Anticipation and Hyperincursion in Belief Formation Based on Evidence

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Abstract
Characteristics of anticipation are considered in belief formation based on crisp evidence. In this paper, anticipation is understood as an ability of the system affording us some useful guide to seek further pieces of evidence in order to give a solution for a given problem with which we are concerned. Then two kinds of anticipation are examined in the belief formation systems depending on whether evidence is incomplete or contradictory, respectively. This examination assumes the closed-world assumption on belief formation. Further kind of anticipation is pointed out under the open-world assumption.
Keywords : Anticipation, hyperincursion, belief formation, Dempster-Shafer theory of evidence, logic of belief

1 Introduction

The notion of (strong) anticipation has been advocated and developed by Prof. Daniel M. Dubois (cf. [4, 5]) and, recently, many researchers have paid attention to the notion in various kinds of systems like biosystems and physical systems. In our previous article[10], we tried to make consideration on what kind of anticipation is found in belief formation based on evidence, where we use the term 'evidence' in the sense of Dempster-Shafer theory (cf. Shafer[12]). There, first, we constructed a belief formation system using fuzzy-measure-based possible-worlds models[6, 7, 8] based on evidence along time. The purpose of collecting evidence is to have a more limited number of possible worlds, so that we can obtain some conviction with respect to a given problem with which we are concerned. Ultimately, if possible, we want one uniquely limited world, which describes a solution of a given problem. Then, degrees of such limitation on the set of possible worlds suggest us which evidence we should search for at the next step. Our conclusion in the previous article[10] was that an ability of the systems affording such suggestion was a characteristic of anticipation in the belief formation systems.

Now, unfortunately, we feel that the former conclusion is insufficient. In our previous work, we also pointed out another problem in the proposed belief formation systems. That is, the previous way of forming belief does not work in cases that pieces of evidence are themselves inconsistent. For example, Shafer[12](p.223), in fact, pointed out that such *dissonance* is always a symptom of some mistake in assessing the evidence. Thus we need some way of extracting a subset or subsets of pieces of evidence from the original collection using, e.g., *maximal consistency* like in, e.g., [9].

In this article, based on such observation, we introduce two kinds of anticipation, which correspond to, respectively, *incompleteness* and *inconsistency* of evidence. For our discussion being clear, we confine ourselves to belief formation based on *crisp* evidence.

2 Belief Model Based on Crisp Evidence

In [10], we used belief models based on evidence, where pieces of evidence are represented by *basic probability assignments* in Dempster-Shafer theory (cf. [12]). As mentioned, however, in Introduction, we will confine ourselves to belief formation based on *crisp* evidence in order that we can capture the essential point of anticipation in the kind of belief formation systems.

Given a *finite* or *countably infinite* set of atomic sentences $\mathcal{P}$, a language $\mathcal{L}(\mathcal{P})$ for logic of belief is formed in the usual way from $\mathcal{P}$ with the well-known propositional operators such as $\top$ (the truth constant), $\bot$ (the falsity constant), $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (material implication), $\leftrightarrow$ (equivalence), and two modal operators B ((*weak*) *belief*) and C (*conviction*, or *firm belief*).

Definition 1 A *belief model based on crisp evidence* is defined as

$$\mathcal{M} = <W, B>, \tag{1}$$

where

1. W is the power set of $\mathcal{P}$, i.e., $W = 2^{\mathcal{P}}$,
2. B is a non-empty subset in W. ■

Any element in W is called a *possible world*, or simply, *world.*

For a world w in W and an atomic sentence p in $\mathcal{P}$, the truth condition for atomic sentences is given by

$$\mathcal{M}, w \models p \text{ (read as '}p\text{ is true at }w\text{ in }\mathcal{M}\text{')} \text{ iff } p \in w. \tag{2}$$

This definition can be extended in the usual way for any compound sentence that includes logical connectives except for the two modal operators. Given a belief model $\mathcal{M}$, truth conditions for belief and conviction sentences is defined using the subset B in $\mathcal{M}$ in the following way.

Definition 2 Given a belief model $\mathcal{M} = <W, B>$ based on crisp evidence,

$$\mathcal{M}, w \models \mathsf{B}p \quad \text{iff} \quad ||p||^{\mathcal{M}} \cap B \neq \emptyset, \tag{3}$$
$$\mathcal{M}, w \models \mathsf{C}p \quad \text{iff} \quad B \subseteq ||p||^{\mathcal{M}}, \tag{4}$$

where $||p||^{\mathcal{M}} \stackrel{\text{def}}{=} \{w \mid \mathcal{M}, w \models p\}$, which is called the *truth set*, or *proposition* of p in $\mathcal{M}$. ∎

A sentence p is said to be *valid* in $\mathcal{M}$, written $\mathcal{M} \models p$, when $\mathcal{M}, w \models p$ for every world w in W. By the definition, we can understand B as the possibilistic counterpart of C:

$$\mathcal{M} \models \mathsf{B}p \leftrightarrow \neg\mathsf{C}\neg p. \tag{5}$$

So the readers may think we do not have to introduce B. Note that, however, the equivalence in formula (5) no longer holds in the general non-crisp setting of evidence as in our previous work[10].

3 Belief Formation System Based on Crisp Evidence

3.1 Universe and Elementary Possible Worlds

Let $\mathcal{P}$ be a finite or countably infinite set of atomic sentences with which we possibly deal in a given problem that we should form belief. And let

$$\mathcal{W} \stackrel{\text{def}}{=} 2^{\mathcal{P}}. \tag{6}$$

We call $\mathcal{W}$ the *universe* with respect to the problem. An element in $\mathcal{W}$ is called an *elementary* possible world.

Example 3 For some murder case, say, in Japan, let

$$\mathcal{D} = \{x_1, x_2, \cdots, x_n\}$$

be a finite set of persons who live now in Japan. For a person x_i, let p_i mean that x_i is a criminal for the case and let

$$\mathcal{P} = \{\mathsf{p}_1, \mathsf{p}_2, \cdots, \mathsf{p}_n\}.$$

Since $\mathcal{W} = 2^{\mathcal{P}}$, any subset in $\mathcal{P}$ can be an elementary world like, for instance,

1. $\mathcal{P}$ itself is the world where all of $x_1, \cdots, x_n$ are criminals.

2. $\{\mathsf{p}_1, \mathsf{p}_2\}$ is the world where both x_1 and x_2 are criminals (and the others are not).

3. $\{\mathsf{p}_1\}$ is the world where only x_1 is a criminal (and the others are not).

4. $\emptyset$ is the world where none of $x_1, \cdots, x_n$ are criminals.

and so on. ■

In general, given a set of atomic sentences $\mathcal{P} = \{\mathsf{p}_1, \cdots, \mathsf{p}_m\}$, for a possible world w in $2^{\mathcal{P}}$, following Carnap[2], we can define its *state description* p_w as the following conjunctive sentence:

$$p_w = \pi_1 \wedge \cdots \wedge \pi_m, \tag{7}$$

where

$$\pi_i = \begin{cases} \mathsf{p}_i, & \text{if } \mathsf{p}_i \in w, \\ \neg \mathsf{p}_i, & \text{otherwise.} \end{cases}$$

Example 4 In our murder case, we have

1. $p_{\mathcal{P}} = \mathsf{p}_1 \wedge \cdots \wedge \mathsf{p}_n$
2. $p_{\{\mathsf{p}_1, \mathsf{p}_2\}} = \mathsf{p}_1 \wedge \mathsf{p}_2 \wedge \neg \mathsf{p}_3 \wedge \cdots \wedge \neg \mathsf{p}_n$
3. $p_{\{\mathsf{p}_1\}} = \mathsf{p}_1 \wedge \neg \mathsf{p}_2 \wedge \cdots \wedge \neg \mathsf{p}_n$
4. $p_{\emptyset} = \neg \mathsf{p}_1 \wedge \cdots \wedge \neg \mathsf{p}_n$

and so on. ■

3.2 Belief Formation Systems

We can formulate in the following way a belief formation system $\mathcal{S}$ with crisp evidence.

Definition 5 A *belief formation system $\mathcal{S}$ based on crisp evidence* is

$$\mathcal{S} \stackrel{\text{def}}{=} \{S_t \mid t = 0, 1, 2, \cdots\}, \tag{8}$$

where

1. $S_t = <E_t, \mathcal{M}_t>$,
2. E_t is a finite set of non-modal sentences given at t,
3. $\mathcal{M}_t = <W_t, B_t>$ is a belief model based on crisp evidence at a point of time t. ■

At each time t, given E_t, the set of atomic sentences that appear in E_t is defined by

$$\mathcal{P}^t \stackrel{\text{def}}{=} \mathcal{P} \cap \bigcup_{p \in E_t} \text{sub}(p), \tag{9}$$

where sub(p) is the set of subsentences of p. Then the following set defined by

$$\mathcal{P}_t \stackrel{\text{def}}{=} \bigcup_{0 \leq t' \leq t} \mathcal{P}^{t'} \tag{10}$$

represents the set of all atomic sentences that have occurred until t.

Since, given $\mathcal{P}_t$ at t, we do not have to concern the atomic sentences in $\mathcal{P} \setminus \mathcal{P}_t$, what we should deal with is the set of worlds that are related with the atomic sentences in $\mathcal{P}_t$. This idea is realized by introducing the quotient set

$$W_t \stackrel{\text{def}}{=} \mathcal{W}/R_t, \tag{11}$$

where R_t defined by

$$wR_tw' \text{ iff } [\text{for any } \mathsf{p} \in \mathcal{P}_t \ (\mathsf{p} \in w \text{ iff } \mathsf{p} \in w')], \tag{12}$$

which is obviously an equivalence relation on W_t. Because $\mathcal{P}_t$ is a finite set, so is W_t and, in fact, it is isomorphic with $2^{\mathcal{P}_t}$:

$$W_t \simeq 2^{\mathcal{P}_t}. \tag{13}$$

In the following, by this isomorphism, we identify W_t with $2^{\mathcal{P}_t}$.

The set B_t is defined as the set of possible worlds where any sentence which we have obtained as evidence is true:

$$B_t \stackrel{\text{def}}{=} \cap \{\|p\|^{\mathcal{M}_t} \mid p \in \bigcup_{0 \leq t' \leq t} E_{t'}\}. \tag{14}$$

Some examples will be illustrated later.

The purpose of forming belief by collecting evidence is to have a more limited number of worlds as possible as we can, so that we can obtain some conviction with respect to the problem with which we are concerned. A more limited number of worlds means *less* uncertainty with respect to the given problem. Thus the ultimate objective of belief formation systems is to reach the unique world w in W_t such that $B_t = \{w\}$.

Then we can introduce the well-known quantity of information with normalization as an index of such limitation:

$$I(S_t) \stackrel{\text{def}}{=} -\frac{1}{|\mathcal{P}_t|} \log_2 \frac{|B_t|}{|W_t|}. \tag{15}$$

By this definition, if there is no information (totally unknown), then $B_t = W_t$, so $I(S_t) = 0$. If we reach one world as such limitation, then $I(S_t) = 1$.

3.3 Initial State

In advance, we must describe the initial state

$$S_0 = <E_0, \mathcal{M}_0>$$

at $t = 0$. There are two possible formulations as the initial state for a given problem.

In the first formulation, we regards the initial state as a *totally unknown* one as in our previous article[10]. Hence we may take $E_0 = \emptyset$, which means no evidence and thus totally unknown. Then the set of world W_0 at $t = 0$ is defined by

$$W_0 = 2^{\mathcal{P}_0}, \tag{16}$$

where $\mathcal{P}_0 = \mathcal{P}$, so W_0 is just the universe $\mathcal{W}$. The set B_0 is defined as the set of possible worlds where any sentence in E_0 is true:

$$B_0 \stackrel{\text{def}}{=} \cap \{||p||^{\mathcal{M}_0} \mid p \in E_0\}. \tag{17}$$

So, actually, $B_0 = W_0$ when $E_0 = \emptyset$.

In the second formulation, we regard the initial state and thereby definitions of E_0 and $\mathcal{P}_0$ as being affected by a way of establishing a framework of a given problem. When E_0 is not an empty set, any piece of evidence in E_0 should have a special status in comparison with other pieces of evidence which shall be later obtained by observation and so on. We shall illustrate this kind of formulation.

Example 6 In our murder case, the following fundamental assumption should have been made, in advance, before starting belief formation:

p_0^1: There exists a criminal for the case

If this assumption does not hold, forming belief in the case does not make any sense. Although we treat it as one of pieces of evidence, it should have the special status because it is related to the essential point of the given problem, that is, investigation of a criminal for the case. The assumption p_0^1, in general, can be identified with that the set of criminals is not empty, that is,

$$p_0^1 = \neg p_\emptyset.$$

Note that, as we shall see later, the sentence p_0^1 has actually different interpretations depending on the set of atomic sentences at each time. Thus

$$E_0 = \{p_0^1\}.$$

Now, suppose we have two suspects x_1 and x_2. Then, besides the two, for a while, we do not have to pay attention to others in the set D of persons. Thus we can take the set of atomic sentences at the initial state as

$$\mathcal{P}_0 = \{\mathsf{p}_1, \mathsf{p}_2\}.$$

Then, a relation R_0 is defined by

$$wR_0w' \text{ iff } [(\mathsf{p}_1 \in w \text{ iff } \mathsf{p}_1 \in w') \text{ and } (\mathsf{p}_2 \in w \text{ iff } \mathsf{p}_2 \in w')],$$

thus we have the following four worlds

$$W_0 = \mathcal{W}/R_0 = \{w_0^1, w_0^2, w_0^3, w_0^4\}$$

is obtained where each element is an equivalence class as follows shown with its state description:

$w_0^1 = \{w \in \mathcal{W} \mid \mathsf{p}_1 \in w, \mathsf{p}_2 \in w\} \simeq \{\mathsf{p}_1, \mathsf{p}_2\}$,
with $p_{w_0^1} = \mathsf{p}_1 \wedge \mathsf{p}_2$. (Both x_1 and x_2 are criminals.)

$w_0^2 = \{w \in \mathcal{W} \mid \mathsf{p}_1 \in w, \mathsf{p}_2 \notin w\} \simeq \{\mathsf{p}_1\}$,
with $p_{w_0^2} = \mathsf{p}_1 \wedge \neg \mathsf{p}_2$. ($x_1$ is a criminal but not x_2.)

$w_0^3 = \{w \in \mathcal{W} \mid \mathsf{p}_1 \notin w, \mathsf{p}_2 \in w\} \simeq \{\mathsf{p}_2\}$,
with $p_{w_0^3} = \neg \mathsf{p}_1 \wedge \mathsf{p}_2$. ($x_1$ is not a criminal but x_2 is.)

$w_0^4 = \{w \in \mathcal{W} \mid \mathsf{p}_1 \notin w, \mathsf{p}_2 \notin w\} \simeq \emptyset$,
with $p_{w_0^4} = \neg \mathsf{p}_1 \wedge \neg \mathsf{p}_2$. (Neither x_1 nor x_2 is a criminal.)

Exactly one of those four worlds is expected to be a partial description of the actual world, that is, the solution of the case.

In the model, note that $||p_\emptyset||^{\mathcal{M}_0} = \{w_0^4\}$. Then we can make the following computation:

$$||p_0^1||^{\mathcal{M}_0} = ||\neg p_\emptyset||^{\mathcal{M}_0} = W_0 \setminus ||p_\emptyset||^{\mathcal{M}_0} = W_0 \setminus \{w_0^4\} = \{w_0^1, w_0^2, w_0^3\} = ||\mathsf{p}_1 \vee \mathsf{p}_2||^{\mathcal{M}_0},$$

thus the following equivalence holds:

$$\mathcal{M}_0 \models p_0^1 \leftrightarrow (\mathsf{p}_1 \vee \mathsf{p}_2).$$

Finally, we have

$$B_0 = ||p_0^1||^{\mathcal{M}_0} = \{w_0^1, w_0^2, w_0^3\}.$$

and

$$I(S_0) = -\frac{1}{2} \log_2 \frac{3}{4} \approx 0.21. \quad \blacksquare$$

4 Anticipation in Belief Formation Systems

In this article, we understand *anticipation* in the belief formation system as an ability of the system affording us some useful guide to seek further pieces of evidence in order to give a solution for a given problem with which we are concerned. The main point is not that we give the system some goal from the outside, but that the system can create some potential solutions by itself.

There are two cases where we need anticipation in belief formation:

1. Evidence is *incomplete.*
2. Evidence is *contradictory.*

In this paper, we call them *anticipation of type I,* and *of type II,* respectively.

4.1 Incomplete Evidence: Anticipation of Type I

In this article, evidence is said to be *incomplete* at t if the resulting set B_t of worlds is not a singleton. Thus further pieces of evidence may provide us a more limited set $B_{t'}$ of worlds at $t'(>t)$.

Example 7 At $t=1$, let us assume we have the evidence that supports

- p_1^1: The case is committed by an individual.

Since, at $t=1$, we do not have explicitly any other atomic sentence besides ones in $\mathcal{P}_0$, we have $\mathcal{P}_1 = \mathcal{P}_0$ and thus $W_1 = W_0$. So we have two singletons, $\{\mathsf{p}_1\}$ and $\{\mathsf{p}_2\}$, each of which represents an individual, thus

$$\|p_1^1\|^{\mathcal{M}_1} = \|p_{\{\mathsf{p}_1\}} \vee p_{\{\mathsf{p}_2\}}\|^{\mathcal{M}_1} = \|(\mathsf{p}_1 \wedge \neg \mathsf{p}_2) \vee (\neg \mathsf{p}_1 \wedge \mathsf{p}_2)\|^{\mathcal{M}_1} = \{w_1^2, w_1^3\}.$$

Then

$$B_1 = \|p_0^1\|^{\mathcal{M}_1} \cap \|p_1^1\|^{\mathcal{M}_1} = \{w_1^2, w_1^3\},$$

and

$$I(S_1) = -\frac{1}{2}\log_2\frac{2}{4} = 0.5. \quad \blacksquare$$

It is, in general, insufficient that we are negatively waiting for new evidence. We should try to look for new evidence in a positive way. The anticipation of type I was already pointed out in our previous article[10]. This type of anticipation requires us to look for further evidence *outside* the current system. The ability of anticipation of type I would prevent us from making *random* search for new evidence. In the following example, we describe the point by introducing an index, which was not dealt with in our previous article[10],

Example 8 In our system at $t = 1$, we have the following two potential solutions:

$w_1^2 = \{w \in \mathcal{W} \mid \mathsf{p}_1 \in w, \mathsf{p}_2 \notin w\} \simeq \{\mathsf{p}_1\}$,
with $p_{w_1^2} = \mathsf{p}_1 \wedge \neg \mathsf{p}_2$. ($x_1$ is a criminal but not x_2.)

$w_1^3 = \{w \in \mathcal{W} \mid \mathsf{p}_1 \notin w, \mathsf{p}_2 \in w\} \simeq \{\mathsf{p}_2\}$,
with $p_{w_1^3} = \neg \mathsf{p}_1 \wedge \mathsf{p}_2$. ($x_1$ is not a criminal but x_2 is.)

To reach the first potential solution $\{w_1^2\}$, we must look up all of possible pieces of evidence $\tilde{p}$ such that the intersection of $\|\tilde{p}\|^{\mathcal{M}_1}$ and B_1 becomes $\{w_1^2\}$. They are the following three:

1. $\tilde{p}_1^1 = \mathsf{p}_1$: x_1 is a criminal. ($\|\mathsf{p}_1\|^{\mathcal{M}_1} = \{w_1^1, w_1^2\}$)
2. $\tilde{p}_1^2 = \neg \mathsf{p}_2$: x_2 is not a criminal. ($\|\neg \mathsf{p}_2\|^{\mathcal{M}_1} = \{w_1^2, w_1^4\}$)
3. $\tilde{p}_1^3 = \mathsf{p}_2 \rightarrow \mathsf{p}_1$: Whenever x_2 commits a crime, he always makes it with x_1. ($\|\mathsf{p}_2 \rightarrow \mathsf{p}_1\|^{\mathcal{M}_1} = \{w_1^1, w_1^2, w_1^4\}$)

To begin with, which pieces of evidence should we try to look for ? We should choose weaker evidence because we can easily seek such evidence to reach the same solution with less effort. For the purpose, the quantity of information is also useful. Here we introduce the following index using the quantity:

$$e(X) \stackrel{\text{def}}{=} 1 - \left(-\frac{1}{|P_t|} \log_2 \frac{|X|}{|W_t|}\right).$$

Then we have

1. $e(\|\mathsf{p}_1\|^{\mathcal{M}_1}) = 1 - (-\frac{1}{2} \log_2 \frac{2}{4}) = 0.5$
2. $e(\|\neg \mathsf{p}_2\|^{\mathcal{M}_1}) = 1 - (-\frac{1}{2} \log_2 \frac{2}{4}) = 0.5$
3. $e(\|\mathsf{p}_2 \rightarrow \mathsf{p}_1\|^{\mathcal{M}_1}) = 1 - (-\frac{1}{2} \log_2 \frac{3}{4}) \approx 0.79$

Thus we can seek evidence in the following order

$$\tilde{p}_1^3 \succ \{\tilde{p}_1^1, \tilde{p}_1^2\},$$

thus, by this anticipation, the belief formation system can suggest us to look for $\tilde{p}_1^3$, that is, **'Investigate whether x_2 always makes it with x_1 or not'.** Note that, intuitively, the first and second ones are not so useful because they are themselves almost nearly the potential solution.

Similarly, to reach the second potential solution $\{w_1^3\}$, possible pieces of evidence are

1. $\tilde{p}_2^1 = \neg(\mathsf{p}_1 \leftrightarrow \mathsf{p}_2) = (\mathsf{p}_1 \wedge \neg \mathsf{p}_2) \vee (\neg \mathsf{p}_1 \wedge \mathsf{p}_2)$: Either x_1 or x_2 is a criminal. ($\|\neg(\mathsf{p}_1 \leftrightarrow \mathsf{p}_2)\|^{\mathcal{M}_1} = \{w_1^2, w_1^3\}$)

2. $\tilde{p_2^2} = \neg \mathsf{p}_1$: x_1 is not a criminal. $(||\neg \mathsf{p}_1||^{\mathcal{M}_1} = \{w_1^3, w_1^4\})$

3. $\tilde{p_2^3} = \neg(\mathsf{p}_1 \wedge \mathsf{p}_2)$: It is not the case that both x_1 and x_2 are criminals together. $(||\neg(\mathsf{p}_1 \wedge \mathsf{p}_2)||^{\mathcal{M}_1} = \{w_1^2, w_1^3, w_1^4\})$

Then we can seek evidence in the following order

$$\tilde{p_2^3} \succ \{\tilde{p_2^1}, \tilde{p_2^2}\}.$$

Thus, by this anticipation, the belief formation system can suggest us to look for $\tilde{p_2^3}$, that is, **'Investigate whether x_1 and x_2 are always criminals together or not'.** ■

We have multiple potential solutions by anticipation, so the belief formation systems with anticipation of type I are *hyperincursive*.

4.2 Contradictory Evidence: Anticipation of Type II

In this article, evidence is said to be *contradictory* at t if the resulting set B_t of worlds is empty. In contradictory cases, we can also find anticipation we call *type II*. It requires us to reexamine the evidence obtained *inside* the current belief formation system.

Example 9 At $t = 2$, assume that we obtain the following two pieces of evidence:

- $p_2^1 = \mathsf{p}_1 \rightarrow \mathsf{p}_2$: If x_1 commits a crime, then he always makes it with x_2.
- $p_2^2 = \neg \mathsf{p}_2$: x_2 has an alibi.

Thus, $E_2 = \{p_2^1, p_2^2\}$. Then, $\mathcal{P}_2 = \mathcal{P}_1$ and $W_2 = W_1$. Their propositions in $\mathcal{M}_2$ are, respectively,

$$||p_2^1||^{\mathcal{M}_2} = ||\mathsf{p}_1 \rightarrow \mathsf{p}_2||^{\mathcal{M}_2} = \{w_2^1, w_2^3, w_2^4\}$$
$$||p_2^2||^{\mathcal{M}_2} = ||\neg \mathsf{p}_2||^{\mathcal{M}_2} = \{w_2^2, w_2^4\}$$

Thus, we have

$$B_2 = ||p_0^1||^{\mathcal{M}_2} \cap ||p_1^1||^{\mathcal{M}_2} \cap ||p_2^1||^{\mathcal{M}_2} \cap ||p_2^2||^{\mathcal{M}_2} = \emptyset.$$

One promising way is to choose maximal consistent subsets from $E_0 \cup E_1 \cup E_2$ such that they contain p_0^1 because, as already explained, p_0^1 has the special status. Then we have the following two subsets:

1. $E' = \{p_0^1, p_1^1, p_2^1\}$, where $\cap\{||p||^{\mathcal{M}_2} \mid p \in E'\} = \{w_2^3\}$.

2. $E'' = \{p_0^1, p_1^1, p_2^2\}$, where $\cap\{||p||^{\mathcal{M}_2} \mid p \in E''\} = \{w_2^3\}$.

The former means that, by this anticipation, the belief formation system requires us to reexamine p_2^2, that is, **'Reexamine whether x_2 has truly an alibi or not'.** If the evidence is disproved, then the solution is w_2^2, that is, x_1 is a criminal. The latter means that, by this anticipation, the belief formation system requires us to reexamine p_2^1, that is, **'Reexamine whether x_1 always makes it with x_2 or not'.** If the evidence is disproved, then the goal is w_2^3, that is, x_3 is a criminal. ■

Again we have multiple potential solutions by anticipation of type II, hence the belief formation systems with anticipation of type II are also *hyperincursive.*

4.3 Another Kind of Anticipation

The discussion of anticipation of type II in the previous section presupposes the *closed-world assumption*, which means that a criminal should be in the current set of suspects at each time. So, the system tries to find a solution within the set of evidence obtained by each time.

Here let us consider what happens if we adopt the *open-world assumption*, which means that a criminal may be outside in the current set of suspects. For example, at $t = 3$, suppose we have the third person who is also a suspect. Then we add the following atomic sentence:

p_3: x_3 is a criminal.

Thus, the set of possible worlds should be reconstructed in the following way. Since $\mathcal{P}_3 = \{\mathsf{p}_1, \mathsf{p}_2, \mathsf{p}_3\}$, the following eight worlds are logically possible:

$$
\begin{array}{ll}
w_2^1 = \{\mathsf{p}_1, \mathsf{p}_2, \mathsf{p}_3\} & (p_{w_1^1} = \ \ \mathsf{p}_1 \wedge \ \mathsf{p}_2 \wedge \ \mathsf{p}_3) \\
w_2^2 = \{\mathsf{p}_1, \mathsf{p}_2\} & (p_{w_1^2} = \ \ \mathsf{p}_1 \wedge \ \mathsf{p}_2 \wedge \neg \mathsf{p}_3) \\
w_2^3 = \{\mathsf{p}_1, \mathsf{p}_3\} & (p_{w_1^3} = \ \ \mathsf{p}_1 \wedge \neg \mathsf{p}_2 \wedge \ \mathsf{p}_3) \\
w_2^4 = \{\mathsf{p}_1\} & (p_{w_1^4} = \ \ \mathsf{p}_1 \wedge \neg \mathsf{p}_2 \wedge \neg \mathsf{p}_3) \\
w_2^5 = \{\mathsf{p}_2, \mathsf{p}_3\} & (p_{w_1^5} = \neg \mathsf{p}_1 \wedge \ \mathsf{p}_2 \wedge \ \mathsf{p}_3) \\
w_2^6 = \{\mathsf{p}_2\} & (p_{w_1^6} = \neg \mathsf{p}_1 \wedge \ \mathsf{p}_2 \wedge \neg \mathsf{p}_3) \\
w_2^7 = \{\mathsf{p}_3\} & (p_{w_1^7} = \neg \mathsf{p}_1 \wedge \neg \mathsf{p}_2 \wedge \ \mathsf{p}_3) \\
w_2^8 = \ \emptyset & (p_{w_1^8} = \neg \mathsf{p}_1 \wedge \neg \mathsf{p}_2 \wedge \neg \mathsf{p}_3 = p_\emptyset)
\end{array}
$$

Thus we have

$$W_3 = \{w_3^1, w_3^2, w_3^3, w_3^4, w_3^5, w_3^6, w_3^7, w_3^8\}$$

is obtained. Each piece of evidence is reinterpreted in the new set of worlds.

1. $||p_0^1||^{\mathcal{M}_3} = ||\neg p_\emptyset||^{\mathcal{M}_3} = W_2 \setminus ||p_\emptyset||^{\mathcal{M}_3} = \{w_3^1, w_3^2, w_3^3, w_3^4, w_3^5, w_3^6, w_3^7\}$
$= ||\mathsf{p}_1 \vee \mathsf{p}_2 \vee \mathsf{p}_3||^{\mathcal{M}_3}$.

2. $||p_1^1||^{\mathcal{M}_3} = ||p_{\{\mathsf{p}_1\}} \vee p_{\{\mathsf{p}_2\}} \vee p_{\{\mathsf{p}_3\}}||^{\mathcal{M}_2} = \{w_2^4, w_2^6, w_2^7\}$.

3. $||p_2^1||^{\mathcal{M}_3} = ||\mathsf{p}_1 \to \mathsf{p}_2||^{\mathcal{M}_3} = \{w_3^1, w_3^2, w_3^5, w_3^6, w_3^7, w_3^8\}$.

4. $||p_2^2||^{\mathcal{M}_3} = ||\neg\mathsf{p}_2||^{\mathcal{M}_3} = \{w_3^3, w_3^4, w_3^7, w_3^8\}$.

Thus, we have

$$B_3 = ||p_0^1||^{\mathcal{M}_3} \cap ||p_1^1||^{\mathcal{M}_3} \cap ||p_2^1||^{\mathcal{M}_3} \cap ||p_2^2||^{\mathcal{M}_3} = \{w_2^7\} \neq \emptyset.$$

Now the contradictory state is solved. We tentatively call this kind of anticipation *type III.* Then, by anticipation of type III, the system requires us that **'Seek the third person',** with a reconstruction of the current framework itself for the problem.

5 Concluding Remarks

In this article, based on our previous work[10], we further made discussion on several characteristics of anticipation in belief formation. Thereby we found that there were at least three kinds of anticipation. In particular, the anticipation of type II and of type III in belief formation seem to be related to *paraconsistent* and *dialectical* logics[11, 1]. We hope to make further discussion in a forthcoming paper.

Acknowledgments. We would like to be very grateful to the referee for his relevant and deep comments. The first author was partially supported by Grant-in-Aid No.12878054 for Exploratory Research of the Japan Society for the Promotion of Science of Japan.

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