# COMPUTATIONAL COMPLEXITY OF SOLVING COMPLEX PROBLEMS

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The notion of computational complexity is looked at as if it were a road along which we encounter several cognitive and operational obstacles. Some of these are difficult to anticipate, to prevent or solve. The study focuses on cases where the need to anticipate and solve is unavoidable. Three categories of such obstacles are discussed.

The first refers to the precise meaning of 'cognitive complexity' and what kind of problems does it rise. Here the discussion is centred on extending tractability of problems by means of identifying and solving circularities whenever they occur. The second enlarges on the accurate translation or transfer of the cognitive complexity into the computational one. The third obstacle is encountered when tractability, and circularity in particular, become so unavoidable that they need be anticipated which, in turn, is a prerequisite of designing the research programmes of particularly complex problems.

The problems considered to be extremely complex are those generated by 'scientifically-based human intervention'. Because of this, and the above technicalities, these problems are anything but disciplinary. A more precise idea of what means 'interdisciplinary' as a counterpart of 'undisciplinary' is given.

The study aims at correcting a *status quo* in which complexity-related tractability and circularity are either neglected or mistakenly regarded as a technicality such as the quantitative limit of computation. The far-reaching aim is to reconsider the legitimacy and the terms in which we think of a 'science of complexity', and the extent to which cybernetics - third-order cybernetics, eventually - becomes the very core of this science.

Keywords: complexity; circularity; tractability ; problem-solving; undisciplinarity.

# 1. The Model 1.1. Conceptual Setting

Four central concepts are defined in this subchapter: 'intervention as a particular case of action', 'inter- vs. undisciplinarity', 'circularity', and 'problem(-solving)'.

To be able to design a feasible research programme of an extremely complex problem we need to concern ourselves with those sources of complexity that are unpredictable. Supposedly, there are two main forms of unpredictability. One is **human intervention** as the most acute form of action. Problems such as environmental degradation, large scale and long lasting poverty, multi-faced social segregation and selfreproductive inequality are all man-made. Our interest in deciphering the essence of

International Journal of Computing Anticipatory Systems, Volume 1, 1998 Ed. by D. M. Dubois, Publ. by CHAOS, Liège, Belgium. ISSN 1373-5411 ISBN 2-9600179-1-9 intervention derives from the necessity to limit the occurence of iatrogene problems, i.e, to try and solve a problem without generating other problems.

In order to change the nature of intervention such as to minimise its unwanted effects, we have to draw our attention to what makes it possible for intervention to be unpredictable in general, and error unpredictable in particular. The point of view here is that many interventions ending up in deeper, sometimes unsolvable problems are generated by human action that is: (a) either (partly) inconsistent with a 'reasonable' goal, or (b) when pursuing a 'reasonable' goal, this is detrimental to another party involved, or (c) despite of the goal being reasonable, this is indivisible such as to accommodate conflicting interests in achieving it. Here 'reasonable' means: it can be explained on the grounds of readable interests, but not necessarily understood or accepted as such.

In understanding and modelling intervention, science has a double-sided role. On the one side, there is about establishing a more general standard of what is 'reasonable' in scientific terms, on the other, there is about diminishing unpredictability by enlarging knowledge. As is known, enormous progress has been achieved in increasing science's capacity of prediction. Action, however, remains about as unpredictable as centuries ago. Besides, it seems that the more scientifically based intervention becomes, the more difficult to solve the problems that emerge. An example is genetic manipulation. If one think manipulation would solve some genetic diseases without paying attention to the yet unpredictable problems genetic manipulation may generate, then the same will happen time and again - we create a scientific base for intervention, but the consequences of that intervention are not scientifically looked for, and sometimes completely forgotten.

Therefore, this study's aim is to improve on one of the species that intervention takes, namely the anticipation of obstacles in the way of researching the problems whose nature and complexity are such that they cannot be error-free solved. That is the least science can do till the time will come when no political decision will be implemented unless science has the chance to research all the possible futures of that intervention.

But, to be able to advance 'complete-scientifically-based intervention', science itself has to make a radical change in the pattern in which interventions are researched - it has to become **undisciplinary**. Neither the problems requiring intervention, nor the design of interventions themselves can be handled disciplinarily. The scientific community has agreed on that the research has to become '**interdisciplinary**', but the way in which interdisciplinarity is applied proved to be just another variant of disciplinarity (Messer-Davidow, 1993). The interdisciplinary approach continues to be thought of in terms of co-operation between disciplines. The main concern goes to accommodate specialists and their disciplines in a project, and not to the possibility that the required competence has to be fundamentally different from any existing disciplinary competence.

If this argument is epistemologically too abstract, then let be considered its 'empirical' side. The main (only) reason why some problems are incorrectly solved or are even declared unsolvable is that a disciplinary competence (regardless of the amount of contributing disciplines) appears to be insufficient. If the competence would be sufficient, the problem would be solved. When a competence is found insufficient, scientists search for 'repairs'. The first attempt goes to render the competence at hand sufficient. Whether such corrections consist of calling in other disciplines or changing the relationship between disciplines is less relevant here. Important is that not all these attempts are successful, which means that not in the number of attempts resides the finding of a sufficient competence. Here science has to take into consideration the possibility that disciplinarity itself might have limits, might be insufficient in solving certain problems. The conclusion - an attempt has to be made in fundamentally constructing a competence outside disciplinarity. That is, we are challenged to think of crossing or even defying disciplinary frontiers and disciplinarity itself, if necessary.

Maintaining the disciplinary framework despite its insufficiency is an interference with the ontological complexity, and that is why it is a non-scientific (not acieved through solving) simplification. This generates much of the hidden incorrectness. It depends on scientists to anticipate it, because this is not an unpredictable.

About as unpredictable as intervention is, however, the occurrence of **circularities** among some of the 'technical' problems that science has to solve in order to be able to solve the problem put to science. For reasons developed under 1.2. and 1.3., circularity becomes the most important factor that may be used/misused while taking under control complexity and because of this, the study deepens into modalities of making circularity identifiable and predictable.

Here circularity refers, firstly, to solvability being a function of tractability, and tractability being a partial function of the form that the solution is expected to take. Secondly, tractability depends, also partly, on the formal structure and composition of the problem we are about to solve (Bechtel and Richardson, 1993). Thirdly, both the formal structure and composition should be defined, which implies producing knowledge about them *via* decomposition and structure analysis. There are cases where these routes cannot be followed linearly. For instance, depending on a *prima facie* idea about the solution a certain problem might receive, that problem will display different ways of decomposition and different structures associated to its components. Generally, the space of a complex problem does get in-formed in more than one way, which means that the researcher interferes with the shape and true configuration of the problem's space. In conclusion, research is also an intervention.

The excellence of this study lies on attacking the prejudice according to which research is an innocent intervention, if intervention at all. The point of view advanced here is that research has a change potential that is far deeper than the policymakers capacity of inflicting change. Therefore, science's intervention has to be improved on before asking intervention to keep itself close to a 'rational' goal.

On the other hand, there are problems whose tractability can be univocally defined whereas their solvability seems unpredictable. Hence, circularity also refers to the turnings back to a previous order of circularity so as to put order in the subsequent levels, and 'order' is meant as a succession of tractable sequences (Lunca, 1995).

To be able to find a way out of circularity, the study defines the first-, second-, and third-order circularity formally, which enables the identification of the correspondent sets of problems implying multi-levelled circular dependencies. Because of these two senses in which circularity is taken, we are dealing with a species of cybernetics that refers to 'control' to a lesser extent than to non-linear chains of feed-backs and feed-forwards within an outlined system of (mutual) dependencies. This suggests roughly that circularity is a cybernetic as well as a system-theoretic concept (Klir, 1991).

To summarise, man-made intervention, undisciplinarity, and circularity are the three pillars so far on which this study is built. There is, however a fourth pillar, namely the notion of **problem** whose meanings depend on where and by whom the problem is signalised. First, a problem, **P**, is any non-trivial situation conflicting the need to reach a next stage in the existence of a non-simple entity (such as a community facing, say, some severe degradation of its natural or built environment). Thus **P** is as non-trivial as it makes it unsolvable by administration, policymakers, and the like. In searching for a solution, **S**, science is confronted with formal and technical difficulties of its own such as finding a suitable method, or language. Solving this kind of difficulties constitutes a distinguished category of problems, i.e., **problems of science**, **Φ**. Circularity occurs whenever the solution to a **Φ** depends on solving a previously occurred **Φ** that has been left unsolved or neglected, or given with an incorrect/revisable solution. All cases implying circularity will be noted  $\Xi$  so as to separate them from the other **Φ**<sub>1</sub>,...,**n**.

In the increasingly self-sufficient domain of **problem-solving**, both senses of 'problem' as distinguished above are studied, although the problems of science are seen as matters for the philosophy of science. Up to now, no serious attempt has been made to correlate the study of problems in the both above senses. Any way, problem-solving became the nest of many, rather divergent, directions of thought, the least of which are quite distant from the research field based on the already classical 'human problem-solving' (Newell and Simon, 1972). Two important turning points unfolded in the recent years. One proceeds from solving problems that raise major challenges to the limits of knowledge patterns, as they are continuously (re-)settled. A leading authority in finding solving patterns is D.R. Hofstandter (1985) and, in general, Minski's or MIT school of Artificial Intelligence focusing on solving process computation (Pylyshyn, 1984).

The other turn is to the pragmatics involved in that scientists are anxious to give increasingly precise instruments of problem-solving to managers, i.e., to those who are most likely to be confronted with nasty combinations of common-sense, routinely social and scientific problems (Flood, 1995; Flood and Romm, 1996). Here it is worth drawing the attention to that managers, whatever skilful, are not scientists, and some problems cannot be solved unless fundamental research is accommodated. As pointed above, the approximately scientific decision-making may generate really unsolvable problems.

#### 1.2. Identifying First-, and Second-order Circularities

Let be considered a non-trivial  $\mathbf{P}$  asking for an  $\mathbf{S}$  that may point at two directions of intervention: that of conserving the existence of an affected entity as it was, or that of enabling that entity to evolve towards a next coming stage. Specifically,  $\mathbf{S}$  may be either an **RS** (return to the stage prior to  $\mathbf{P}$ ), or an **SF** (reach a future stage as a superior, or temporarily problem-free state). To the extent to which an entity is aware of being faced with a **P**, that **P** receives a formulation in which there are included general features of the expected **S** such that one is able to estimate whether it would be an **RS** or an **SF**, and whether it is for science to decide which one shall be adopted. Technically, formulating **P** means generating a text consisting of a number of sentences that are grouped in three categories as they refer to:

- *P1* the description of a problematic situation as an obliterate mechanism that is populated with objects/things/facts, (some of) their properties, (some of the) relations among objects, and (some of the) relations among objects' properties;
- **P2** the identification (in a story-like representation) of the critical area(s) or point(s) where the mechanism is disabled to function as previously, or expectedly; and
- **P3** figure out much of an S in terms of changes to be operated on the critical area(s) leading to the identification of loci of control (Bechtel and Richardson, 1993).

In the hypothesis that the affected entity is the best in signalising what is detrimental to itself,  $P_1$  consists, as already mentioned, of systematically identifying the objects (**o**), their properties (**p**), and the relations (**r**) among objects (**r**<sub>o</sub>) and among properties (**r**<sub>p</sub>) in the space of **P**. Suppose that through systematic observations, all **o**'s, **r**<sub>o</sub>'s, and **r**<sub>p</sub>'s of concern for formulating **P** are identified. Yet, this might not be enough for a scientific formulation and even less for identifying  $P_2$  accurately (as it almost always is the case). Therefore to  $P_1$  a first group of  $\Phi$ , is associated that concerns finding not only what the **o**'s, **r**'s, and **p**'s are precisely, but also finding the scientific terms, **u**, of representing each of them. Here the danger threatens that a disciplinary perspective will prevail if this provides well-defined terms. Taking for granted these definitions in any circumstance is the easiest, but neither the most direct, nor accurate path to tractability.

Solving the  $\Phi_1$  also concerns finding those o's, r's or p's that cannot be termed as conveniently as to build meaningful sentences, and how many unknown terms,  $\mu$ , are there in question-form sentences such as: 'Knowing that x, what is y ?'. Specifically, within  $\Phi_1$ , the following operations has to be worked out:

- Define each object, **o**, as an outlined universe of properties and relations,  $o\{p, r_o, r_p\}$ , the limits of which are established to the extent they are relevant for formulating **P** consistently with an intended/expected **S**. Because the scientific formulation takes into account both the relevance and consistency, for each  $o\{p, r_o, r_p\}$  a representation  $\Omega\{\alpha, \beta, \gamma\}$  is associated. That is, a translation takes place.
- Identify when a property,  $\alpha$ , is relevant and consistent with an intended S, but it is not as simple as to be translated into a univocal variable. When a property refers to an object,  $\Omega$ , through another property, this is a meta-, or a relational property,  $\gamma$ .
- Separate  $\Omega{\alpha, \beta}$ 's from  $\Omega{\gamma}$ 's, and universes such as  $\Omega{\alpha, \beta, \gamma}$  or  $\Omega{\gamma(\alpha, )}\beta$ .
- Indicate when the unknown terms,  $\mu$ , might possibly refer to an  $\Omega$ ,  $\alpha$ ,  $\beta$  or  $\gamma$ .

In so far as  $\Phi_1$  is about terming, we may suppose that it can be kept undivided regardless of how many steps are involved there. If so, then terming  $\Omega$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$  and

specifying the  $\mu$  terms may receive four forms according to whether the intended S is an **RS** or an **SF**, and to whether or not it is for science to decide which one will be adopted. At this point, a first circularity,  $\Xi_1$ , occurs, as far as choosing/deciding over an S needs at least two next coming stages to be accomplished and then returning to this step. That is:

#### $\Xi_1 = \langle \mathbf{RS} / \mathbf{RF} \rangle$ where S is either scientifically serached or otherwise (1)

For the scientific formulation of  $P_2$ , the  $\Phi_1$ 's that were concerned with terming are repeated till we find out which terms are altered after P emerged. Identifying the difference(s) between ante P and P is specific to  $\Phi_2$ , which contains, additionally, a resolution as to whether the form of S has also to be decided by scientific means. During the solving of  $\Phi_2$ , a circularity,  $\Xi_2$  is identified, that is not as linear as the previous one. Precisely,  $\Phi_1$  might need to be revised, if  $\Phi_2$  becomes unsolvable. To prevent insolvability, some of the terms are to be reconsidered, which implies either a matter of terminology only, or a matter of language as a system of representing and generating knowledge. This turning back generates the second circularity,  $\Xi_2$ , which is double-levelled, i.e., at one level  $\Phi_1$  is contained as it is, at the other,  $\Phi_1$  is reformulated, which leads to an  $\Phi_1$ ':

$$\Xi_2 \mid \Phi_2((\Phi_1)/(\Phi_1')), \tag{2}$$

where the connector  ${}_{1}^{\prime}$  is to be established for every case either as a direct relation or as a mediate one. The type of equation, usually a differential one, is to be established too.

For specifying  $P_3$  scientifically, the correspondent  $\Phi_3$  should answer to <u>two</u> <u>different categories of questions</u>, which is the reason why  $\Phi_3$  cannot be kept undivided, so that, for a safe handling, there are rather a group  $\Phi_3$ , and a  $\Phi_4$  one.

The first category of questions refers to whether there is complete knowledge for deciding/choosing the form of S, and subsequently, whether RS became impossible because of some irreversible alteration of some of the  $\Omega\{\alpha, \beta\}$ 's or  $\Omega\{\alpha, \beta, \gamma\}$ 's. To answer this category of questions,  $\Phi_3$  should transform the  $\mu$  terms into known terms, **u** (up to specifying to which  $\Omega$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$  do they refer). This transformation should, in principle, become part of the S, or more precisely, part of the scientific formulation of S. In addition,  $\Phi_3$  has to contain the difference(s) between *ante* **P** and **P** as established throughout solving  $\Phi_2$ . Accordingly, there is a linear circularity:

 $\Xi_3 \mid \Phi_3(\Phi_2)$ , and a double-levelled circularity:  $\Xi_4 \mid \Phi_3\{\Phi_2((\Phi_1)/(\Phi_1'))\}$  (3,4)

The latter may be brought to a linear form (Johnson, 1990). Here 'linear circularity' designates a circularity transferable, almost directly, into a linear function.

<u>The second set of questions</u> refers to what changes are there necessary, and what is to be submitted to change for delivering an S. So, it might be necessary to change or eliminate some of the o's, r's, or p's. The plausible case is that intending to eliminate a p, for example, leads to a change in the stand of some r's or  $r_0$ 's which, in turn, requires that the changing relatedness of o's, r's and p's has to be kept under control.

If **RS** is the chosen alternative, then  $\Phi_4$  consists either of reducing the difference(s) between  $\Phi_1$  and  $\Phi_1$ ' by their mutual reduction (in the procedural sense), or of reconstructing  $\Omega{\alpha, \beta}$ 's, and  $\Omega{\alpha, \beta, \gamma}$ 's as they were before **P** occurred. Then,

$$\Phi_4\{(\Phi_1' \to \Phi_1), (\Phi_2)\} \cong \Xi_5 \tag{5}$$

The connector  $\cong$  means that a mediator factor/system is to be taken into account in order to decide whether an equation is the accurate expression of the relation at hand.

If **RS** cannot be chosen because it became impossible, then the difference(s)  $(\Phi_1)/(\Phi_1')$  need to be termed in the terminology and language that were used for terming  $\Omega$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ . Then,

$$\{(\Phi_1)/(\Phi_1')\} \to \mu \to \mathbf{u},\tag{6}$$

which implies turning back to  $\Phi_3$  with the consequence that each of the  $\Xi_3$ ,  $\Xi_4$ ,  $\Xi_5$ , or pairs of them may return into play. As a result, the sixth circularity takes the form of

$$\{\Xi_6 \mid \Xi_{6-n}((\Phi_1)/(\Phi_1'), \mu')\} \to \mathbf{u}$$
(7)

Here  $\Xi_{6-n}$  is a meta-variable made out of any of the  $\Xi_3$ ,  $\Xi_4$ ,  $\Xi_5$ ,  $\Xi_{34}$ ,  $\Xi_{45}$ ,  $\Xi_{35}$ ,  $\Xi_{345}$ , and  $\mu'$  is a new estimate for some of the  $\mu$ 's. The  $\Xi_6$  circularity is no longer a double-levelled one. It takes the form of a tree (Grim, 1991) with two variants according to the answer to the if-question above, which comes to an entire set of  $\Phi$ 's, namely  $\Phi_5$ . In fact,  $\mu'$  implies identifying and solving as many re-iterations as possible, and the construction of the approach's format in both the semantic and syntactic senses.

At this point, one will try to find out where does the second-order circularity intervenes. Certainly, the sixth circularity is a second-order one, but it would rather be considered that at least one second-order circularity is generated by the need to re-term some of the universes  $\Omega{\alpha, \beta, \gamma}$  in order to accomplish  $P_2$  as conveniently as to enable  $P_3$ . The basic assumption here is that an order of circularity does generate a form of circularity. As a result, some of these forms are of the third-order kind, and perhaps cannot be reduced to a second-order kind. A less generalisable way of separating the first-order circularities from the second-order ones is that of isolating the first circularity we meet and cannot solve. To the extent to which we are able to explain why a given circularity cannot be solved, this explanation in itself (Churchland, 1991) provides the difference that excludes a circularity from the first-order kind. A good explanation should be one that will be upheld for the difference between second-, and third-order, i.e., to be as general as to serve in both cases. That is what will be tried below.

#### **1.2. Third-order Circularities**

Defining these forms of circularity and the ways one is embedded into another do not cover all the impediments science has while solving the problems of the sort defined earlier. The solution to circularity uncovers, however, another set of  $\Phi$ , namely the semantic and formal incompatibilities among disciplinarily specialised methods and languages. Throughout experiencing programmes for solving circularities as defined above, I have realised that the group of problems referring specifically to solving incompatibilities need to be separated and worked out as such. These are namely so complex and different by nature that research in its own rights has to be accommodated.

This is because both the formulation that the entity gives to its  $\mathbf{P}$ , and the scientific formulation of that  $\mathbf{P}$  should make use of terms, sentences and relations between them. They form the fundamental units of a scientific (formal) language. As noticed earlier, this language could not qualify as formal if it would have the function of representing only. When two referential systems compete in determining the meaning of a term, it is most likely that an incompatibility between the two occurs (Stankey, 1994). The solution to this incompatibility is either unifying the two referential systems (if possible), or creating a referential system that accommodates not simply the comprehensive meaning, but the meaning most adequate to that  $\mathbf{P}$ . In both cases, the referential system, i.e., the language/formalism, fulfils the function of knowledge generator as well.

This bi-functional language,  $\ell$ , has to be constructed with respect to its semantics and syntax. The requirement for construing  $\ell$  holds even in the eventuality that one will adopt the way of integrating two or more disciplinary referential systems or working with so-called hybrid languages (Pahre, 1996). Later on, a simplified description of the requirements that the semantics and the syntax of such a language should satisfy will be given. The simplification consists of that it only specifies the kind of relations among terms that are at the basis of rules for relating terms and sentences.

Before defining the sets of rule-givers relations, it is worth explaining why this set of circularities cannot be reduced to a second-order kind. As argued earlier, the kind of **P**'s at issue are not signalised by a competence in using scientific terms to formulate an experienced situation. The difference between the way the affected entity formulates its **P**, and the formulation given by science, thus by 'the knowing competence' (or knower) is not only semantic, but formal too. More precisely, it is for science to decide what is relevant for handling **P** (in terms of  $\Omega$ ,  $\beta$ , and  $\alpha$ ), and what **S** will be consistent with the scientific rationale of having **P** solved. But, it is for the affected entity to agree on the actual consistency according to the entity's interest. The interest of the knower in solving that **P** might be to generalise the case of that **P** and its (alternative) **S**'s so as to issue a model and to theorise upon this model.

These two interests are as many rationales, and a rational supposition is that they are not necessarily conflicting, but complementary. If so, then the knower will find always that formulation of S, which is meaningful and makes sense for the user of the solution. This means that the knower should be aware of his/her interest and intention, as well as the interest and intention of the affected entity. Here then, we have two, rather different, *intentionalities* (Ecco, 1990), and only the vehicle of one of them, namely the knower's, can handle them both so as to keep them in complementary relation. It is in this way that the increase in 'rationality' of the entity's goal will become achievable.

Needless to stress that the interference of the knower's intentionality determines not only the complexity of the entire solving process, but particularly when and why a **P** is declared intractable, or unsolvable. For example, getting away with a  $\Xi$  without

solving it might artificially enlarge tractability. As a result, a next  $\Xi$  will be even more difficult to solve, and in place of enlarging tractability we only obtain a postponed intractability. In other words, the knower must intentionally solve all  $\Phi$ 's in order to avoid intractability, first of all, and the premature simplification (e.g., disciplinary framing), secondly. In order to do so, the knower should formulate and solve a new and large set of  $\Phi$ 's that determine the competence in solving the sets of  $\Phi$ 's defined above.

This new set,  $\Phi_6$ , consists of choosing/creating  $\ell$  as a referential system of representing-generating knowledge. The table below, of relationships involving terms and sentences, is meant to emphasise the way in which  $\ell$  becomes qualified in generating knowledge necessary to transform the  $\mu$  terms into **u** terms, and to allow for answering to question-form sentences. The table encounters as many relations among semantic units as necessary to draw the difference between basic (sometimes even axiomatic) relations and the first-, or second-derived relations. These relations express forms of semanticity, and, at the same time, they generate semanticity. As Hofstadter put it, "Semantics is an emergent quality of complex syntax" (1985, p.445). The syntactic ordering is given by a linear dimension consisting of the alternative: either **direct** or **mediated** (Lunca, 1996, pp.312-327). The reading pattern of this table is: 'A relation x is of the type  $a_{1,...,8}$ , in which the relates are  $b_{1,2}$ , that are  $c_{1,2}$  occurring between  $d_{1,2,3}$ .'

a) RELATIONS, $\beta$	17	in v	vhic	h th	e re	lates	s are	kn	own	, u,	uni	its:	or at	leas	t one
b	) _		te	rms			or		sent	ence	s		of th	he r	elates
$\rightarrow$ c) d)		direct			mediated			direct			mediated		is an µ term identifiable as		
		between		between			between			between					
<b>OF:</b> ↓	Ω	α	Iγ	Ω	α	1 Y	Ω	α	1 Y	Ω	α	1 Y	Ω	a	1 2
EQUIVALENCE, states in- a-set inclusion/exclusion														1	
<b>COMPLEMENTARITY</b> , asymmetric operation			12												
(IR)REDUCIBILITY of p's, or o's via p's		1										1			
<b>CONDITIONALITY</b> "If $x_1, \ldots, x_n$ , then y"															
<b>CAUSALITY</b> " $\forall x \rightarrow y$ "															
UNRELATED OR ACCIDENTAL CO-OCCURENCES							STIPULATIVE CLOSURE					INDETER- MINANCY			

# Figure 1. Types of semantic and syntactic relationships

When designing a research programme, some sentences will be given with the function of axioms. For these sentences, the relations above do not hold if the other relate is also assigned as axiom. Conversely, if two axioms are not independent, but somewhat related, at least one of them cannot fulfil the function of axiom and should not

be accounted for (Leeuwen, 1990; Grim, 1991). One of the axioms is, obviously, the **stipulation** case that makes it possible for the entire semantics to be closed. It is the semantic closure that makes the semantic system workable and allows for the design of the research programme (Lunca, 1996).

The axiom of semantic closure is analogous to the axiom of choice in set theory, and because the semantic system is based on such a type of axiom, the issues derived from working with a closed semantic system are sets rather than classes. If designing the research programme requires the identification and reduction of cases of circularity, then this is made possible by considering the reducibility of some of the properties of the objects involved in the problematic situation. Accordingly, it is by means of changing or eliminating some of these properties that we can transform the problematic situation into a non-problematic, or another one. Each object or property-related object(s) is/are sets. What we need is to be able to perform operations, and not classifications. The argument here is that, on the one hand, sets permit a number of operations larger than classes do, and on the other, classes are usually determined by non-axiomatic rules. The semantic complexity is too high to renounce to the functions that an axiom ensures.

When the language (consistent with the relations above) takes the form of a semantic system,  $\ell$  ought to be formalised under an axiomatic programme in a next stage. This allows for the unification of an eventual hybrid language, i.e., a sort of formal language built on a number of disciplinarily specialised languages plus a natural language. The latter is, in principle, the language in which the entity affected by the problem at issue describes it, and claims a solution (into the natural language of the user). In any event, unification is in itself a modality of solving some of the semantic incompatibilities, but the safest way is that of construing an  $\ell$  fully consistent with **P**.

To the  $\Phi_6$  only one circularity,  $\Xi_7$ , is associated, but this takes several forms according to how many subsequent rules the language needs to derive, which in turn, depends on how complex **P** to be solved is. In fact,  $\ell$  is incrementally achievable which means that when encountering a second, third, etc., circularity, a preliminary  $\ell$  is already at work. By now, it might be clearer why  $\Xi_7$  cannot be reduced to second-order circularity. On the contrary, some of the second-order circularities might be reduced to a third-order one, resulting from this that the **overall solvability is, actually, a case for two-successive-orders-cybernetics -** a formulation that I consider better than 'second-order cybernetics' because 'the second' becomes 'the first' when re-iterating the first.

As is well known, it depends on the scientists the re-iteration of the first-order, or better said, of the first occurring circularity up to the point of obtaining an appropriate linearity of tractable sequences. To the extent to which solvability is 'goal-and-task-goaland-task' dependent, an amount of two-successive-order-cybernetics will persist (will be residual) as far as the entire solving process is concerned.

In addition, what is called second-order cybernetics cannot be conceived and worked out unless one will construe the system that mediates between the two orders cybernetics, a mediator whose essence is semantic-formal. It is within this mediator system that the position of the knower is conceivable. In the case of the problems discussed above, the mediator system becomes so large and important in itself (it enables/disables tractability) that it pushes the  $\Phi_6$ 's altogether into another-order-circularity. Hence the idea of successively re-iterated orders that may be put like this:



#### Figure 4. Stages of achieving *l*

# 1.4. A Simplified Configuration of the Model

The findings so far can be summarised by a graphic representation. This is a simplification in the sense that each  $\Phi$  and correspondent  $\Xi$  are considered as a node in a graph regardless of how many operations are included in each of them The configuration given below is hypothetical to the extent to which a particular **P** may not encounter all the  $\Phi$ 's and accordingly, not all  $\Xi$ 's, or relatively other ways of relating  $\Phi$ 's and  $\Xi$ 's.



Figure 3. Hypothetical configuration of dependencies.

Expectedly, this route has to be made at least twice in order to reach the kind of formal isomorphism between  $o\{p, r_o, r_p\}$  and  $\Omega\{\alpha, \beta, \gamma\}$  that will enable as much linearity as possible. Yet, we say that the circuit is complete when we have produced knowledge such that we are able to obtain a content-based homogeneity that is (can be) translated into a fare linearity. This linearity may theoretically take the form of repeatedly embedded functions such as:

$$\Phi_{i}(\Phi_{n-1}(\Phi_{n-2}(\Phi_{n-3}(\Phi_{n-4}(\Phi_{n-5}(\Phi_{n-6})))))), \qquad (8)$$

In each simply-embedded function, any of the  $\Phi$ 's may occur. As far as the sequence  $\Xi_3 \leftrightarrow \Xi_4$  is concerned, it remains irreducible to one node if one of the two develops a separate relation with at least another node, which is highly likely.

The critical question arises at this point as to what shall we do if we cannot obtain a 'fair linearity'. One possible answer is that abandoning the attempt to solve the problem at issue will be senseless to the extent to which solutions as to how to work with non-linearity are available (Dubois and Resconi, 1992). Reducing non-linearity to linearity must, yet, be aimed at throughout next steps. This will enlarge the sequence preparing tractability, but will ease the treatment/approach, and will decrease the problem's chance to be declared unsolvable. It might be, however, that one circularity resists getting solved or contained into a tractable non-linear sequence, as in the case of some differential equations that have no analytical solution, but may have an algebraic one. If this will be the case (and it may be met quite frequently), then we will probably give up, or will give a try by eliminating the deadlocked sequence from the circuit. This implies necessarily re-writing the configuration of the chains or trees and, above all, assuming the risk of an approximate or even incorrect solution.

# 2. The Translation of the Cognitive Complexity into Research and Computation Programmes

An intrinsic part of the translation of the cognitive complexity into research and computational programmes consists of checking the cognitive complexity against the ontological complexity. The reason hereby is that the undisciplinary nature of **P** is an ontological matter, first of all, and not simply one resulting from whatever may happen between disciplines. Therefore, the cognitive complexity depends on both the content-formal circularities, and the  $\Phi$ 's due to the incompatibility between disciplinary languages-methods. To satisfy both these conditionals, the meaning of complexity needs to be extended so as to comprise a feature designating **heterogeneity**.

The substance of **heterogeneity** is semantical (Hale, 1987), and it emerges to the extent to which variables cannot be constructed other than in different semantic systems, thus not in a single one. It might be that from all the semantic systems we need, only a few are in use, so that we have to build an additional/new system. This occurs, in turn, as a result of dealing with properties,  $\mathbf{p}$ , that are irreducible to each other even if these properties were 'attached' or are the descriptors of one and the same object,  $\mathbf{o}$ .

Now, because here homogeneity/heterogeneity are taken as features of semantic nature, the **p**'s and **o**'s are not the real things, are not ontological entities, but the terms in which we designate them. Then the idea is that we often cannot define an  $\Omega$  in terms of its properties,  $\alpha$ , by remaining within a single semantic system as long as that  $\Omega$  is not as disciplinary as the known disciplinary objects (Hale, 1987; Lunca 1996). More over, because a problem is more complex than an object, we cannot entrust a problem to a disciplinary or multi-disciplinary approach if in the space of that problem we find one object that cannot be termed in a single (disciplinary) semantic system. An example is technology assessment where 'the technical facility' and 'the social' (represented by users and their social environment) are two terms totally different by nature, and yet, they have to be submitted to a treatment that has to be unitary so as to issue an assessment. The science's response to this problem was to strengthen the disciplinary character of the field 'technology assessment' in which 'the social' has been simplified beyond recognition. In other words, content and formal features of a class of objects have been sacrificed for the seek of reporting a new discipline.

That form of complexity that is generated within the ontological level, and passes through the phase of cognitive complexity is the form that we translate into the research and computational complexity. These levels or phases emerge when several causal chains compete in explaining something. Some of the chains provoke circular dependencies between properties accounted for (Johnson, 1990). The number of variables and their mutual relatedness is the indirect way in which circularity gets expressed. It is at this point that the order of rationality of the knowing instance intervenes, the result of which is expected to be a reduction of complexity and heterogeneity.

I am aware that issues referring to homogeneity-heterogeneity and forms/degrees of complexity belong to a larger scientific agenda. The above assumptions are meant to restrict these matters to what is relevant in this context. Homogeneity and heterogeneity are, for instance, poles of a continuum, and the proportion of their inseparability in the middle section is difficult to determine, which is why these are matters vulnerable to (inaccurate) interpretations. Of utmost relevance here is the question of why complexity cannot do, by itself, the task of separating disciplinary from undisciplinary problems. Well, because when we research a problem, we place it into an appropriate system of components, and in this format, we do not research 'the complexity', but 'how complex' that system is. This means that we refer to 'complex' as to a meta-property expressing some accumulation of properties as they were taken over by variables.

"Complexity (in the epistemological and methodological sense) is thus associated with systems, that is, some abstractions distinguished on objects that reflect the way in which the objects are interacted with. Systems, however, have many different facets, each represented by one of the epistemological categories of systems and, possibly, by some methodological distinctions within the category" (Klir, 1991, p.115).

To be able to render  $\mathbf{P}$  tractable, we need to submit all the detectable properties to a reduction procedure of the form that physics uses for the reduction of meta-, or supervenient properties to complex-univocal or even simple properties. Reduction is complete when it becomes possible that to each property one or a small number of variables is/are associated. When two or more variables are found to be associated to a property, these have to be formally independent. Any formal overlap between two variables indicates that these variables were either incorrectly reduced, or require to be both reduced to a third, more comprehensive, variable. Generally, the reduction procedure applied to a set of both physical and non-physical properties has as a prerequisite the construction of the semantic and formal system of the problem's space. Given the amount of unknown terms referring to properties, and the involvement of meta-properties that are partially known, we cannot assume whether these might be physical properties. Therefore, the semantic system that we use while applying a reduction becomes analogic to the domain of facts of a theory. This is why it takes a great deal of constructivism in order to avoid premature simplification – reduction does not mean simplification, but specification (Dogan and Pahre, 1990).

The requirements ordered into the two sets below are both substantial and formal

Set A	Set B
$\rightarrow$ <b>a</b> <sub>1</sub> Identify $\Omega$ as a specification of <b>P</b> . $\rightarrow$	b <sub>1</sub> Define the languages in which both $\Omega$ and <b>P</b> have been described while being signalised. $\downarrow$ $\downarrow$
a <sub>2</sub> Describe the universe represented by P in terms of objects/facts, proper- → ties, related facts and their properties	b <sub>2</sub> Verify if the known $\rightarrow$ b <sub>3</sub> Identify the kind terms are appropriate and sufficient; and then identify unknown terms.
→ $a_3$ Organise the knowledge issued to → identify the sub-P's, and the configu- ration of the space of P in terms of heterogeneity and formal complexity.	<ul> <li>b<sub>5</sub> Identify all the known Ω's, α's, and γ's; and operationalise the question answering.</li> <li>1</li> <li>1</li> <li>b<sub>6</sub> Construe the language able to accommodate</li> <li>the unknown terms into question-form sentences which become sub-objectives to be researched:</li> </ul>
<b>a</b> <sub>4</sub> Assess the immediate problematic context of <b>P</b> . A better specification of <b>P</b> emerges as a result of its contextua- lisation, and so the knowledge about S that is required by <b>P</b> is obtained.	$\downarrow \qquad \downarrow$ - <b>b</b> <sub>7</sub> <b>Reassess</b> the meaning $\leftarrow$ <b>b</b> <sub>8</sub> <b>Construe</b> the se- mantics and syntax to enable the meaning of the question-sentences to come through. $\downarrow \qquad \downarrow \qquad \uparrow$ - <b>b</b> <sub>9</sub> <b>Establish</b> the methodological and formal requirements to question answering and choose

Figure 4. Requirements for designing interdisciplinary research

At this point, let it be supposed that the formulation of requirements does not rise obstacles of principle (such as true/untrue), but of further specification. For example,  $b_2$ may be split off into three relatively distinct requirements. In other words, this schema may be enriched through a more analytic formulation. If so, then what becomes really important is to see whether the succession of steps does not generate problems of operationalisation. The connection between every two steps, and the overall succession of steps are, in fact, also ascribable under the heading 'requirements', and these cannot be simply re-formulated more analytically. They need, instead, to be rendered functional.

At the first reading of Figure 4, two kinds of connections designated by implicitness or conditionality can be observed. Thus, we deal with a number of downward connections starting from a1 and b1, and a number of feed-backs. Some of the latter are upward connections within Set B, whereas the others are either onward connections, or upward and onward connections between Set A and Set B. Because of the partial overlap between these two latter forms of feed-backs, we have to consider, actually, a third set, Set C (connections) that becomes about as important as the previous two. Here the question arises of whether every connection is necessary, and if not, then which of them may be optional. It might be so if we would be working in a disciplinary competence where the need of terming/re-terming, and of identifying unknown terms preoccupies to a lesser extent. When we need to construe a competence undisciplinarily, the design of the research programme becomes considerably larger through termingidentification-terming steps, and that is why the connections established within this sequence become so numerous (Mitchell and Hofstandter, 1991). Thus, it is this Set C that enables the computation route (below), where the double-lined arrows represent both forms of feed-backs, most of which are links between Set A and Set B.



Figure 5. The logical schema of computation

It appears that the number of upward/onward feed-backs is about as large as the number of downward connections. But the significant finding is that only the downward connections can be treated as being linear. This schema is then a first indication of the computational complexity increased by processing the non-linear links. This puts forwards the need to find a procedure of reducing the non-linearity to sequential linearity through conserving the cognitive complexity that has lead to non-linearity. If the cognitive content would not be 'complex', it would be expressed linearly.

Of course, the safest way of reduction to linearity consists of solving the  $\Phi$ 's contained in each requirement in the Figure 4. Another way is to insert in the computer programme instructions as to where to go back when a bi-conditional point (node in a tree) is met. Finding and choosing these ways is helped by the construction of the tree that will follow. Here we can highlight those nodes to which the computer might be forced to turn several, actually, indefinitely many times. These repeated returns to a previous node are vicious, or, more precisely, they are that sort of circularities that cannot be left unsolved, hence the need to pass through the phase of tree construction.

$$b_{1} [b_{2} \leftrightarrow b_{3}] \rightarrow b_{4} \rightarrow b_{5} \rightarrow b_{6} [b_{7} \leftrightarrow b_{8}] \rightarrow b_{9} \rightarrow a_{1}/ \rightarrow a_{4} \rightarrow a_{1}$$

$$a_{1}$$

$$[b_{2} \leftrightarrow b_{3}] \rightarrow b_{4} \rightarrow b_{5} \rightarrow b_{6} [b_{7} \leftrightarrow b_{8}] \rightarrow b_{9} \rightarrow a_{1}/ \rightarrow a_{4} \rightarrow a_{1}$$

$$a_{4} \rightarrow a_{1}$$

$$b_{6} \rightarrow [b_{7} \leftrightarrow b_{8}] \rightarrow b_{9} \rightarrow a_{1}/ \rightarrow a_{4} \rightarrow a_{1}$$

$$b_{1} \rightarrow [b_{2} \leftrightarrow b_{3}] \rightarrow \dots \rightarrow b_{9} \rightarrow a_{1}/ \rightarrow a_{4} \rightarrow a_{1}$$

$$a_{4} \rightarrow a_{1}$$

$$b_{6} \rightarrow \dots \rightarrow b_{9} \rightarrow a_{1}/ \rightarrow a_{4} \rightarrow a_{1}$$

$$a_{2}$$

$$a_{3}$$

$$b_{6} \rightarrow \dots \rightarrow b_{9} \rightarrow a_{1}/ \rightarrow a_{4} \rightarrow a_{1}$$

$$a_{4} \rightarrow a_{1}$$

$$a_{4} \rightarrow a_{1}$$

$$a_{4} \rightarrow a_{1} \dots$$

Figure 6. Sources of circularity in the tree.

As one may observe in Figure 6, the overall modality to reduce the tree to a limited number of linear sequences is to solve a node such that we would can eliminate it together with the links in which it is involved. Then the computer receives the prompt <done> which annulled the instruction <go back to...>. It sound simple, and if we arrive at this point, it is simple indeed – what we have to do is to choose an appropriate programming language and programme. These languages and programmes are so advanced nowadays that after finding or adapting one, we may receive feed-backs referring to possible inadvertent expressions of the non-linearity due to the considered cognitive complexity called by Mitchell and Hofstandter (1991) "emergent computation".

Suppose we are facing the computing phase. The concise order of solving runs as follows. We have to start with the simplest links, which are  $\mathbf{b}_2 \leftrightarrow \mathbf{b}_3$ , and  $\mathbf{b}_7 \leftrightarrow \mathbf{b}_8$ . Then

the feed-backs within Set B come, followed by those from Set A to Set B. The most difficult to work out are the connections generated in  $\mathbf{a}_2$  (in case the links from this one have not been reduced), and the connections sent forwards by  $\mathbf{a}_3$  and  $\mathbf{a}_4$ . The difficulty here resides in that even if the node to which they send a feed-back would be eliminated, the links are still transmitted to the next node in Set B. The annulment of these connections is only made possible when the latter  $\mathbf{a}$ 's are solved either completely, or to such an extent that we are able to stipulate the case of 'convenient/sufficient completeness'. To be able to achieve completeness, whether in its strong or weak forms, repeated work out of every node is required. This is why these are circularities, computational circularities, to be precise. The last and most difficult circularities are the vicious ones, namely the feed-backs  $\mathbf{b}_9 \rightarrow \mathbf{a}_4 \rightarrow \mathbf{a}_1$ , and  $\mathbf{b}_9 \rightarrow \mathbf{a}_1$ .

The solution to the latter circularities is usually taken to be a strict matter of computational complexity, that many scientists think of as a technicality (Leeuwen, 1990). I consider, however, that in so far as the computational complexity is the result of translating the cognitive complexity into a programming language, we have to keep both forms of complexity in close correlation in order to solve circularities. In this scope, we can, for instance, soften some requirements, in particular those represented by nodes sending two or more feed-backs.

Imposing strong assumptions on working out a node, or assuming stipulations on finishing out a node, might be an alternative way to the above. Either these alternatives are useful in reducing the links sent to, and by  $a_3$ . For example, if, after a number of returns to  $a_3$ , we cannot obtain any further specification to the 'heterogeneity' of **P** at hand, then the solution does not resides on repeating the sequence, but on determining the degree of heterogeneity heuristically. The heuristic determination works similarly to reasoning with incomplete knowledge. It can by applied to solve, for example, the block made out of  $b_2 \rightarrow b_5$ , and the block  $b_6 \rightarrow b_8$ , and then inserting  $a_3$  between them. The feed-backs from  $b_9$  allow for being annulled by a simulation, which is meant to uncover formal contradictions among methodological requirements. This is precisely why we need a **meta-theoretical** frame in  $b_9$ . Its role is to settle the frame in which contradiction can be detected qualitatively (Denzin and Lincoln, 1994).

\* :

In the **conclusive remarks** hereafter, only three matters will be taken up – some limitations of this study, and two theses, which I consider to be definitory for the deep transformation that science is passing through in this decade.

As far as the limitations are concerned, the most serious refer to that the study takes every possible opportunity to avoid simplification, which is why it might not be easily accessible. Another limitation consists of the way in which cognitive and computational complexity have been related to each other, and the latter translated from the first. This way represents, in essence, an attempt at preventing the unpredictable, namely the third-order circularity, to occur. Suppose that conducting research on enlarging tractability enables us to anticipate the first-, and second-order circularities, and this anticipation in itself gives substance to what is generally called first-, and secondorder cybernetics. If it would appear that a third-order circularity emerges, then this cannot be anticipated in the way the other two were. Solving the first two forms of circularity completely might eventually, prevent the occurrence of the third. To the extent to which this is an almost 'natural' consequence of the first-, and second-orders ones, the intention to prevent the third-order to get manifest is epistemologically wrong. Prevention, in this case, is a bad thing to do in so far as science is precisely supposed to try everything and make predictable the yet unpredicted.

The study does, however, contribute to the reduction of the unpredictable by enlarging the sequence responsible with anticipating the possibility that the obstacles represented by circularities may become deadlocks. In addition, each point where the danger of premature and not-through-solving simplification threatens has been signalized, and the consequences thereby have been underlined.

Hopefully, the study succeeds in advocating in favour of a few crucial ideas, among which two are particularly important. The first refers to the fact that many problem-solving cases are superficially declared as interdisciplinary while their very nature remains ambiguous and outside a rigorous/explicit operationalisation. The point the study made on this matter is that non-simple and heterogeneous P's generate most of the  $\Phi$ 's referring to the creation of full compatibility between that P as it is, and the language-method in which we seek for a solution. When for solving a P, there are disciplinary language-methods available, the requirement of creating compatibility remains, and, even if fulfilled - it does not exclude *per se* the need for a newly created (meta-)language-method.

On the other hand, because most of the non-simple/heterogeneous P's are iatrogene, it is advocated that even 'typically disciplinary' P's do contain undisciplinary components. Whenever such undisciplinary components are met, and in particular when they occur abundantly, crossing or defying disciplinary epistemological frontiers becomes a necessity. Scientists have been trying about two decades to find 'a method of interdisciplinarity' (Dogan, 1994; Gibbons, 1994), whereas what we have to find (actually, to build) is the programme that will endorse 'a method' which, in turn, will be neither 'of interdisciplinarity' nor 'of undisciplinarity' (if any), but of the highly complex-heterogeneous P to be solved.

The second thesis deserving to be emphasized refers to the finding that two, somewhat similar, tendencies – one towards interdisciplinarisation and the other towards the unification of cybernetics – have followed parallel developments. In the study it is argued that, in fact, these tendencies do converge to such an extent that it is rather strange how it happened that it remained unobserved for so long. Indeed, the fundamental  $\Phi$ 's that both cybernetics and interdisciplinarisation struggle to solve are very much the same. It is the observer that continues to behave disciplinarily even when second-order observations are conducted. On this ground then, the study had tried to bring the two tendencies-fields to bear to their common goal of solving.

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