformation of a two dimensional subluminal referential frame (ORF-2) into a two dimensional superluminal referential frame (TRF-2), with the rotation operator O of angle $\pi/2$, such as

$$\begin{pmatrix} \tilde{x}^0 \\ \tilde{x}^1 \end{pmatrix} = \begin{pmatrix} 0 & e^{i\frac{\pi}{2}} \\ e^{i\frac{\pi}{2}} & 0 \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix} \tag{36}$$

R. DUTHEIL has introduced an IMF from an intermediate rotation position between an ORF and a TRF, which has a rotation angle of $\pi/4$

$$\begin{aligned} e^{-i\frac{\pi}{4}}\tilde{x}^0 &= e^{i\frac{\pi}{4}}x^1 \\ e^{-i\frac{\pi}{4}}\tilde{x}^1 &= e^{i\frac{\pi}{4}}x^0 \end{aligned} \tag{37}$$

so with a mere algebraic method he has deduced a two dimensional IMF. Using the intermediate variables α^0, α^1

$$\begin{aligned} \alpha^0 &= e^{i\frac{\pi}{4}}x^0 \\ \alpha^1 &= e^{i\frac{\pi}{4}}x^1 \end{aligned} \tag{38}$$

and taking the complex conjugate of α^1 (the ORF coordinate x^1 is real)

$$\alpha^{1*} = e^{-i\frac{\pi}{4}}x^1 \tag{39}$$

he has written the following complex expression

$$\alpha^0 + \alpha^{1*} = \tfrac{1}{\sqrt{2}}(ct + x) + \tfrac{i}{\sqrt{2}}(ct - x) \tag{40}$$

in which the real components are the coordinates of a two dimensional IMF

$$\begin{aligned} \tau &= \tfrac{1}{\sqrt{2}}(ct + x) \\ \zeta &= \tfrac{1}{\sqrt{2}}(ct - x) \end{aligned} \tag{41}$$

In this he has introduced two light cone coordinates.

2.3 Discussion of the second definition of an IMF

Because of the properties of the SO(2;C) group, R. DUTHEIL has considered that infinite momentum frames have only two inherent light cone coordinates.

R. DUTHEIL has considered τ, ζ and τ', ζ' as being inherent coordinates, but from the example of K→K' transformation given above, it is clear that their definition as linear combinations of t, x and t', x', in equations 63, 64 is related to the Lorentz transformation in equation 5, i.e. to the translation of K' along the x axis. So the τ, ζ, τ', ζ' coordinates are not inherent to the IMF, but they are only inherent to the translation direction of K' along the x axis of K. Taking the two light cone coordinates τ, ζ defined by

$$\begin{aligned} \tau &= \tfrac{1}{\sqrt{2}}(ct + x) \\ \zeta &= \tfrac{1}{\sqrt{2}}(ct - x) \end{aligned} \tag{42}$$

R. DUTHEIL has added the two ordinary coordinates of K and K'

$$\begin{aligned} y' &= y \\ z' &= z \end{aligned} \tag{43}$$

to complete the IMF to four coordinates. So the referential $\Omega(\tau,\zeta,y,z)$ which he used is a just mixture of two IMF cordinates (τ,ζ) and two ORF coordinates (y,z). In the definition of such a referential $\Omega(\tau,\zeta,y,z)$ one space axis has been privileged (here the x axis) by the K $\rightarrow$ K' translation.

No space direction should be privileged, because physical space is supposed to be isotropic. I think that the three space coordinates should be similarly considered in an IMF. So the R. DUTHEIL's method is not able to provide a four dimensional IMF, and it is limited to the framework of the special theory of Relativity.

2.4 Third definition of an infinite momentum frame

R. DUTHEIL and G. NIBART [3] have proposed to generalize the concept of IMF to four light cone coordinates, taking the new coordinate ξ^{μ} notation as follows

$$\xi^{0} = \tau \quad \xi^{1} = \zeta \tag{44}$$

They have named « IMF-2 » any infinite momentum frame having only two light cone coordinates, so the coordinates system $\{\tau,\zeta\}$ and the referential $\Omega(\tau,\zeta,y,z)$ are both IMF-2.

They have shown [3] it is possible to define an infinite momentum frame having four light cone coordinates from any ORF (or from any TRF), which have an equivalent metrics. They have named it « IMF-4 » and they have given an example where the τ-coordinate is a linear combination of the ORF space coordinates (with the same weight) and of the ORF time coordinate. They also have given an example of an « IMF-4 » built from a TRF.

2.5 Discussion of the third definition of an IMF

It is possible to define a four dimensional infinite momentum frame (IMF-4) in relation to both an ORF and a TRF [3]. However all IMF-4 transformations are not Lorentz invariant, because ORF and TRF correspond to two different Lorentz groups.

R. DUTHEIL has reduced his IMF framework to only two dimensions to satisfy the Lorentz invariance requirement. So the generalization of an IMF to four dimensions is in contradiction with this requirement.

A two dimensional IMF has been deduced from the transformation of a two dimensional ORF into a two dimensional TRF, represented by the equation 36 which produces a space time permutation.

The duality between space and time in extending the usual Lorentz group to superluminal transformations is well known. The generalization of infinite momentum frames to more than two space dimensions cannot avoid this problem. A transformation of a four dimensional ORF into a four dimensional TRF would produce a similar space time permutation, which is not possible because of the mismatch between the number of space and time dimensions.

A six dimensional space-time manifold has been proposed by several authors [8,9,10] to extend the Lorentz transformation to superluminal velocities, and L. MARCHILDON, A. F. ANTIPPA, and A. E. EVERETT [11] have analyzed the proposals of these authors. A timeless six dimensional manifold has been proposed by G. NIBART

[12] to build a relativistic model of a particle antiparticle pair with relativist E.P.R. correlations.

In a next work we will introduce a six dimensional infinite momentum frame. But before doing the generalization to six dimensions, we first have to propose a new definition of an infinite momentum frame (see section 4).

3 Relativist Transformations of an Infinite Momentum Frame

We call « IMF Transformation » a transformation of an infinite momentum frame which corresponds to either a subluminal Lorentz transformation or a superluminal Lorentz transformation. It is always possible to deduce expressions of an IMF transformation from a subluminal or superluminal Lorentz transformation, if we know the defintion of the IMF coordinates. Here we give an example of how we can deduce an IMF-2 transformation from a special Lorentz transformation, in both the subluminal case and the superluminal case.

3.1 The two metrics of an IMF-2

From the two light cone coordinates defined in equation 42 we can write the two ORF coordinates as

$$\begin{aligned} ct &= \tfrac{1}{\sqrt{2}}(\tau+\zeta) \\ x &= \tfrac{1}{\sqrt{2}}(\tau-\zeta) \end{aligned} \tag{45}$$

and we obtain a relation between the interval in IMF-2 and in ORF-2

$$c^2t^2 - x^2 = 2\tau\zeta \tag{46}$$

so we can deduce the corresponding metrics of an IMF-2

$$ds^2 = c^2dt^2 - dx^2 = 2\,d\tau\,d\zeta \tag{47}$$

Similarly R. DUTHEIL has deduced [2] the tachyonic coordinates of a TRF-2 from the tachyonic light cone coordinates

$$\begin{aligned} c\tilde{t} &= \tfrac{1}{\sqrt{2}}(\tilde{\tau}+\tilde{\zeta}) \\ \tilde{x} &= \tfrac{1}{\sqrt{2}}(\tilde{\zeta}-\tilde{\tau}) \end{aligned} \tag{48}$$

and he has shown that the tachyonic light cone coordinates can be related to the same IMF-2 with:

$$\begin{aligned} \tilde{\tau} &= \zeta \\ \tilde{\zeta} &= -\tau \end{aligned} \tag{49}$$

so he has deduced the corresponding tachyonic metrics in the IMF-2

$$\tilde{x}^2 - c^2\tilde{t}^2 = -2\tilde{\tau}\tilde{\zeta} = 2\tau\zeta \tag{50}$$

i.e.

$$ds^2 = d\tilde{x}^2 - c^2d\tilde{t}^2 = -2\,d\tilde{\tau}\,d\tilde{\zeta} = 2\,d\tau\,d\zeta \tag{51}$$

3.2 The timelike region and the spacelike region in an IMF-2

A subluminal velocity related to an ORF-2 has a timelike interval, such as

$$c^2 t^2 - x^2 > 0 \quad \Leftrightarrow \quad 2\tau\zeta > 0 \tag{52}$$

so the corresponding timelike region in an IMF-2 is defined by the condition

$$\tau > 0 \quad \text{and} \quad \zeta > 0 \tag{53}$$

or the condition

$$\tau < 0 \quad \text{and} \quad \zeta < 0 \tag{54}$$

A superluminal velocity related to a TRF-2 has a spacelike interval, such as

$$\tilde{x}^2 - c^2\tilde{t}^2 > 0 \quad \Leftrightarrow \quad -2\tilde{\tau}\tilde{\zeta} > 0 \tag{55}$$

so the corresponding spacelike region in an IMF-2 is defined by the condition

$$\tilde{\tau} > 0 \quad \text{and} \quad \tilde{\zeta} < 0 \tag{56}$$

or the condition

$$\tilde{\tau} < 0 \quad \text{and} \quad \tilde{\zeta} > 0 \tag{57}$$

It is very strange that R. DUTHEIL has restricted the timelike region of an IMF-2 to the only condition 53 where the two light cone coordinates are both positive. Similarly R. DUTHEIL has strangely restricted the spacelike region of an IMF-2 to the only condition 56 where the tachyonic $\tilde{\tau}$ coordinate is positive. He did not explain why he has rejected conditions 54 and 56, but I think it is to be related to the time arrow.

According to equation 49, the conditions 56 and 57 are respectively equivalent to

$$\tau > 0 \quad \text{and} \quad \zeta > 0 \tag{58}$$

or

$$\tau < 0 \quad \text{and} \quad \zeta < 0 \tag{59}$$

This shows that the region of an IMF-2 corresponding to the spacelike region represented by a TRF-2 is identical to the region of the IMF-2 corresponding to the timelike region represented by an ORF-2. So in an IMF-2 timelike and spacelike regions are not separated.

3.3 Subluminal IMF-2 transformations

We can easily deduce an IMF-2 transformation equation from an expression of a subluminal special Lorentz transformation. Let us consider again the system of equations 5

$$\begin{aligned} x' &= x\cos\theta + i\,ct\sin\theta \\ ct' &= i\,x\sin\theta + ct\cos\theta \end{aligned} \tag{60}$$

Adding and subtracting them we get the equivalent system of equations

$$\begin{aligned} x' + ct' &= (x + ct)(\cos\theta + i\sin\theta) \\ x' - ct' &= (x - ct)(\cos\theta - i\sin\theta) \end{aligned} \tag{61}$$

From equations 7 we obtain the following system of equations

$$\cos\theta + i\sin\theta = \frac{1-\beta}{\sqrt{1-\beta^2}}$$
$$\cos\theta - i\sin\theta = \frac{1+\beta}{\sqrt{1-\beta^2}} \tag{62}$$

The system of equations 61 contain two light cone coordinates defined from K as

$$\tau = \tfrac{1}{\sqrt{2}}(ct + x)$$
$$\zeta = \tfrac{1}{\sqrt{2}}(ct - x) \tag{63}$$

and from K' as

$$\tau' = \tfrac{1}{\sqrt{2}}(ct' + x')$$
$$\zeta' = \tfrac{1}{\sqrt{2}}(ct' - x') \tag{64}$$

thus the subluminal special Lorentz transformation of an IMF-2 can be deduced from the system of equations 61, i.e.

$$\tau' = \tau\frac{1-\beta}{\sqrt{1-\beta^2}} \tag{65}$$

$$\zeta' = \zeta\frac{1+\beta}{\sqrt{1-\beta^2}} \tag{66}$$

which are equivalent to

$$\tau' = \tau\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \tag{67}$$

$$\zeta' = \zeta\frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} \tag{68}$$

because 1-β and 1+β are always positive.

Remark: we see that the relativist singularity has not been removed, but in an IMF-2 the both coordinates τ', ζ' are not simultaneously divergent. When taking the subluminal limit $\beta \to +1$, τ' is not divergent as shown below

$$\tau' \to 0$$
$$\zeta' \to \pm\infty \tag{69}$$

and when taking the limit $\beta \to -1$, ζ' is not divergent as shown below

$$\tau' \to \pm\infty$$
$$\zeta' \to 0 \tag{70}$$

3.4 Superluminal IMF-2 transformation

We can similarly deduce an IMF-2 transformation equation from an expression of a superluminal special Lorentz transformation. Let us consider a mass particle related to its proper tachyonic referential frame $\widetilde{\mathrm{K}}$, and an other superluminal referential frame

$\widetilde{\mathrm{K}}'$ moving along the $\widetilde{x}$ axis with a velocity $\widetilde{v}$ which is a little greater than the light velocity.

The superluminal special Lorentz transformation $\widetilde{\mathrm{K}} \to \widetilde{\mathrm{K}}'$ may be represented by complex rotation equations which are similar to equations system 5 (except the tilda notation of tachyonic coordinates)

$$\begin{aligned} \widetilde{x}' &= \widetilde{x}\cos\varphi + i\,c\widetilde{t}\sin\varphi \\ c\widetilde{t}' &= i\,\widetilde{x}\sin\varphi + c\widetilde{t}\cos\varphi \end{aligned} \tag{71}$$

with a complex rotation defined by the following equations

$$\begin{aligned} \cos\varphi &= \frac{\widetilde{\beta}}{\sqrt{\widetilde{\beta}^2 - 1}} \\ \sin\varphi &= \frac{i}{\sqrt{\widetilde{\beta}^2 - 1}} \end{aligned} \tag{72}$$

where

$$\widetilde{\beta} = \frac{\widetilde{v}}{c} \tag{73}$$

and having an imaginary angle φ

$$\varphi = \mathrm{Arctg}\left(\frac{i}{\widetilde{\beta}}\right) \tag{74}$$

<u>Remark</u>: In equations 72 the cosinus depends on the sign of $\widetilde{\beta}$, i.e. it depends on the direction of the referential K'.

Adding and subtracting equations 72 we get (similarly to equations 61)

$$\begin{aligned} \widetilde{x}' + c\widetilde{t}' &= \left(\widetilde{x} + c\widetilde{t}\right)\left(\cos\varphi + i\sin\varphi\right) \\ \widetilde{x}' - c\widetilde{t}' &= \left(\widetilde{x} - c\widetilde{t}\right)\left(\cos\varphi - i\sin\varphi\right) \end{aligned} \tag{75}$$

From equations 72 we obtain the following equations system

$$\begin{aligned} \cos\varphi + i\sin\varphi &= \frac{\widetilde{\beta} - 1}{\sqrt{\widetilde{\beta}^2 - 1}} \\ \cos\varphi - i\sin\varphi &= \frac{\widetilde{\beta} + 1}{\sqrt{\widetilde{\beta}^2 - 1}} \end{aligned} \tag{76}$$

The equations 75 contain implicitly two light cone coordinates defined from $\widetilde{\mathrm{K}}$ as

$$\begin{aligned} \widetilde{\tau} &= \tfrac{1}{\sqrt{2}}\left(c\widetilde{t} + \widetilde{x}\right) \\ \widetilde{\zeta} &= \tfrac{1}{\sqrt{2}}\left(c\widetilde{t} - \widetilde{x}\right) \end{aligned} \tag{77}$$

and from $\widetilde{\mathrm{K}}'$ as

$$\begin{aligned} \widetilde{\tau}' &= \tfrac{1}{\sqrt{2}}\left(c\widetilde{t}' + \widetilde{x}'\right) \\ \widetilde{\zeta}' &= \tfrac{1}{\sqrt{2}}\left(c\widetilde{t}' - \widetilde{x}'\right) \end{aligned} \tag{78}$$

thus the superluminal special Lorentz transformation can be expressed from equation 75, in an IMF-2 as

$$\tilde{\tau}' = \tilde{\tau}\frac{\tilde{\beta}-1}{\sqrt{\tilde{\beta}^2-1}} \tag{79}$$

$$\tilde{\zeta}' = \tilde{\zeta}\frac{\tilde{\beta}+1}{\sqrt{\tilde{\beta}^2-1}} \tag{80}$$

Remark: we see that the relativist singularity has not been removed, but in an IMF-2 the both coordinates $\tilde{\tau}'$, $\tilde{\zeta}'$ are not simultaneously divergent. When taking the superluminal limit $\tilde{\beta} \to +1$, we have

$$\tilde{\beta} > 1 \quad \Rightarrow \quad \tilde{\beta}-1 > 0 \quad \tilde{\beta}+1 > 0 \tag{81}$$

and then

$$\begin{aligned} \tilde{\tau}' &= \tilde{\tau}\frac{\sqrt{\tilde{\beta}-1}}{\sqrt{\tilde{\beta}+1}} \to 0 \\ \tilde{\zeta}' &= \tilde{\zeta}\frac{\sqrt{\tilde{\beta}+1}}{\sqrt{\tilde{\beta}-1}} \to \pm\infty \end{aligned} \tag{82}$$

so $\tilde{\tau}'$ is not divergent. When taking the superluminal limit $\tilde{\beta} \to -1$, we have

$$\tilde{\beta} < -1 \quad \Rightarrow \quad \tilde{\beta}+1 < 0 \quad \tilde{\beta}-1 < 0 \tag{83}$$

and then

$$\begin{aligned} \tilde{\tau}' &= -\tilde{\tau}\frac{\sqrt{-\tilde{\beta}+1}}{\sqrt{-\tilde{\beta}-1}} \to \pm\infty \\ \tilde{\zeta}' &= -\tilde{\zeta}\frac{\sqrt{-\tilde{\beta}-1}}{\sqrt{-\tilde{\beta}+1}} \to 0 \end{aligned} \tag{84}$$

so $\tilde{\xi}'$ is not divergent.

3.5 Conclusion about relativist transformations of IMF-2

While special Lorentz transformations of ordinary referential frames (ORF) or tachyonic referential frames (TRF) are complex rotations, the corresponding relativist transformations of infinite momentum frames appear to be a change of scale: coordinates are just multiplied by a function of the relative velocity.

4 A new concept of an infinite momentum frame

We propose here to generalize the concept of IMF to any number of dimensions, with a new definition of infinite momentum frames.

4.1 Preliminary considerations about IMF-2 metrics

We have shown in section 3.1 that the IMF-2 and ORF-2 metrics are such as

$$ds^2 = c^2 dt^2 - dx^2 = 2\, d\tau\, d\zeta \tag{85}$$

We may write the IMF-2 metrics as

$$ds^2 = \eta_{ij}\, d\xi^i d\xi^j \tag{86}$$

with the following tensor notation of the IMF-2 coordinates

$$\xi^0 = \tau \quad \xi^1 = \zeta \tag{87}$$

so we can see that the IMF-2 metrics tensor has a null diagonal

$$\eta_{jk} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{88}$$

and that the two basis vectors ε_0, ε_1 have a null square:

$$\begin{aligned} \varepsilon_0 \cdot \varepsilon_0 &= \eta_{00} = 0 \\ \varepsilon_1 \cdot \varepsilon_1 &= \eta_{11} = 0 \end{aligned} \tag{89}$$

Consequently all basis vectors of an IMF-2 are isotropic.

4.2 A new definition of an infinite momentum frame

We may define an infinite momentum frame as a system of any number of light cone coordinates. The most general equation of a light cone with any number of dimensions, is

$$ds^2 = 0 \tag{90}$$

All basis vectors of such an IMF belong to the light cone, i.e. all basis vectors are isotropic, thus the metrics tensor has a null diagonal, as shown below

$$\varepsilon_\mu \cdot \varepsilon_\mu = \eta_{\mu\mu} = 0 \qquad (\mu = 0,1,\ldots n) \tag{91}$$

For example we may propose the following IMF-4 metrics tensor corresponding to a four dimensional pseudo Euclidean space, and it might be

$$\eta_{\mu\nu} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{92}$$

These considerations has led us to propose the following definition: ***An infinite momentum frame is a referential frame generated by any number of isotropic basis vectors.***

Remarks: We may imagine an IMF-n with more than 4 dimensions. All IMF coordinate axis have a « space-time nature ».

5 A Relativist Quantum Theory in Infinite Momentum Frames

5.1 The Klein, Gordon and Fock equations in IMF-2

Considering the momentum p of the timelike region in an ORF and the momentum π in the corresponding IMF-2, R. DUTHEIL has shown [2] that in the timelike region we have

$$p^{02} - p^2 = \pi^0 \pi + \pi \pi^0 \tag{93}$$

then he has deduced [2] the following Klein, Gordon and Fock equation of a subluminal boson

$$\left(\partial_\tau \partial_\zeta + \partial_\zeta \partial_\tau\right)\psi = \chi^2 \psi \tag{94}$$

where the Klein, Gordon and Fock constant is

$$\chi = \frac{m_0 c}{\hbar} \tag{95}$$

and he has deduced [2] the Klein, Gordon and Fock equation of a superluminal boson in the same IMF-2

$$\left(\partial_\tau \partial_\zeta + \partial_\zeta \partial_\tau\right)\widetilde{\psi} = -\chi^2 \widetilde{\psi} \tag{96}$$

5.2 The Klein, Gordon and Fock equations in IMF-4

R. DUTHEIL did not propose a generalization of the Klein, Gordon and Fock equations to a four dimensional infinite momentum frame having (IMF-4). Using an IMF-4 metrics tensor, for example the metrics tensor (equation 92), we may generalize the Klein, Gordon and Fock equations to four dimensions, so we have for subluminal bosons

$$\left(\eta^{\mu\nu}\partial_\mu \partial_\nu - \chi^2\right)\psi \tag{97}$$

and for superluminal bosons

$$\left(\eta^{\mu\nu}\partial_\mu \partial_\nu + \chi^2\right)\widetilde{\psi} \tag{98}$$

using the coordinates ζ^μ of the same IMF-4.

5.3 The Dirac equations in IMF-2

Applying the classical method of linearization to the Klein, Gordon and Fock equation 94, R. DUTHEIL has deduced [2] the following Dirac equation of a subluminal fermion

$$\left(\gamma_0 \partial_0 + \gamma_1 \partial_1 - \chi\right)\psi = 0 \tag{99}$$

and also the Dirac equation of a superluminal fermion in the same IMF-2

$$\left(i\gamma_0 \partial_0 + i\gamma_1 \partial_1 + \chi\right)\widetilde{\psi} = 0 \tag{100}$$

or

$$\left(\gamma_0 \partial_0 + \gamma_1 \partial_1 - i\,\chi\right)\widetilde{\psi} = 0 \tag{101}$$

where χ is the Klein, Gordon and Fock constant (equation 95) and where the Dirac matrix γ_0, γ_1 in IMF-2 satisfy the relation

$$\gamma_j \gamma_k + \gamma_k \gamma_j = 2\eta_{jk} \qquad (j,k = 0,1) \tag{102}$$

with the IMF-2 metrics tensor expressed by equation 88.

To show that a superluminal fermion is equivalent to an anti-fermion having an opposite electric charge [3], and to show that a pair of a superluminal fermion and a subluminal fermion is equivalent to a photon [2,4,5], R. DUTHEIL has used the Dirac equations 99, 101 expressed in an in IMF-2. His demonstration is not very pertinent, because an electromagnetic field cannot be considered within a two dimensional space (IMF-2) and thus his demonstration of the fermion equivalence is only valid without electromagnetic field.

5.4 The Dirac equations in IMF-4

To generalize the Dirac equations to four dimensions, R. DUTHEIL has later proposed [4] to use a « curvilinear coordinates system » having the following metrics

$$ds^2 = 2\, d\xi^0\, \eta_{0i}\, d\xi^i \tag{103}$$

he has obtained Dirac equations generalized to four dimensions, one for a subluminal fermion

$$\left(\gamma_\mu \frac{D}{D\xi_\mu} - \chi\right)\psi = 0 \tag{104}$$

and one for a superluminal fermion

$$\left(\gamma_\mu \frac{D}{D\xi_\mu} - i\,\chi\right)\tilde{\psi} = 0 \tag{105}$$

where D represents the covariant derivatives.

Considering the coordinates tensor notation 87, we see that the metrics defined by the equation 103 is a generalization of the IMF-2 metrics to four dimensions, but R. DUTHEIL did not present his « curvilinear coordinates system » as being an infinite momentum frame.

Moreover he did not demonstrate that the Dirac equations 104, 105 are Lorentz invariant in the same « curvilinear coordinates system ».

The generalization of the Dirac equations to IMF-4 with our method would not require the introduction of covariant derivatives. However the equation 102 can be extended to an IMF-4 as

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\eta_{\mu\nu} \qquad (\mu,\nu = 0,1,2,3) \tag{106}$$

Remark: in the framework of the special theory of Relativity the IMF metrics tensor $\eta_{\mu\nu}$ is just composed of constants as -1, 0, +1, but in the framework of the general theory of Relativity, the components of the IMF metrics tensor $\eta_{\mu\nu}$ are IMF gravitation potentials.

So the question arises: should Dirac matrix depend on the gravitation field ?

6 Conclusion

Infinite momentum frames (IMF) are intrinsic referential frames of photons, and they can be used to describe simultaneously subluminal particles, superluminal particles and photons in the same referential frame.

The introdution of the IMF metrics tensor in the Klein, Gordon and Fock equations, the relation between IMF metrics tensor and the IMF Dirac matrix allows us to expect a future generalization of the relativist quantum theory within the framework of the general theory of Relativity.

According to a more general concept, an infinite momentum frame with any number of dimensions should be buitlt on isotropic basis vectors. Using this definition in a next work, we will introduce a six dimensional infinite momentum frame (IMF-6) in relation to the usual ORF-4 and the tachyonic TRF-4.

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