Extended Detrended Fluctuation Analysis for financial data

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Abstract

A method to sort out temporal correlations in financial data within the Detrended Fluctuation Analysis (DFA) statistical method is used. Both linear and cubic detrendings are considered. Our findings are surprisingly similar to those for DNA sequences which appeared as a mosaic of coding and non-coding patches.

Keywords: time series, fluctuations, fractal, economy, Hurst exponent

1. Introduction

Recently, there have been several reports that financial fluctuations may display long-range power-law correlations [1] similarly to turbulent [2] and self-organized critical systems [3]. If this is true, this should allow physicists to dream about economy modeling and work on predictability. However, others reported that economic fluctuations have features close to decorrelated

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sequences [4, 5]. Different statistical techniques have been used up to now: wavelets [6], Lévy statistics [1, 4, 7], discrete scale invariance [8], Brownian motion [5], a.s.o. We present here a method to sort out temporal correlations in financial data within the *Detrended Fluctuation Analysis* (DFA) statistical method [9]. The latter has demonstrated its usefulness for the investigations of long-range power-law correlations in DNA sequences [10] as might be searched for in economics. Our findings are surprisingly similar to those for DNA sequences which appeared as a mosaic of coding and non-coding *patches* [10].

2. Linear and Cubic Detrending

The DFA technique consists in dividing a random variable sequence $y(n)$ of length N into N/t nonoverlapping boxes, each containing t points. Then, the local trend in each box is defined. Two cases will be hereby examined. Usually a linear trend $z(n)$ is assumed in a box, as

$$z(n) = an + b, \tag{1}$$

such that the parameters a and b are estimated through a linear least-square fit of the data points in that box. The process is repeated for all boxes. The detrended fluctuation function $F(t)$ is then calculated following

$$F(t)^2 = \frac{1}{t}\sum_{n=kt+1}^{(k+1)t} (y(n) - z(n))^2, \qquad k = 0, 1, 2, \cdots, (\frac{N}{t} - 1) \tag{2}$$

Averaging $F(t)$ over the N/t intervals gives the fluctuations $\langle F(t)\rangle$ as a function of t. If the $y(n)$ data are random uncorrelated variables or short range correlated variables, the behavior is expected to be a power law

$$\langle F\rangle \sim t^{\alpha} \tag{3}$$

with an exponent 1/2 [9]. An exponent $\alpha \neq 1/2$ in a certain range of t values implies the existence of long-range correlations in that time interval as e.g. in the fractional Brownian motion [11]. Correlations and anticorrelations correspond to $\alpha > 1/2$ and $\alpha < 1/2$ respectively. In fact, the exponent α is the Hurst exponent often denoted H in the literature. In terms of the Hurst exponent H related to the spectral exponent β of a signal by $\beta = 2H + 1$, we can talk about pink noise or black noise depending whether $H < 1/2$ or $H > 1/2$. Black noise is related to long-memory effects (persistence), and pink noise to anti-persistence. One has often found that most natural processes are persistent, with long-memory effects. Pink noise ($H < 1/2$) occurs when relaxation and dissipative processes are dominant over the external influences and perturbations.

Notice that it can happen that in a specific box, the linear trend might be way-off from the overall intuitive trend, henceforth shorter scale fluctuations might be missed if the box size becomes quite larger than the intrinsic short time fluctuation scale of the signal. Therefore it is of interest to consider another assumed trend, like a cubic one

$$z(n) = cn^3 + dn^2 + en + f, \tag{4}$$

In the following we will call $F_1(t)$ and $F_3(t)$ respectively the fluctuation functions (Eq.(2)) derived from the linear (1) or cubic (3) trend, and similarly for the other functions of interest when appropriate.

The main advantages of the DFA over other techniques like Fourier transform, or R/S methods [12, 13] are that (i) local and large scale trends are avoided, and (ii) local correlations can be easily probed as we will see below. In economic data like stock exchange and currency fluctuations, long or short scale trends are *a posteriori* obvious and are of common evidence. The DFA method allows one to avoid such trend effects which can be considered as the envelope of the signal and mask interesting details. Thus, we expect that DFA will allow a better understanding of apparently complex economic signals [13, 14].

3. USD/DEM evolution

We have considered the economic evolution of the DEM/USD currency exchange rate from Jan. 1, 1980 till Oct. 15, 1996. This represents $N = 4383$ data points. The week-ends and holidays are obviously not considered even though political events can occur during week-ends. The jagged evolution of the DEM/USD currency is presented in Figure 1. At the time scale of the figure, large trends are clearly observed as e.g. the devaluation of the USD between 1985 and 1988.

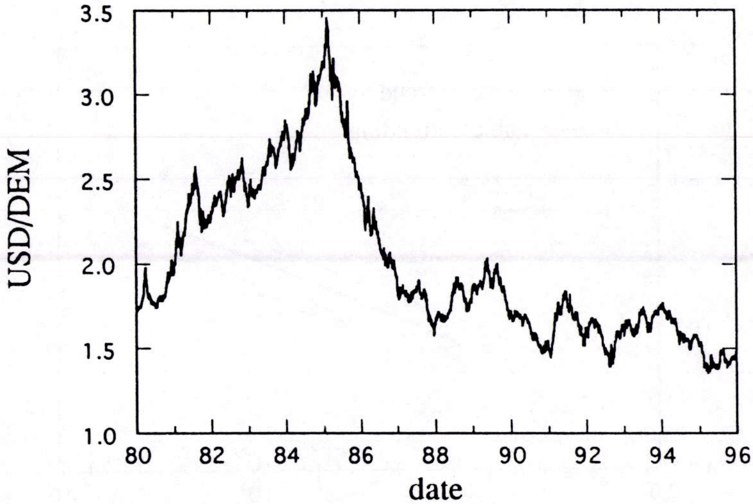

Figure 1 — The jagged evolution of the USD/DEM currency exchange rate from January 1st, 1980 to October 15th, 1996 representing 4383 data points.

In Figure 2, a log-log plot of the fluctuation functions $\langle F_1(t) \rangle$ and $\langle F_3(t) \rangle$ are shown for the whole data of Figure 1. The dashed line is the expected fluctuation fonction for the Brownian motion. The $\langle F_1(t) \rangle$ function is very close to a power law with an exponent $\alpha_1 = 0.54 \pm 0.01$ holding over two decades in time, i.e. from about one week to about 2 years. This results clearly support the existence of long-range power-law correlations in currency fluctuations *whatever the trend*. The $\langle F_3(t) \rangle$ function is also close to a power law with an exponent $\alpha_3 = 0.56 \pm 0.01$. The values of these exponents are different but they are significatively different from the Brownian motion case ($\alpha = 1/2$). These power laws are signatures of a propagation of *information* across the economic system during very long times. For time scales above 2 years, a crossover is however observed and is indicated by an arrow. This crossover suggests that correlated sequences have a characteristic duration of ca. 2 years along the whole currency exchange market evolution at least for the case studied here over 16 years.

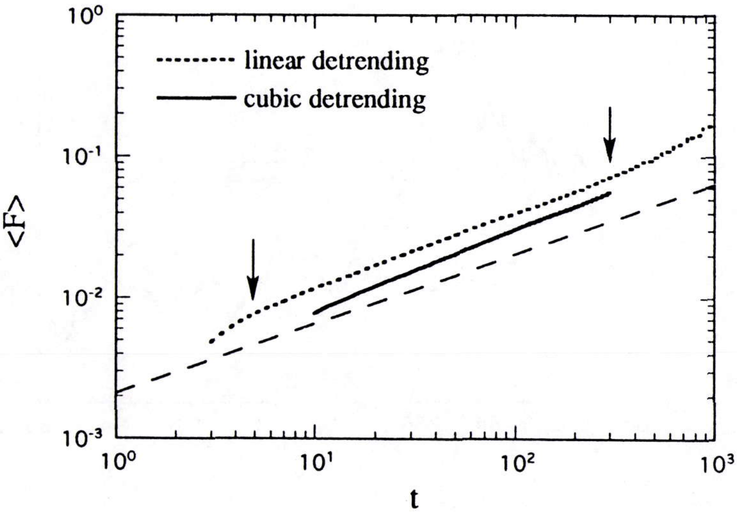

Figure 2 — The log-log plot of the $\langle F_1(t) \rangle$ and $\langle F_3(t) \rangle$ fluctuation functions showing time scale invariance from one week to one year. The crossovers are denoted by arrows. The dashed line crossing the graph is the Brownian motion theoretical case, i.e. a power law with an exponent 1/2.

Conversely, it is also of interest to check whether there are decorrelated sequences. In order to prove or probe the existence of *correlated and decorrelated financial sequences*, we first construct a so-called observation box (a probe) of "length" 2 year placed at the beginning of the data, and we calculate α_1 and α_3 for the data contained in that box. Then, we move this box by 20 data points (i.e. 4 weeks) toward the right along the time sequence and again calculate α. Iterating this procedure for the 1980–1996 evolution of the USD/DEM ratio, we obtain a "local measurement" of the degree of "long-range correlations". The results are shown in Figure 3 at various typical times. Even within (not indicated) error bars, we clearly observe that the α exponent values much vary with the date. The α_1 exponent value is mostly above 1/2. However, both around spring 1983 and spring 1987, the exponent α_1 has a sharp minimum below 1/2. The second event is the most dramatic one. The values of α_3 are not systematically lower nor higher than those of α_1. Values are intertwinned. They are however very close to each other, and both *alpha*-curves exhibit the same features.

It should be noted that we have tested both linear and cubic detrendings on well defined self-affine functions like fractional Brownian motions. We found that the cubic detrending provides the best estimate of α for these artificial data. The fact that α_3 presents the same features that α_1 means that the α variations in Figure 3 are significative.

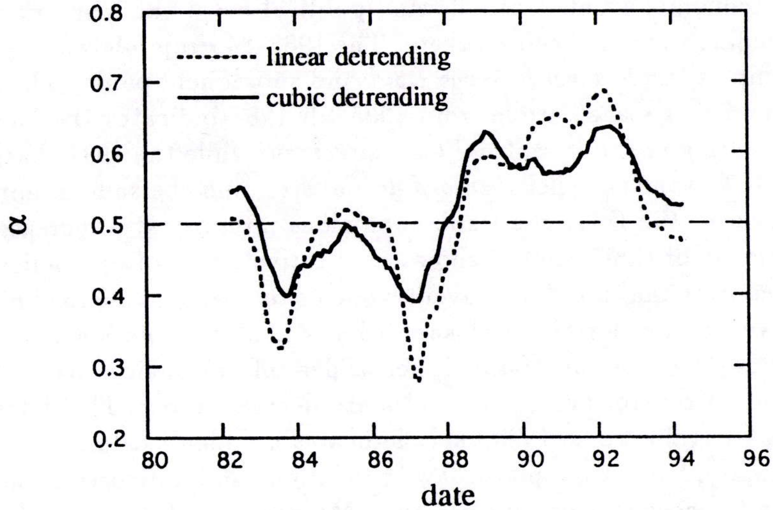

Figure 3 — The evolution of the local values of α_1 and α_3 estimated with the DFA technique for boxes of 2 year size.

4. Conclusion

The presence of persistent and antipersistent sequences is similar to what is also observed along DNA sequences where the α exponent has sharp variations [10]. It drops below 1/2 in so-called non-coding regions but remains above 1/2 in coding regions. By analogy, our findings suggest that for *some yet unknown reason* currency markets have different activity regions not immediately seen on signals like the exchange currency ones discussed here above. Just like for DNA, currency markets can loose control over the *information propagation* at specific moments. It should be noted that sharp variations in sequences observed around 1983 and 1987 respectively were not directly visible from the data in Figure 1 and would likely be missed by R/S and Fourier analysis [13].

Therefore the above features can be tentatively put into perspective with respect to economic events [15] following political events and/or some panic storm spreading over financial markets. The 1983–84 drop follows the *Volker* chairmanship at the *Federal Reserve Bank* and subsequent panic. The α turn over, rebound and steady value from 1985 till 1987 indicates the "need for control" resulting in the so-called *Plaza agreement* while the 1987 sharp drop can be related to the so-called "*accord du Louvre*", and the sudden minimum and turn over to the Deutsche Bank anomalous interest rates increase.

The α behavior thus seems to show quantitatively that after a policy move for better control and in order to avoid panic or to heal a crisis, the "market" nevertheless searches for the best subtle holes in the regulation in order to avoid the most severe constraints, then slides off the policy main stream, and sets out of control before new rules are decided upon. The detrending independent–α value since 1993 likely indicates a difficult calm.

In summary, long-range power law correlations and anticorrelations have been shown to occur in economic systems. Moreover, we have quantified that some sequences appear where the economic system looses some control over information propagation. It seems that these features can be associated with real economic events and policies. Cubic and linear detrendings provide close quantitative results.

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