Are There «Sequential Descriptions» in Physical Systems Analogous to DNA Ribbons in Living Cells ?

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Abstract

According to recent developments of arithmetical relators, the «sequential descriptions» of some physical systems might have some analogy with DNA ribbons in living cells. But such descriptions might have a physical existence only in «space-time-imbrication» reference frames. After recalling several properties of arithmetical relators, we explain in the framework of this formalism why these descriptions disappear when an arbitrary observer tries to observe them.

A stabilized arithmetical relator (AR) expresses in a structural way the adaptation of a natural system to its environment. Underlying ideal structures of the model appear as Lie structures. The coupling between a system and its environment involves a notion of time which usually cannot be reduced to classical Galilean or relativistic time. It evokes the numbering of successive symbols in a narrative. The structure of this narrative that gives rise to anticipatory aspects is destroyed by an arbitrary observation. A sequential description contains objective and non-objective elements and depends of the level of imbrication.

At macroscopic scale, the objective part of a sequential description is extracted by means of a «Peaceable Working Observer». He does not perturb the system because he knows and respects the structure of imbrication. Then, in simple cases, roles of system and observer can be permuted. The displacement of a rudimentary system in two-dimensional space or the non quantum harmonic oscillator are typical examples; these models use degenerate AR's. Surprising results appear when the narrative is dealing with a system moving in three-dimensional physical space: the solution involves either the use of spinors or an algebraic extension of the formalism.

However, the notion of «Peaceable Working Observer» is inadequate at the quantum level. We propose to extract objective descriptions by means of a «spinorial pilot» who works at the imbrication level $K/2$ when the physical system is described at the imbrication level K. This approach is applied to the quantum harmonic oscillator. Our starting point is nilpotent Lie algebra of dimension 3. A rule of recurrence is deduced from the sequential description. This rule is applied at the level K to an arbitrary point of a «linearized» one-dimensional lattice. Different «linearized» states, belonging to the same level K, are associated only to one point. Without additional information, this rule cannot give automatically certain states at other level $K' > K$. The «non deterministic part» of the spreading of states, deduced from this rule, appears to be a sequential description of the physical interaction. The deterministic part is a discrete approximation of a derivative operator. The whole description may be considered as a discretization of

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the Schrödinger equation. This conclusion results from a tedious analysis using an extension of Morse-Thue sequences. Bases of the formalism are presented in two books *Relateurs arithmétiques,* volumes 1 and 2 (Editions Belrepère, 1997). Some details are given in a recent book *Objectivité et pilotage spinoriel,* (same publisher, 2000, 443 pages). But there is no information in this book about relations between nilpotent Lie algebra and our model of quantum harmonic oscillator, nor is a comparison between non quantum and quantum models. These points are new.

Following these results, we suggest an analogy between peculiar sequential descriptions at quantum scale and DNA ribbons in living cells. This other point is new too. However, such a suggestion is deduced from arithmetical relators describing the environment by only one variable. The presence of an arbitrary observer increases the number of environment variables and destroys the imbrication structure. On the contrary, if this imbrication structure is preserved by hard chemical and geometrical constraints and if a peculiar observer respects them too, the «sequential description» might be observed. There is no inconsistency with the results of quantum physics because this peculiar observer can observe only what he knew before.

Keywords : Arithmetical relators; environment; Lie structures; fractals; anticipatory systems.

1 Introduction

The existence of «sequential descriptions» in physical systems, that could be analogous to DNA ribbons in living cells, has been suggested by a formalism which basically expresses the adaptation of a system to its environment. This formalism, named arithmetical relators, has been developed since 1973 by a research group at the Ecole Nationale Supérieure de Techniques Avancées (Paris, France). Our aim was to apprehend the behaviour and the evolution of natural systems, especially living ones, by mixing main results of physical theories, formal languages and computer techniques. Gradually, we have understood that the classical notion of differential element has to be prohibited at the starting point, because a differential element cannot contain a description of the whole as a cell of a living multi-cellular organism does it. Thus, we have abandoned the hypotheses of continuity and derivability.

Several mathematical theories have had a great influence on our approach, especially the theory of self-reproducing automata (Von Neumann, 1966) and the geometry of numbers (Minkowski, 1891). Arithmetical relators have been developed step by step in the framework of the General Systems Theory (Klir, 1969) by using an opening-closing dilemma. For this reason, most of the concepts introduced by this open formalism are new. At the present time, they are connected to root systems of Lie algebras, Catastrophe theory (R. Thom, 1972) and fractals (Mandelbrot, 1975) but cannot be reduced to the concepts defined in these theories.

The main theorems have been proved by several scientists, notably by Cl. Vallet, J.-P. Luminet, L. Nottale, M. Ferré, J. Chastang, F. Chauvet and Ph. Riot. A new terminology has been introduced only if necessary. Their work has been progressively presented in about hundred papers by twenty scientists and published in scientific journals or proceedings of scientific congress between 1971 and 1991. In order to simplify the collection of these papers and their study, we have gathered them with additional information in two books ([RA1] Moulin *et al.* 1997) and ([RA2] Moulin *et* *al.*, 1997), which can be bought in bookshops. We cannot summarize these books in 2 or 3 pages. For this reason, we mention pages of [RA1] and [RA2] to read for more details. Recent developments introduce only four new concepts, named «Peaceable Working Observer», «Spinorial Pilot», «Structural Loop» and «Space-Time-Imbrication reference frame». Some of their properties are given in a book ([RA3] Moulin, 2000), recently published and entitled *Objectivité et pilotage spinoriel*.

However, many scientists ignore this formalism and some of them would reduce the basic concepts to what they know. This is impossible unless they distort the meaning of these concepts. For instance, a Basic Cyclic Relator (BCR) has strong relations with the Killing form of a root system of Lie algebra, but it cannot be reduced to it because several BCR's are generally associated to a Dynkin graph. A BCR is supported by an extreme form (Coxeter, 1951) but it cannot be identified with an extreme form because it takes into account initial values and sequences of processes.

For applications in physics, biology and medicine, we use some new principles because classical space-time reference frames are not at the starting point of our approach. They are:

- the «structural adaptation of a system to its environment»,
- the «objectivity of sequential descriptions»,
- the existence of «structural loops in Space-Time-Imbrication references frames».

The third principle involves numerous and coherent applications in physics and biology at different level of organization, from the Planck level to the cosmic level. New effects are coming from coherent and non-classical relations between several levels of organization. At the limit, there is a connection with a fractal approach, concerning not only the anatomy of natural systems and their behaviour, but also the underlying structure of space-time, which is locally replaced by «Space-Time-Imbrication reference frames». Some analogies appear with the concept of fractal space-time (Nottale, 1993) and the theory of scale relativity (Nottale, 1998), due to Laurent Nottale. This astrophycisist worked in our group in 1980 and his fundamental results are often quoted. But, at the present time, his beautiful research on scale relativity is disconnected from his previous studies on arithmetical relators.

There is another reason why our approach may have new possibilities. Basically, we use two different links with Lie structure (that plays a fundamental role in physics). The first link deals with Basic Cyclic Relators. An underlying BCR describes an ideal structure of a system that is disconnected from its environment. This link is rigorous and placed at a very abstract level. A part of the quadratic form that supports the arithmetic relator may be consider as a Killing form for a peculiar choice of basis vectors. It can differ from the choice associated with the Dynkin graph of a Lie algebra. Therefore, the arithmetical properties of a BCR suggests the use of a differential formalism.

The second link is a consequence of the adaptation to the environment. It is placed at a very concrete level in opposition with the previous abstract level. We assume that the quadratic form locally defines the metric of a discrete model of space-time. The states of the system evolve by means of sequences of reflections. For peculiar sequences and for peculiar values of certain parameters, the states of the system are gathered in clusters, made up of sub-clusters, which themselves are made up of sub-sub-clusters and so on. A generalization of Lie structures, including fractal aspects, may appear by this way.

Basically, we assume a deep connection between both meanings of the quadratic form that supports an arithmetical relator. The key-point is the cristallographic meaning of the Weyl group of a root system of Lie algebra. This connection leads to «structural loops in

Space-Time-Imbrication references frames» and gives the reason why we dare suggest the existence of *«sequential descriptions»* in inanimate matter. In a first part, we recall some properties of arithmetical relators. The next part gives some consequences of the *«objectivity of sequential descriptions»* at the macroscopic scale. The last part is focused on the quantum harmonic oscillator.

2 Bases of Arithmetical Relators

2.1 Open Definition

A quadratic arithmetical relator is characterized by three data: a quadratic form with integer coefficients, named *support*, which defines the metric $g_{ij} = e_i \bullet e_j$; a set of *reflections* $\mathcal{H}_i$, which preserve this form; and *initial values* which are integers. The variables Z^i take only integer values. $\mathcal{H}_i$ is associated to the basis vector e_i and transforms Z^i into Z^{i*}, other components Z^j for $j \neq i$ being unchanged. Practically, Z^{i*} is given by the second root of the equation of second order (in Z^i), which is deduced from the support when other components Z^j, for $j \neq i$, are considered as constant. Z^{i*} is computed from the sum of roots since Z^i is known. The successive states evolve from the initial state by means of sequences of reflections. This «definition» is open because coefficients of the support, sequences of reflections and arithmetical structure of initial values are not completely defined. We complete progressively the model by a procedure of classification.

2.2 BCR and Root System of a Lie Algebra

A quadratic arithmetical relator is a BCR if the components of successive states are always integers whatever the initial values and the sequences of reflections will be. Moreover, if the same sequence of reflections is repeated over and over again, a BCR always produces a cyclic result. A BCR is directly associated to the root system of a Lie algebra. However, given the root system of a Lie algebra, usually we can deduce several BCR's from this root system [RA1, 6-7 and 60-73].

2.3 Stabilized Quadratic Arithmetical Relator (AR)

Stabilized quadratic arithmetical relators (AR) describe very simple systems adapted to their environment. We do not use any longer the symbols Z^i because the support of an AR contains two parts: one for the *underlying BCR* with main variables X^{μ}, which are related to the system, and the other for the *environment BCR* with variables V^{α}. The global support does not characterize a BCR because there are coupling terms proportional to products $X^{\mu}V^{\alpha}$ and additional peculiar terms which distort the metric of the underlying BCR. The initial state evolves by means of reflections related to the main variables or to the environment. The reflections related to the main variables are privileged. A reflection is never immediately repeated because reflections are involutions. A reflection related to the environment gives *always* integer components, but reflections related to the system may produces a *non-integer* component. In this case, the AR stops; in other words, there is a *blocking*.

In this case, the AR comes back to its previous state and applies a product of reflections related to the environment, named *call to the environment*. If the coupling and additional terms of the support have been chosen according to a fundamental theorem (Vallet *et al.*, 1976), summarized in footnote and developed in [RA1, 74-170], a new application of the previous reflection related to the system gives integer components. In other words, the blocking has been *suppressed* by this call to the environment.

The simplest support with two main variables X, Y and one environment variable V can be written as follows[1]:

$$\{P(X^2+Y^2) + D(AX+V)(BY+V)\} = \{-\}_o \quad (1)$$

P, D, A, B are non-null integers. $P > 0$. D, A, B have no common factor with P. According to a theorem of (Ferré, 1984) [RA1, 171-192], (X^2+Y^2) is the support of the *underlying BCR*, which is associated to the root system of Lie algebra $A_1 \oplus A_1$. The support is simplified for two reasons: the coefficients of X^2 and Y^2 are identical; and there is no coupling term XY in the support of the underlying BCR). The *canonical initial values* (2) are defined by the imbrication theorem (Luminet, 1980), [RA1, 197-235]). In the simplest case, these values introduce the exponent K, equal to the maximum number of main reflections that can be alternatively applied without call to the environment.

$$X(0) = x_o P^K, \quad Y(0) = y_o P^K, \quad V(0) = v_o P^K \quad (2)$$

The second simplification is due to the choice of the *class (0) of functioning* (defined in the proof of the imbrication theorem). The AR works in class (0) if the parameters x_o, y_o, v_o and coupling coefficients A, B respect the following blocking conditions:

$$(Ax_o + v_o) \not\equiv 0 \ (mod\ P), \qquad (By_o + v_o) \not\equiv 0 \ (mod\ P), \quad (3)$$

$$(A^2+B^2) \not\equiv 0 \ (mod\ P), \quad (4)$$

2.4 Notions of Time

The notion of adaptation to the environment is «open defined» without reference to the classical notion of space and time. Our models of natural systems may be presented as an harmonious coupling between two Lie structures. However, the result is not a Lie structure. It takes into account a narrative aspect.

1) A set of AR's may statically describe the external anatomy of a system at a particular level of organization. Each of them may be considered as a cell of a multicellular

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[1] The proof may be summarized as follows : A general quadratic form with three variables X, Y, V can be written : $PX^2+QY^2+\lambda XY+A'XV + B'YV+DV^2$. A reflection $\mathscr{H}_x$ transforms X into : $X^* = -X - (\lambda Y+A'V)/P$; Y and V are unchanged. X^* exists only if P is a non-null integer; it always can be chosen positive. For our example, we introduce the following simplification $Q = P$. A reflection $\mathscr{H}_v$ transform V into $V^* = -V - (A'X+B'Y)/D$; X and Y are unchanged. Since V is an environment variable, $\mathscr{H}_v$ always gives an integer result. Therefore A' and B' are divisible by D and can be written : $A'=DA$, $B'=DB$. The support becomes : $P(X^2+Y^2)+\lambda XY+D(AXV+BYV+V^2)$. According to the definition, main reflections $\mathscr{H}_x$ and $\mathscr{H}_y$ are alternatively applied until the components are integers. Let's suppose $\mathscr{H}_x$ has given an integer result and let's suppose the next reflection $\mathscr{H}_y$ $:Y^*=-Y-(\lambda X+DBV)/P$ (X and V unchanged) gives a non-integer result. Thus, $(\lambda X+DBV) \not\equiv 0\ (mod\ P)$. The AR comes back to the previous state (obtained just after $\mathscr{H}_x$). Since $\mathscr{H}_x$ has been applied and is an involution, $(\lambda Y+DAV) \equiv 0\ (mod\ P)$. We apply $\mathscr{H}_v$: V is transformed into $V^* = -V-AX-BY$ (X, Y unchanged). The AR is stabilized if $\mathscr{H}_y$ produce an integer result. Therefore, $(\lambda X+DBV^*) = (\lambda-DAB)X-DBV-DB^2Y$ must be divisible by P. Now, if A has no common factor with P, the inverse A^{-1} exists in the ring $\mathbb{Z}/(P\mathbb{Z})$. We can multiply $(\lambda Y+DAV) \equiv 0\ (mod\ P)$ by A^{-1} and $(\lambda X+DBV^*) \equiv (\lambda-DAB)(X+A^{-1}BY)\ (mod\ P)$. If $(\lambda-DAB) \equiv 0\ (mod\ P)$, the result is an integer. The same reasoning is applied when a blocking appears during $\mathscr{H}_x$. If B has no common factor with P, the condition $(\lambda-DAB) \equiv 0\ (mod\ P)$ guarantees the efficiency of $\mathscr{H}_v$. The simplest solution of this congruence is $\lambda = DAB$. Thus, the support becomes the expression (1).

organism. Its state depends on the choice of a peculiar sequence of reflections. For one given sequence and for one value of a particular variable playing the role of discrete *time coordinate* named X^4, this set of AR's generates a *«parallel description»* of the system. A change of this time coordinate produces another static parallel description. Typical examples are flower-singularities and insect-singularities obtained in 1979 by Cl. Vallet [RA2, 781-816]. Some of them are reproduced on the cover of volumes [RA1], [RA2].
2) A set of AR's may dynamically describe the internal behaviour of a system at a particular level of organization, between two successive values of the discrete time coordinate. This «local» dynamics is due to *coherent changes* of sequences of reflections. These changes may be interpreted as the expression of an *internal dynamical time*.
3) The coherence is due to an AR which sequentially describes the structure of the whole and introduces an imbrication of structure loops between different levels of organization. An integer parameter, named τ, *numbers* successive steps of this description, *i.e.* successive symbols or words of a narrative. τ appears to be a *narrative-time*.

2.5 Consequences of the Opening

1) A «sequential description» contains two informational parts. Only one may be considered as objective. The other part cannot be eliminated by means of a classical procedure because we study a «sequential» description, not a «parallel» one.
2) The structure of «sequential description» strongly depends on the number of environment variables. The simplest cyclic functionings bring to light regular imbrications. They appear with a single environment variable V. If the support contains several environment variables V^{α}, the functioning becomes extremely complicated and evokes chaotic behaviour unless new exponents K_{α}, associated with V^{α}, take the same value K. In this peculiar case, one may observe again regular imbrications.

To apply arithmetical relators in physics or in biology, we sketch two procedures. One of them is based on biquadratic AR's and leads to applications in mechanics at macroscopic scale; our starting point is the cristallographic meaning of a root system. The other procedure, using «linearized» AR's, finds applications at quantum scale. If we choose nilpotent Lie algebra of dimension 3 as the starting point, this approach leads to a new modelling of quantum harmonic oscillator, which is linked to the research of (W. Schempp, 1986) and of (D. Dubois, 1998, 1999). A link between results at the macroscopic and the quantum scale suggests properties which might be connected to applications of quantum holography, studied by (P. Marcer, W. Schempp, 2000).

3 Objective Part of a «Sequential Description»

The extraction of objective «sequential descriptions» at macroscopic scale is based on two theorems of L. Nottale, dealing with the conjugate of an AR (Nottale, 1982), [RA2, 501-548] and the notion of reduced sequence (Nottale 1981), [RA2, 423-469].

3.1 Notion of «Peaceable Working Observer»

We select objective states by means of a strange observer, named *«Peaceable Working Observer»*, who is represented by the conjugate of an AR and has the following properties:

- He does not perturb the system.
- He executes calls to the environment in «synchronism» with the system, *i.e.* his sequential functioning is identical to the one of the system.

- At each step of functioning, he selects a state of the system as *objective* if this state and his own state are located at the same place.

3.2 Emergence of an Objective Dyadic Tree

An AR $\mathfrak{R}$ with the support (1) is the first draft of a simple model of a macroscopic system. *A priori,* we attribute a crystallographic meaning to this support. The part of (1) that does not contain V defines the metric of the two-dimensional space where the system is located by coordinates X, Y.

According to L. Nottale, the conjugate AR $\mathfrak{R}'$ has the following support:

$$\{P(X'^2+Y'^2) + D(BX'+V')(AY'+V')\} = \{-\}_o \qquad (5)$$

It is easily deduced from (1): X, Y, V are respectively replaced by X', Y', V'. The coupling coefficients A and B are permuted. $\mathfrak{R}'$ represents the Peaceable Working Observer (PWO) of the system. We assume that the system and its PWO are located at the same geometrical point in a common reference frame when starts the objective description. This problem has been called *resetting* by M. Ferré. There are several basic possibilities. Each of them has a specific application in physics (Nottale, 1982) or in biology for applications to plant morphogenesis (Ferré, Le Guyader, 1984, 1998).

In practice, the PWO has a resetting defined by the following transformation $\mathcal{N}$:

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \mathcal{N} \begin{pmatrix} X' \\ Y' \end{pmatrix} \quad \text{with } \mathcal{N} = \frac{1}{\overline{\overline{P}}} \begin{pmatrix} AB(2P-DB^2) & P(A^2-B^2) \\ -P(A^2-B^2) & AB(2P-DA^2) \end{pmatrix} \; ; \quad V = V'$$

$$\mathcal{N}^{-1} = \frac{1}{\overline{\overline{P}}} \begin{pmatrix} AB(2P-DA^2) & -P(A^2-B^2) \\ P(A^2-B^2) & AB(2P-DB^2) \end{pmatrix} , \quad \overline{\overline{P}} = P(A^2+B^2) - DA^2B^2 \qquad (6)$$

The initial canonical values of $\mathfrak{R}'$ must be integers. Therefore, the initial canonical values of $\mathfrak{R}$ are proportional to $\overline{\overline{P}}$ since $\mathcal{N}^{-1}$ involve a division by $\overline{\overline{P}}$:

$$X(0) = x_o\, \overline{\overline{P}}\, P^K, \;\; Y(0) = y_o\, \overline{\overline{P}}\, P^K, \;\; V(0) = v_o\, \overline{\overline{P}}\, P^K \qquad (7)$$

L. Nottale proved that $\mathfrak{R}$ and $\mathfrak{R}'$ have the same class of functioning. Moreover, $\mathcal{N}^{-1}$ preserves the second member $\{-\}_o$. Let τ be the number of main reflections from the initial states. At every step τ, there is a couple of states produced by $\mathfrak{R}$ and $\mathfrak{R}'$. From step to step, the states of a couple are located at the same point in the common reference frame. In this case, the PWO can perceive the presence of the system. For this reason, this kind of couple is named *objective* in the framework of sequential descriptions. According to (3), (4) and (6), $\mathfrak{R}$ and $\mathfrak{R}'$ have a cyclic behaviour and belong to the same class (0) of functioning. For the underlying BCR defined by (1), all the states produced by $\mathfrak{R}$ or $\mathfrak{R}'$ are structured in a 3-adic tree, but the common states get organized in a dyadic tree (this point is proved in [RA3, 145-154]). This structure is directly displayed by $\mathfrak{R}$ and $\mathfrak{R}'$ for adequate values of $P, D, A, B, x_o, y_o, v_o$. *A priori,* this model may have many applications in botanic (ramification or nervation points) or anatomy of certain parts of animals (dendritic trees for instance). However, at first sight, it does not seem to deal with the description of simple physical systems, such as small grains of matter moving in space.

3.3 Displacement of a Pattern

To transpose in a «sequential description» what appears as a translation in a «parallel description», we add an additional assumption by using degenerate supports.Their effect is typical if the number of variables remains unchanged, because this choice drastically reduces the role of the environment (it is integrated inside the system). The condition of degeneracy, expressed by the determinant of (1) equal to 0,

$$P=(A^2+B^2) \quad , \quad D=4 \tag{8}$$

involves a change of the class of functioning because the blocking condition (4) is transgressed. L. Nottale solved this problem in [RA2, 635-686] for all the underlying BCR with two variables. There is no new blocking condition but (3) must be respected. The functioning becomes pseudocyclic and degenerated itself. This class of functioning is named *degenerate class (γ).* From a mathematical point of view, this new kind of AR produces orthogonal transvections (Artin, 1962, p. 158). According to a suggestion of L. Nottale, we transform «sequential descriptions» into «parallel description» as follows: Every successive pseudo-cycle describes the system as a pattern of states. The same pattern appears at successive mean positions. The jerky displacement of the pattern is due to a sequence of pseudocycles, each of them being considered as indivisible and repeated again and again. One indivisible pseudocycle defines the «Elementary Structured Displacement», ESD in abbreviated form.

This kind of ESD is the simplest one. It replaced the couple «mass-point & differential element» in elementary mechanics. Our formalism gives rise to other kinds of ESD too. One obtains them by increasing the number of variables. For instance, some limit ESD have a fractal content (J. Chastang, F. Chauvet [RA2, 687-779]). In the present paper, we do not examine the pertinence of these ESD's in physics. Our aim is limited to some remarks about the content of an EDS associated with degenerate class (γ).

3.4 Influence of the Parity of *K*

Let's apply the extraction of the objective pattern from the content of the ESD by using PWO technique. The result depends of the parity of *K*.

If K is *even,* the direct AR and its conjugate have common points in all the pseudo-cycles if the initial canonical states are located at the same point for $\tau = 0$. This property is proved in [RA3, 358-361]. In this particular case, the PWO technique may be replaced by a very simplified procedure: we need only to pick up a state every other time from the starting point (*i.e.* one point out of two). The common pattern is moving at constant velocity.

If K is odd and if the initial canonical states (defined for $\tau = 0$) are located at the same point, both patterns (one produced by the direct AR and the other by its conjugate) move in *opposite* directions. For $K = 3$, there are only two common points during the first pseudo-cycle. The result is very different if the states obtained for $\tau = 1$ are located at the same point: Both patterns move in the same direction, the initial canonical states appear to be different, and there are common points in each pseudo-cycle [RA3, 360-361].

Several canonical initial values that comply with the same second member may be «simultaneously» considered. Thus, we can define a finite set of AR's which have the same support, belong to the same class of functioning and introduce the same exponent *K*. They are therefore synchronous. A basic problem is dealing with the location of the different objective states in a common reference frame. The result highly depends on the parity of K. If K is even, the ESD is proportional to (Bx_o+Ay_o). It does not take into account v_o. If K is odd, the ESD is proportional to $(Ax_o+By_o+2v_o)$ [RA2, 666-668].

The physical interpretation of these results needs an extension in (3+1)-dimensional space. This problem is difficult because the existence of the conjugate involves either the use of algebraic integers or the introduction of additional quaternary quadratic forms associated to spinors of 3-dimensional space. This difficulty may be easily explained. If the support contains four main variables and one environment variable, the environment term of (1) is replaced by the product $4(A_1X_1+A_2X_2+A_3X_3+V)(BY+V)$ when the partition (3+1) is preserved. In the support of the conjugate, the corresponding term becomes in a simple case:

$$\left(\sqrt{\frac{B^2}{a^2}}\,(A_1X_1'+A_2X_2'+A_2X_2')+V'\right)\left(\sqrt{\frac{a^2}{B^2}}\;BY'+V'\right)\;;a^2=(A_1)^2+(A_1)^2+(A_1)^2 \quad (9)$$

If $A_2 = A_3 = 0$, the expression (9) leads to permute A and B. But a square root may appear if all the coupling are not null, because a^2 is not the square of an integer in general cases. Some details are given in [RA3, 399-433]. This problem will be analyzed in a next volume dealing with applications of AR's in physics. Nevertheless, we mention some preliminary results at the end of the present paper because they are in relation with the modelling of DNA.

3.5 Non Quantum Harmonic Oscillator

Let's come back to the two-dimensional problem. The system is described by (1), (2), (3) and (8). To simplify the discussion, we choose even values of K. The PWO can be forgotten since objective description is selected by taking one state out of two from the state at $\tau = 0$ (from the canonical initial values). If $(Bx_o+Ay_o) = 0$, the ESD is null. Thus, the AR describes the content of the ESD, in other words the stationnary solution. It is usefull to describe the objective stationnary states in a peculiar reference frame ξ, η, associated to the degenerate support by the following idendity:

$$(A^2+B^2)(X^2+Y^2)+4(AX+V)(BY+V)=\xi^2+\eta^2\;;\xi=AX+BY+2V,\quad \eta=BX+AY \quad (10)$$

We named $(\xi^2+\eta^2)$ the support of the *emergent* BCR, because it reflects the support of the underlying BCR: (X^2+Y^2) and produces a change of the level of description (the environment level compared to the system level). All the states (X, Y, V) are located on the circle $\xi^2+\eta^2 = \{-\}_o$ = constant. If we replace ξ by $\xi' = AX+BY$, the sequence of points (ξ', η) in the same reference frame are located on two circles which are stationnary if $\eta(0) = 0$. The circles are very close if a second condition, dealing with K, is verified.

Thus, in this particular case, the sequences of points ξ, η or ξ', η approximately give the same result. In first approximation, the displacement of this objective state is discret and regular along the circle. The axis ξ' is privileged since the pattern of points ξ', η have a mean displacement along this axis if the stationnary condition is not respected. This dissymetry suggests the following interpretation: ξ or ξ' may be considered as a position in a one-dimensional space; η appears to be a variable proportional to the impulse. Thus, the emergent reference frame becomes a phase space. This model described a pendular motion, which seems to be undamped. But it is not true actually. The stationnary pseudo-cycle contains two calls to the environment. One of them represents a small loss of energy, and the other provides the complementary energy supply.

To sum up, this model of non quantum harmonic oscillator has a degenerated support steming from (1) and produces a stationnary solution with minimum effects of calls to the environment. In the next paragraph, we will show that the quantum harmonic oscillator is based on the same support. But it is «linearized» instead of being degenerate.

4. Model of Quantum Harmonic Oscillator

4.1 Starting Point : Nilpotent Lie Algebra of Dimension 3

In the chapter 7 of (Schempp, 1986, p. 140-167), W. Schempp explains how the nilpotent Lie algebra of dimension 3 leads to quantum harmonic oscillator.Therefore we use this algebra as a starting point. According to J.-P. Serre (1964), «a non-abelian nilpotent Lie algebra of dimension 3 has a basis $\{X_1, X_2, X_3\}$ such that

$$[X_1, X_2] = X_3 \ , \ [X_1, X_3] = 0 \ , \ [X_2, X_3] = 0 \ \text{»}. \tag{11}$$

Let $C^{\lambda}_{\alpha\beta}$ be the structure constants. $\alpha,\beta,\lambda \in \{1,2,3\}$. According to (11),
$C^3_{12} = - C^3_{21} = 1$. The other structure constants $C^{\lambda}_{\alpha\beta}$ are null. (12)

Choice of new constants We use new values of the constants $C^{\lambda}_{\alpha\beta}$ that are close to the previous ones. The complementary terms are proportionnal to the rational number $\varepsilon = D/P$, which can be arbitrarily small. They verify classical relations $C^{\lambda}_{\alpha\beta} = - C^{\lambda}_{\beta\alpha}$ $\alpha,\beta,\lambda \in \{1,2,3\}$ but several relations deduced from Jacobi identity are not rigorously verified. Therefore, the new coefficients do not strictly characterize a Lie algebra. The number of new constants is reduced to the minimum insofar as all the coefficients of the new «Killing form» are different from zero.

$C^3_{12} = -C^3_{21} = 1$, $C^1_{12} = -C^1_{21} = 0$, $C^2_{21} = -C^2_{12} = 0$, $C^1_{13} = -C^1_{31} = 0$, $C^2_{23} = -C^2_{32} = 0$ are unchanged. We put : $C^1_{23} = -C^1_{32} = \varepsilon\ (1+\alpha\,\varepsilon)$, $C^2_{31} = -C^2_{13} = \varepsilon\ (1+\beta\,\varepsilon)$,

$$C^3_{23} = -C^3_{32} = -A\varepsilon \quad , \quad C^3_{31} = -C^3_{13} = -B\varepsilon$$

The coefficients α and β are chosen so that the coefficients g_{11} and g_{22} of the new «Killing form» are equal and independent of A and B.

These values are summarized in three matrices 3X3 giving $C^{\lambda}_{\alpha\beta}$ for λ constant :

$$\lambda = 1 \qquad\qquad \lambda = 2 \qquad\qquad \lambda = 3$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & (\varepsilon+\alpha\varepsilon^2) \\ 0 & -(\varepsilon+\alpha\varepsilon^2) & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 & -(\varepsilon+\beta\varepsilon^2) \\ 0 & 0 & 0 \\ (\varepsilon+\beta\varepsilon^2) & 0 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 1 & B\varepsilon \\ -1 & 0 & -A\varepsilon \\ -B\varepsilon & A\varepsilon & 0 \end{pmatrix} \tag{13}$$

Coefficients of the new «Killing form» These coefficients $g_{\alpha\beta}$ are deduced from the structure constants by means of the following relation, the summation being not subject to restrictions, see for instance (Wybourne, 1974, p. 60).

$$g_{\alpha\beta} = \sum\nolimits_{\lambda,\mu} C^{\lambda}_{\alpha\mu} C^{\mu}_{\beta\lambda} \tag{14}$$

It is more convenient to compute these coefficients (without a computer) by means of three other matrices 3X3 giving $C^{\lambda}_{\alpha\mu}$ for α constant. Index λ locates a row and index μ a column.

$\alpha = 1$; $\alpha = 2$; $\alpha = 3$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -(\varepsilon+\beta\varepsilon^2) \\ 0 & 1 & B\varepsilon \end{pmatrix} ; \begin{pmatrix} 0 & 0 & (\varepsilon+\alpha\varepsilon^2) \\ 0 & 0 & 0 \\ -1 & 0 & -A\varepsilon \end{pmatrix} ; \begin{pmatrix} 0 & -(\varepsilon+\alpha\varepsilon^2) & 0 \\ (\varepsilon+\beta\varepsilon^2) & 0 & 0 \\ -B\varepsilon & A\varepsilon & 0 \end{pmatrix}$$

$$g_{11} = -2\varepsilon + \varepsilon^2(B^2-2\beta) ; g_{22} = -2\varepsilon + \varepsilon^2(A^2-2\alpha) ; g_{33} = -2\varepsilon^2 -2(\alpha+\beta)\varepsilon^3 -2\alpha\beta\varepsilon^4$$
$$(g_{12}+g_{21}) = -2AB\varepsilon^2 \ ; \ (g_{13}+g_{31}) = -2A\varepsilon^2 -2\beta A\varepsilon^3 \ ; \ (g_{23}+g_{32}) = -2B\varepsilon^2 -2\alpha B\varepsilon^3$$

The development of these coefficients is limited to terms in ε^2. We choose α and β so that $g_{11} = g_{22}$. Therefore $\alpha = A^2/2$ and $\beta = B^2/2$. The coefficients of the «Killing form» become :

$$g_{11} = -2\varepsilon \ ; \ g_{22} = -2\varepsilon \ ; \ g_{33} = -2\varepsilon^2$$
$$(g_{12}+g_{21}) = -2AB\varepsilon^2 \ ; \ (g_{13}+g_{31}) = -2A\varepsilon^2 \ ; \ (g_{23}+g_{32}) = -2B\varepsilon^2 \qquad (16)$$

They are divided by -2ε ; ε is replaced by D/P. Finally, the quadratic form is multiplied by P. We assume that the form is equal to a constant. Hence we obtain the support (1) of the stabilized arithmetical relator (X_1 is replaced by X, X_2.by Y and X_3 by V). This AR can be «linearized» since D/P is arbitrarily small.

$$\{P(X^2 + Y^2) + D(AX+V)(BY+V)\} = \text{constant.} \qquad (17)$$

4.2 Some Definitions

Let ***i,r,V*** be respectively the reflections along axis *OX, OY, OV*. According to L. Nottale [RA2, 423-469], we name **0** = ***i*** or ***r***, **1** = ***iVr*** or ***rVi*** , **2** = ***iVrVi*** or ***rViVr***. The symbols **0**, **1**, **2** indicate the number of calls to the environment in these short sequences. A sequence of symbols **0**, **1**, **2** always represents a sequence of reflections ***i***, ***r***, ***V*** where ***i*** and ***r*** are alternated. For instance, **0 2** = ***irViVr*** or ***riVrVi***. Every state can be quickly obtained from the initial canonical state by means of a sequence of symbols **0**, **1**, **2**, possibly completed by an initial and/or final ***V***, named *reduced sequence*. The proof and the algorithm of sequence reduction are based on the following identities:

$$iVrViVr = rViVrVi = V \qquad (18)$$

with $i^2 = r^2 = V^2 = id$ (J.-P. Luminet [RA1, 212-213], M. Ferré [RA1, 175]). The first main reflection of a reduced sequence is identical to the one the AR starts with. A sequence that contains four main reflections (***i*** or ***r***) is called a *quadrisequence* if the first reflection or the last one are never ***V***. There are only 2^3-1 = 7 quadrisequences because of (18). These notions do not require the hypothesis of «linearization».

An AR $\mathfrak{R}$ is *«linearized»* if the development of the components *X, Y, V* is limited to terms of degree 1 in $\varepsilon = D/P$ and if its class of functioning is known. The states are gathered in 8 clusters. In the following, we consider only the cluster C^0 that contains the initial state.

4.3 Extraction of the Most Objective States of a Cluster

Possible use of an ordinary pilot: P takes very high values because D/P is close to 0. Therefore, many problems of computing appear even if K is a relatively small integer. Since the class of functioning of $\mathfrak{R}$ is known, we can use a synchronous arithmetical relator, named «ordinary pilot» 𝔓, which controls the calls to the environment of ℜ. The pilot is working with another positive integer value of parameter P, named P_{pilot}, which is the smallest one. Integers are essentially dealing with 𝔓 and the computations with a floating point may be used for ℜ since the states are «linearized». In other words, we divide the components of a state by P^K, then we select two parts in the development of each component as a function of D/P. Terms of low degree in D/P give the position of the state (approximation with rational numbers). Terms of high degree in D/P are taken into account by the pilot (approximation with p-adic integers).

Key-role of quadrisequences: Every state belonging to C^o can be deduced from the initial canonical state by a sequence of quadrisequences, possibly completed by initial and/or final V. We only consider the sequences which do not contain an initial V (the second half cycle is eliminated in class (0) of functioning), or a final V (this reflection does not change the main variables).

Use of reduced variables: L. Nottale studied the displacement of states in C^o (from the initial canonical state) by using the following reduced variables ([RA2, p. 455] or [RA3, p. 201]):

$$\hat{x} = \frac{X - X(0)}{(DA/P)\,P^K} \quad , \quad \hat{y} = \frac{Y - Y(0)}{(DB/P)\,P^K} \quad , \quad \hat{v} = \frac{V}{P^K} \quad ; \quad \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{v} \end{pmatrix} = \mathscr{A} \begin{pmatrix} DAx_o \\ DBy_o \\ v_o \end{pmatrix} \tag{19}$$

Thus, these reduced components are given by the adimensional components of $\mathscr{A}$ which only depend on the sequence of quadrisequences. These matrices $\mathscr{A}$ are computed from the seven quadrisequences when the sequence of quadrisequences is known. Nevertheless, we draw attention to the rule of multiplication: it is peculiar because the «linearization» does not give the same role to X, Y and to V (the factor of V^2 in the support (17) is very small compared to 1 in front of X^2 or Y^2, see [RA2, 444-453] and [RA3, 218-224]).

Choice of an underlying commutative geometry: The displacements $\hat{x}$, $\hat{y}$ due to the products $\mathscr{A}\mathscr{A}'$ and $\mathscr{A}'\mathscr{A}$ are generally different because $\mathscr{A}$ and $\mathscr{A}'$ do not generally commute. However, this non-commutativity is avoided if

$$|v_o| >> |DAx_o|, \ |DBy_o|. \tag{20}$$

Moreover, if the number of calls to the environment in $\mathscr{A}$ and $\mathscr{A}'$ is even, the global displacement becomes the sum of the vectorial displacements separately due to $\mathscr{A}$ and $\mathscr{A}'$. We adopt these two simplifications.

Selection of basis quadrisequences: According to the definition of the macroscopic sequential objectivity, all the reduced states computed by a «linearized» AR seem to be objective since matrices $\mathscr{A}$ do not depend on A, B, x_o, y_o, v_o. In order to select a subset of the states belonging to C^o, we attribute a basic role to the quadrisequences that produce the non-trivial lowest change $|\delta k|$ of valuation ($|\delta k|$ is equal to 2). This point is deeply studied in [RA3, 185-191]. The idea is to balance both kinds of «linearization», previously mentioned. This rule involves the selection of three quadrisequences: **02**, **11** and **20**. They contain two calls to the environment.

$$\mathbf{02} = \boldsymbol{irViVr} \text{ or } \boldsymbol{riVrVi} \quad , \ \mathbf{11} = \boldsymbol{iVriVr} \text{ or } \boldsymbol{rVirVi} \quad , \ \mathbf{20} = \boldsymbol{iVrVir} \text{ or } \boldsymbol{rViVri} \tag{21}$$

4.4 Use of a Spinorial Pilot

The ordinary pilot 𝔓 is poorly suited to the selection of objective states. Moreover, the computation time is very long. The coding emerging from the previous rule suggests another solution. It drastically reduces the difficulty of computing and the computation time. The idea is to suppress either the last symbol or the first one inside **02**, **11** and **20**. There is no lack of information if the reading direction and the choice of the suppressed symbol are defined once and for all. For instance, if we read from left to right and if we suppress the last symbol, **020211200220** gives without ambiguity **001202**, and we can deduce the original sequence from the compressed one.

However, this one-to-one relation disappears if we ignore the reading direction or the position of the suppressed symbol. This remark becomes important in the following. It has strong relations with the research of (Marcer, Schempp, 2000) on DNA coding and the research of (Dubois, 1998, 1999) on quantum harmonic oscillator. There is also a link with the thoughts of X. Sallantin, a pioneering philosopher who applies a «generalized arithmetic» to the living world (Sallantin, 1975, pp. 99-130).

We show in [RA3, 193-194] that all the quantum objective sequences can be automatically generated by another pilot, named *spinorial pilot* **𝔖𝔭**, working from a canonical initial state defined by exponent *K/2* instead of *K*. There are two possibilities:, **𝔖𝔭** replaces either **0** by **02**, **1** by **11** and **2** by **20**, or 0 by **20**, **1** by **11** and **2** by **02**. In the class (0) of functioning, we take into account all the reduced sequences produced by **𝔖𝔭** during the first half-cycle except those ended by ***V***.

To sum up, the ordinary pilot **𝔓** is eliminated and replaced by a spinorial pilot **𝔖𝔭**. It is strange that the objective part of the quantum states produced by **ℜ** is selected by the set of objective and non-objectve states produced by **𝔖𝔭**. Perhaps this result introduces some elements to solve the following basic problem: «Can we describe certain subjective states by means of a mathematical formalism, which is necessary objective?»

4.5 Space-Time-Imbrication Reference Frames

Level of a state: **𝔖𝔭** can easily associate to every quantum objective sequence a level h and two components $\hat{x}$, $\hat{y}$ related to **ℜ**. This level, which is between 0 and *K/2*, equals the length of the sequence. For instance, if $K = 6$, **𝔖𝔭** produces the following sequence of reduced sequences [RA2, 433]) not ended by ***V*** :

0, 00, 000, 001, 002, 01, 010, 011, 012, 00, 020, ... (22a)

For $h = 3$, we retain successively: **000, 001, 002, 010, 011, 012, 020, 021, 022, 100, 101, 102,** ..., **222**. This sequence represents in basis 3 the beginning of the natural sequence of integers. **𝔖𝔭** deduces the following sequence of the quantum objective sequences of **ℜ** (with the first possibility of coding):

020202, 020211, 020222, 021102, 021111, 021120, 022002,... (22b)

Components $\hat{x}$, $\hat{y}$ **:** In [RA3, 190-231], we prove the following result: Let *a, b, c* be respectively the number of **2**, **1**, **0** in a reduced sequence generated by **𝔖𝔭**. Due to (20), their respective positions have no effect on the «linearized» components $\hat{x}$, $\hat{y}$ of the associated state (generated by **ℜ**). The level h , $\hat{x}$ and $\hat{y}$ depend only on *a, b, c* [RA3, p. 231, Eq. 63]:

$$\hat{x} = a + c \quad ; \quad \hat{y} = -(b + c) \quad \text{with} \quad h = (a + b + c) \tag{23}$$

The components $\hat{x}$, $\hat{y}$ h locate a state in a three-dimensional lattice Λ.

Phase effects: The points generated by **ℜ** *via* **𝔖𝔭** have a phase given by the formula:

$$\phi \equiv [2k - (\hat{x} + \hat{y})] \frac{\pi}{2} \ mod\ (2\pi) \tag{24}$$

They are spread on four *separated* sublattices of Λ. These properties are proved in [RA3, 237-239].Thus, we can define a complex number $\Psi(\hat{x}, \hat{y}, k)$ (with integer components). The norm of $\Psi(\hat{x}, \hat{y}, k)$ is the number of visits at the point $(\hat{x}, \hat{y}, k)$ during the half-cycle. The phase of Ψ equals the phase associated to the point $(\hat{x}, \hat{y}, k)$.

4.6 Displacements in the Lattice Λ

If the first reflection applied to the initial canonical state is *i*, all the quadrisequences start by **i** too. In this case, we add to **02**, **11** and **20** the index *i*. The reading direction is chosen from left to right. The following displacements $(\hat{x}, \hat{y})$ are computed in [RA3, 218-227]: (1,0) for $(\mathbf{0\,2})_i$, (1,–1) for $(\mathbf{1\,1})_i$ and (0,–1) for $(\mathbf{2\,0})_i$. After a half-cycle, all the states are located inside a pyramid and on its faces. The vertex is at the level 0, and the basis at level $K/2$ (let's recall that K is even). This basis is an isoceles right-angled triangle. On the face containing the hypotenuses, the displacement at level h is produced by $[(\mathbf{0\,2})_i]^{h-c}\ [(\mathbf{2\,0})_i]^c$. At the same level, along the side parallel to Ox axis, the displacements are given by $[(\mathbf{1\,1})_i]^b\ [(\mathbf{2\,0})_i]^{h-b}$; and along the third side (parallel to Oy), by $[(\mathbf{0\,2})_i]^a\ [(\mathbf{1\,1})_i]^{h-a}$. In [RA3, 229-232], we prove that 𝕽 (drived by 𝕾𝔭) automatically produces a trinomial distribution in a section parallel to the basis. The distribution is binomial on each of the three faces. The trinomial coefficient $(h;a,b,c)$ gives the number of states located at $(\hat{x}, \hat{y}, h)$ as a function of a,b,c computed from (23):

$$(h;a,b,c) = \frac{h!}{a!\ b!\ c!} \qquad \text{with} \qquad h = (a+b+c) \tag{25}$$

Let s be the centered coordinate along the hypothenuse at the level h. The limit distribution is gaussian for a renormalized abscissa $\zeta = 2s/\sqrt{h}$.

Moreover, the successive points may be alternatively affected by a phase equal to 0 or π $(mod\ 2\pi)$. In this case, the couples (number of states – phase) are alternatively represented by positive or negative integers. Let's consider, at level h, the displacement of a state along the hypothenuse. It is irregular because we ignore the other states appearing inside the pyramid. Fortunately, the use of generalized Morse-Thue sequences [RA3, 274-286] makes easier the analysis of these displacements. An imbrication is brought to light between levels h and h+2. The rule of imbrication is made of two parts: The first one does not depend on h and appears to be deterministic. By taking the phase into account, it is interpreted as a second discrete derivative. The second part seems to be locally «unforeseeable» for an observer who has a short memory. A natural idea is to compute for a given point the sum Σ of local «unforeseeable» changes of position, leading to this point during a half-cycle.

A priori, Σ might be interpreted as the product of displacements by a force proportional to the distance ζ of the point to the center of the distribution. But we cannot obtain with a single coding a limit value proportional to ζ^2 when h becomes greater and greater. Both codings, producing Σ^+ and Σ^-, must be used. The correct potential is associated to $(\Sigma^+ + \Sigma^-)$. Moreover, the classical creation and annihilation operators are deduced from a rule of imbrication between levels h and $(h+1)$ by means of the difference $(\Sigma^+ - \Sigma^-)$ [RA3, 292-300]. This result might lead to anticipatory effects since the second coding is defined by a change of the reading direction. This point of view seems to agree with the use of «Forward and Backward Discrete Derivative» by (Dubois, 1999).
Thanks to phase effect, we can apply successive discret derivative of order n to the basic state. The wave functions of quantum harmonic oscillator are obtained by this way. Thus the AR is used with two basic roles: First, it produces at the limit a gaussian distribution for K becoming greater and greater. Secondly, by taking phase effects into account, it computes the wave functions with finite values of K, which may be small.

5 Conclusion

This modelling of quantum harmonic oscillator is sketched out from an approximation of nilpotent Lie algebra of dimension 3. We use a «linearized» AR, which is drived by a spinorial pilot. The support of the model and the support dealing with the non-quantum harmonic oscillator are identical, but the second one is degenerated. Another «linearization» in the neighbourhood of the degenerated support seems to be useful too [RA2, 482-483]. Therefore, the link between non-quantum and quantum models requires further development. The novelty comes from the underlying dynamics, which may give rise to anticipatory effects if the imbrication structure is preserved (a classical observer destroys this structure). A sequential description may be extracted from the imbrication rule; it is related to the Schrödinger equation, at least for the basic wave function. For this reason, we suggest an analogy with DNA ribbons in living cells.

Moreover, four other models using AR can be linked to nilpotent Lie algebra of dimension 3 by means of two pairs of additional small coefficients of structure. The macroscopic models are based on corresponding supports. They underlying BCR's are different and coupled by pairs. They might be associated to A-T and G-C[2] pairs. The fifth solution is a generalization of the support used in the present paper. It might be associated to uracil in RNA. These remarks seem to be in good agreement with recent research about DNA (Marcer, Schempp, 2000), (Gariaev *et al.*, 2000), and other development (Marcer, Schempp, 1998), (Clement *et al.*, 1999).

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[2] A for *Adenine*, T for *Thymine*, G for *Guanine*, C for *Cytosine*.

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